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A NOTE ON THE SQUEEZE FILM LUBRICATION WITH NON-NEWTONIAN FLUID

N. M. BUJURKE, S. G. BHAVI* AND P. S. HIREMATH

Department of Mathematics, Karnatak University, Dharwad 580003

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A systematic analytical study of squeeze film lubrication between two approaching parallel surfaces is presented by considering second order fluid as lubricant. The closed form expressions for the average pressure distribution, load carrying capacity and time of approach have been obtained. The pressure and load capacity are found to increase significantly as compared to the Newtonian case. The time-height relation indicates that the time of approach for second order fluid is delayed considerably in comparison with the Newtonian fluid of the same viscosity.

NOMENCLATURE

A_1, A_2	=	The first two Rivlin-Ericksen tensors
f_1, f_2	=	functions of h [defined eqn. (23)]
$h(t)$	=	film thickness at time t
$h(t_0)$	=	film thickness at time $t = t_0$ a reference time
$H = \frac{h(t)}{h(t_0)}$	=	Ratio of film thickness at any two times t and t_0
L	=	Characteristic length
m, n	=	Functions of x given by eqns. (11 & 12)
N_n	=	Dimensionless parameter [defined in eqn. (27)]
p	=	Pressure in the film region
P	=	Average pressure in the film region
$\bar{P}$	=	Dimensionless average pressure [defined in (eqn. 24)]
S	=	Stress tensor
T	=	Time of approach
$\bar{T}$	=	Dimensionless time of approach [defined in eqn. (26)]

* Department of Mathematics, S. D. M. Engineering College, Dharwad 580002.

u, v	=	Velocity components in x & y directions respectively
V_0	=	Velocity of approach
V	=	Velocity vector
W	=	Load carrying capacity
$\bar{W}$	=	Dimensionless load carrying capacity (defined in eqn. 25)
x, y	=	Cartesian co-ordinates
$\bar{x}, \bar{y}$	=	Dimennsionless cartesian co-ordinates
$\epsilon = \frac{h}{L}$	=	Dimensionless film thickness
$\mu = \phi_0$	=	Newtonian viscosity
ρ	=	Density
ϕ_i	=	Material constants of the fluid ($i = 0, 1, 2$).

1. Introduction

The mechanism of lubrication of machine parts, that come into direct contact is aimed at the elimination of destructive heating, minimisation of wear and increasing mechanical efficiency. The lubricant prevents direct contact of surfaces with one another and forms uninterrupted fluid film between two mutually opposing parts of machine to which it is applied. When two surfaces containing lubricant in between them approach each other, then the fluid is squeezed resulting in the build up of pressure which helps in avoiding the possible contact of surfaces. This is termed as squeeze film lubrication. The relevant literature on squeeze film lubrication can be found in Moore[1] and Archibald[2]. The lubricative action depends mainly on the material properties of the lubricating fluid. An effective lubricant should possess a preferable degree of viscosity and should be chemically stable and inert towards metals. Most of the commonly used lubricants are thick polymer solutions exhibiting rheological characreristics such as normal stress differences in shear flow. The rheological behaviour of thick polymer solutions can be adequatety described by the constitutive equation for the second order fluids due to Coleman and Noll[3]. The objective of the present article is to study the squeeze film lubrication with second order fluid as lubricant between two parallel plates wherein the upper plate approaches the lower plate with a finite velocity. The influence of lubricant rheology on lubrication characteristics has been examined.

2. Mathematical Formulation and Solution

An incompressible homogeneous second order fluid in motion is governed by Coleman and Noll's constitutive equation

$$S = -pI + \phi_0 A_1 + \phi_1 A_2 + \phi_2 A_1^2 \qquad ...(1)$$

where S is Cauchy stress, $-pI$ the spherical stress due to incompressibility, A_1 and A_2 are the first two Rivlin-Ericksen tensors defined by

$$A_1 = \text{grad } V + (\text{grad } V)^T \qquad \ldots(2a)$$

$$A_2 = \overset{\circ}{A}_1 + A_1 \text{ grad } V + (\text{grad } V)^T A_1 \qquad \ldots(2b)$$

where V is the fluid velocity, ϕ_0, ϕ_1 and ϕ_2 are the coefficients of viscosity, viscoelasticity, and cross viscosity, respectively. The dot in eqn. 2 (b) denotes the material time derivative. We consider the two-dimensional squeeze film of such a second order fluid formed between two parallel plates where the upper plate is approaching the lower plate with a finite velocity V_0. The origin O is chosen at the centre of the lower plate, the x-axis is chosen along the lower plate and y-axis perpendicular to it. Let (u, v) be the velocity components in x, y directions respectively. Let $h(t)$ be the gap width between the two plates at time t and $2L$ be the length of the plates. Hence $V_0 = \frac{dh}{dt}$. The flow in the fluid film is governed by the following equations of continuity and motion[4,5]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \ldots(3)$$

$$\frac{1}{\rho} \frac{\partial p}{\partial x} = \nu \frac{\partial^2 u}{\partial y^2} + \beta \left[3 \frac{\partial^2 u}{\partial y^2} \frac{\partial u}{\partial x} + 4 \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} + u \frac{\partial^3 u}{\partial x\, d y^2} + \frac{\partial^2 v}{\partial y^2} \frac{\partial u}{\partial y} + 2 \frac{\partial v}{\partial y} \frac{\partial^2 u}{\partial y^2} + v \frac{\partial^3 u}{\partial y^3} \right] + 2\gamma \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} \qquad \ldots(4)$$

$$\frac{1}{\rho} \frac{\partial p}{\partial y} = 2 (2\beta + \gamma) \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} \qquad \ldots(5)$$

where p is the pressure and

$$\nu = \phi_0/\rho, \ \beta = \phi_1/\rho, \ \gamma = \phi_2/\rho. \qquad \ldots(6)$$

Equations (3), (4) and (5) are subjected to the following boundary conditions.

$$u = 0, v = 0 \text{ at } y = 0 \qquad \ldots(7a)$$

$$u = 0, v = -V_0 \text{ at } y = h \qquad \ldots(7b)$$

$$p = 0 \quad \text{at } x = \pm L. \qquad \ldots(8)$$

Following Tanner[6], we seek the solution of equations (3) to (5) in the form[7]

$$u(x, y) = m(x) y^2 + n(x) y \qquad \ldots(9)$$

where the functions $m(x)$ and $n(x)$ are to be determined. Integrating the continuity equation (3) after the substitution of the velocity profile given by eqn. (9) and using the boundary conditions (7), we obtain

$$v(x, y) = -\frac{1}{3}\frac{dm}{dx}y^3 - \frac{1}{2}\frac{dn}{dx}y^2 \quad \text{...(10)}$$

$$m(x) = (-6V_0x + C)h^{-3} \quad \text{...(11)}$$

$$n(x) = -(6V_0x + C)h^{-2} \quad \text{...(12)}$$

where C is the arbitrary constant to be determined. Using the equations (9) and (10), eqns. (4) and (5) reduce to

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = 2\nu m + 4(2\beta + \gamma)\frac{d}{dx}(m^2)y^2 + 4(2\beta + \gamma)y\frac{d}{dx}(mn) + \frac{(3\beta + 2\gamma)}{2}\frac{d}{dx}(n^2) \quad \text{...(13)}$$

$$\frac{1}{\rho}\frac{\partial p}{dy} = 4(2\beta + \gamma)m(2my + n). \quad \text{...(14)}$$

Integration of equations (13) and (14) yields

$$\frac{1}{\rho}p(x, y) = 2\nu\int m dx + 4(2\beta + \gamma)m^2y^2 + 4(2\beta + \gamma)mny + \frac{(3\beta + 2\gamma)}{2}n^2 + D \quad \text{...(15)}$$

where D is an arbitrary constant to be determined. The average P of the pressure distribution $p(x, y)$ across the film thickness h is given by

$$P = \frac{1}{h}\int_0^h p(x, y)\, dy \quad \text{...(16)}$$

which, on using eqn. (15), yields

$$\frac{1}{\rho}P = 2\nu\int m dx + \tfrac{4}{3}(2\beta + \gamma)mh(2mh + 3n) + \tfrac{1}{2}(3\beta + 2\gamma)n^2 + D. \quad \text{...(17)}$$

Using equation (8) the eqn. (16) reduces to

$$P = 0 \text{ at } x = \pm L. \quad \text{...(18)}$$

After using the equations (11) and (12) in eqn. (17), the constants C and D satisfying the condition (18) are obtained as

$$C = 0$$

$$D = \frac{6}{h^3}\nu V_0 L^2 - \frac{6}{h^4}(\beta + 2\gamma)V_0^2 L^2 \quad \text{...(19)}$$

and the average presssure distribution P is obtained as

$$\frac{1}{\rho} P = \frac{6\nu V_0}{h^3} (L^2 - x^2) + \frac{6}{h^4} (\beta + 2\gamma) V_0^2 (x^2 - L^2). \quad ...(20)$$

The load carrying capacity W is defined as

$$W = \int_{-L}^{L} \int_{0}^{h} p(x, y)\, dx\, dy. \quad ...(21)$$

Using equations (16) and (20) in (21), we obtain

$$\frac{W}{\rho} = \frac{8V_0 L^3}{h^2} \left[\nu - \frac{V_0}{h} (\beta + 2\gamma) \right]. \quad ..(22)$$

Solving the equation (22) for V_0, the time-height relation for a constant applied load W is obtained in the form

$$T = -2 \int_{h(t_0)}^{h(t)} \frac{f_1(h)\, dh}{f_2(h) + \sqrt{f_2^2(h) - 4W f_1(h)}} \quad ...(23)$$

where

$$f_1(h) = \frac{8 L^3}{h^3} (\phi_1 + 2\phi_2)$$

$$f_2(h) = \frac{8\phi_0 L^3}{h^2}$$

$$T = t - t_0$$

and t_0 is the reference time. The non-dimensional average pressure distribution $\bar{P}$, load carrying capacity $\bar{W}$ and time of approach $\bar{T}$ can be obtained as

$$\bar{P} = \frac{P h^2}{\phi_0 V_0 L} = \frac{6}{\epsilon} (1 - \bar{x}^2)(1 - N_n) \quad ...(24)$$

$$\bar{W} = \frac{Wh}{\phi_0 V_0 L^2} = \frac{8}{\epsilon} (1 - N_n) \quad ...(25)$$

$$\bar{T} = \frac{\phi_0 T}{(\phi_1 + 2\phi_2)} = -\log \left[\frac{1 - (1 - \frac{1}{2} B_n H)^{1/2})}{1 + (1 - \frac{1}{2} B_n H^{1/2})} \right] + \frac{2}{1 + (1 - \frac{1}{2} B_n H)^{1/2}} + \log \left[\frac{1 - (1 - \frac{1}{2} B_n)^{1/2}}{1 + (1 - \frac{1}{2} B_n)^{1/2}} \frac{2}{1 + (1 - \frac{1}{2} B_n)^{1/2}} \right. \quad ...(26)$$

where

$$\bar{x} = x/L$$

$$\epsilon = h/L$$

$$N_n = \frac{(\phi_1 + 2\phi_2) V_0}{\phi_0 h} \qquad \ldots(27)$$

$$H = h(t)/h(t_0)$$

$$B_n = \frac{(\phi_1 + 2\phi_2) W h(t^0)}{\phi_0^2 L^3}.$$

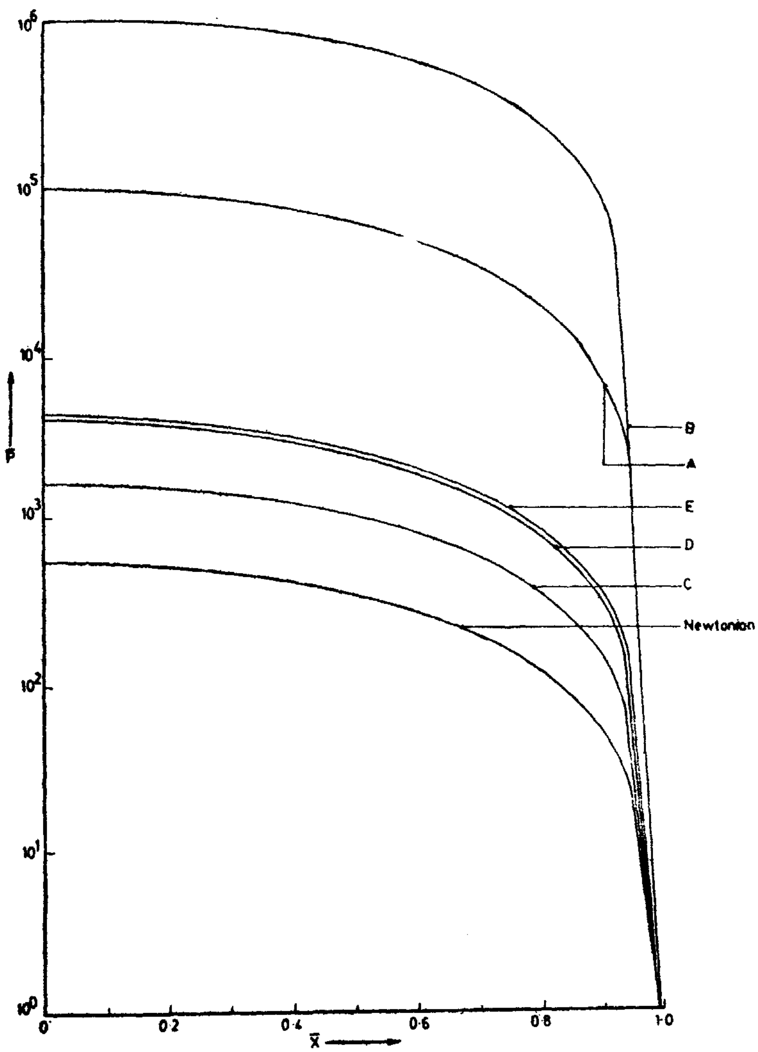

FIG. 1. The dimensionless axial pressure distribution for various fluid samples A — E ($L = 1.0$; $h = 0.01$, $V_0 = -0.5$).

3. Discussion and Conclusion

For the purpose of numerical calculation, we have taken the values of the material parameters from references Lai *et al.*[8]. and Markovitz[9], which are given in Table I.

TABLE I

Fluid	Description	ϕ_0 (*p*)	ϕ_1 (*ps*)	ϕ_2 (*ps*)
A	Normal old human synovial fluid[8]	21.6	—24.1	48.2
B	Normal young human synovial fluid[8]	82	—975	1950
C	Osteoarthritic fluid[8]	2.5	—0.025	0.05
D	Polyisobutylene in cetane[9] 5.4% at 30°C	18.5	−0.2	1.0
E	Polyisobutylene in cetane[9] 5.39% at 30°C	18.5	—0.32	1.12

The Fig. 1 shows the axial pressure distribution. The pressure build up for second order fluids is found to be considerably higher than that for the Newtonian fluid of the

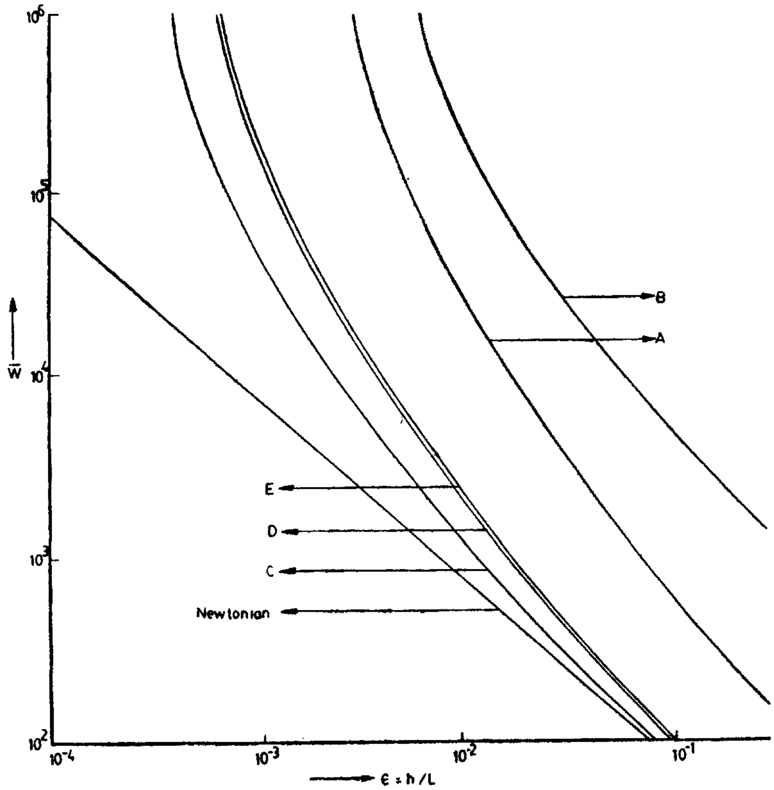

FIG. 2. The dimensionless load carrying capacity $\overline{W}$ *vs* ϵ for various fluid samples *A—E* ($L = 1.0$: $V_0 = -0.5$).

the same viscosity which can be distributed to the normal stress effects. The graph of load carrying capacity $\overline{W}$ versus ϵ is given in the Fig. 2. The load carrying capacity for second order fluids is significantly higher than that for the Newtonian case. Also, from the Fig. 3, we observe that the time of approach for second order fluids is found to be delayed in comparison with that for the Newtonian fluid. From Figs 1—3, it is observed that fluid samples with larger numerical values of ϕ_1, ϕ_2 yield higher load capacity and delayed time of approach compared with smaller values. This implies that the possible contact of lubricating surfaces is delayed for a longer time in case of second order fluid lubricant. Thus, the second-order fluids behave as effective lubricants.

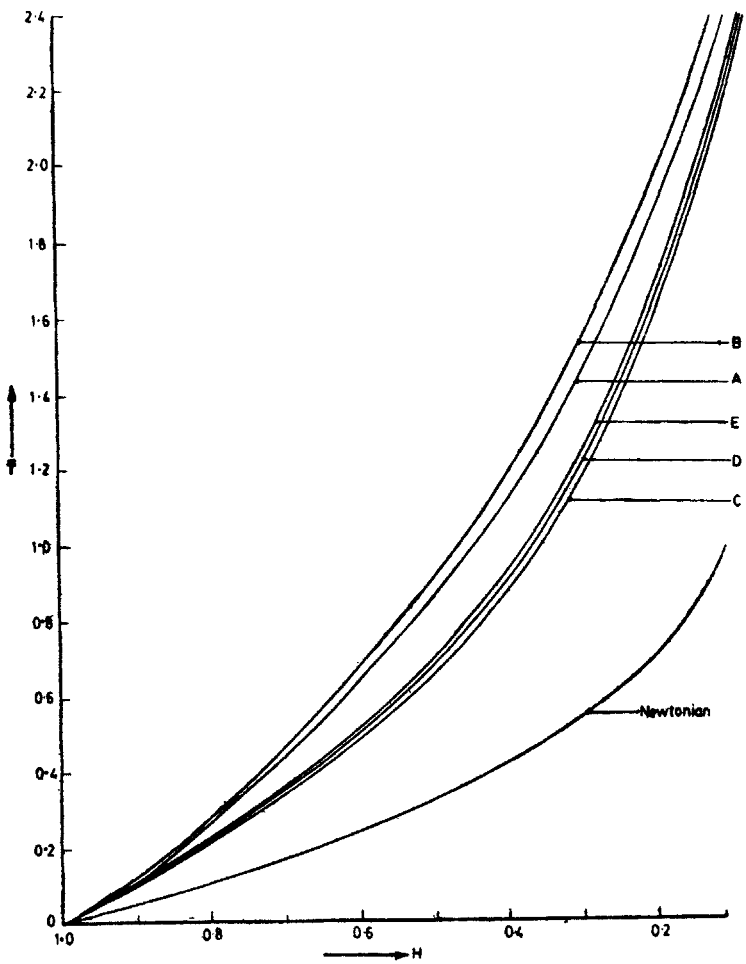

FIG. 3. The time.height relation for varions fluid samples $A - E$ ($L = 1.0$; $h_0 = 0.1$; $W = 100$).

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