

EFFECT OF THERMAL DIFFUSION ON THERMOHALINE INTERLEAVING IN A POROUS MEDIUM DUE TO HORIZONTAL GRADIENTS

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The effect of thermal diffusion on the onset of instability in an unbounded vertically stratified, stable, quiescent thermohaline fluid with compensating horizontal thermal and salinity gradients saturating a porous medium is investigated. It is found that for this system, instability sets in only through stationary mode when the wavelength is large compared to the porosity of the medium. The dependence on thermal diffusion of (a) the maximum growth rate (b) the ratio of fluxes (c) the onset and region of instability, are shown using the Soret parameter, S . For (i) $S = \tau^{-1} - 1$, the system remains stable for small perturbations (ii) $S < -1$, convection sets in even in the absence of the horizontal gradient when both temperature and salinity gradients are stable where τ is the ratio of mass diffusivity to thermal diffusivity. It is also shown that salt gets transported faster than heat when $S < \tau^{-1} - 1$ while transport of heat is more for $S > \tau^{-1} - 1$ for some growth rate of instability.

1. INTRODUCTION

The study of thermohaline instability with thermal diffusion in a fluid saturated porous medium is of importance in geophysics, ground water hydrology, soil sciences, oil extraction, extraction of ores etc., because it is known that the earth's crust is a porous medium saturated by a mixture of different types of fluids like oil, water, gases and molten form of ores or ores dissolved in fluids. Thermal gradient present between the interior and exterior of the earth's crust may help convection to set in. Also the two transport processes (heat and mass transfer) interfere with each other and produce cross phenomena known as thermal diffusion (Soret effect) and diffusion thermo (Dufour effect)^{1,2}. Thermal diffusion is the flux of mass caused by a temperature gradient and diffusion thermo is the flux of heat caused by a concentration gradient. But,

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in general, in liquid mixtures diffusion thermo is negligible for a heat-solute pair. Various authors have analysed Soret driven instability for Newtonian fluid layers³⁻⁵ as well as for fluid saturated porous media⁶⁻¹⁴.

All these authors have ignored the horizontal gradients of temperature and salinity that may be present. The earth's crust made up of large sedimentary materials of different permeabilities and heat capacities¹⁵ causes nonuniform heating of the lower layers resulting in a horizontal temperature gradient. Also, due to the nonuniform distribution of the constituent solutes in the multicomponent system saturating the earth's crust, one encounters horizontal solutal gradients in addition to the vertical ones.

The aim of the present paper is to study the effects of both thermal diffusion and horizontal gradients on thermohaline instability in a fluid-saturated porous medium. Holyer¹⁶ has analysed double-diffusive (thermohaline) interleaving due to horizontal gradients in a Newtonian fluid layer. The present work is based on Holyer's¹⁶ work.

Numerical computations are made by choosing the values of the Soret parameter S at random since it is reported by Hurlle and Jakeman³, Schechter *et al.*⁴, and Legros *et al.*¹⁸, that the sign and magnitude of the Soret coefficient depend on the velocity of flow and/or solute concentration of the system.

2. MATHEMATICAL FORMULATION

Consider an unbounded region of a porous medium saturated with a Boussinesq fluid which is quiescent and which has both temperature and salinity variations. It is assumed that the horizontal temperature and salinity gradients compensate each other in the basic state and hence there is no change in the density in the horizontal direction. The mass flow generated by the temperature gradient is taken into account in the formulation of the problem which is analysed for two-dimensional perturbations. The coordinate axes are chosen with z -axis vertically upwards and x -axis horizontal in the direction of increasing salinity. Only a stable basic state (i. e. where the total density gradient is negative) is considered to study the problem. The governing equations are

$$\frac{\partial u}{\rho x} + \frac{\partial \omega}{\partial z} = 0 \quad \dots (1)$$

$$\frac{1}{\phi} \frac{\partial \vec{q}}{\partial t} + A |\vec{q}| \vec{q} = \frac{-\nabla p}{\rho_0} + \frac{\rho \vec{g}}{\rho_0} - \frac{\nu \vec{q}}{k} \quad \dots (2)$$

where A is a constant dependent on the porosity and the geometrical parameters of the porous medium. This is the Navier-Stokes equation modified for Boussinesq approximation and Darcy law^{9,17}:

$$\frac{\partial T}{\partial t} + M \vec{q} \cdot \nabla T = K \nabla^2 T \quad \dots (3)$$

$$\frac{\partial C}{\partial t} + \vec{q} \cdot \nabla C = D \nabla^2 C + D_1 \nabla^2 T \quad \dots(4)$$

$$\rho = \rho_0 [1 - \alpha_T (T - T_0) + \alpha_c (C - C_0)] \quad \dots(5)$$

where u, w are the horizontal and the vertical components of the velocity \vec{q} ; t, p, ν, ρ are the time, pressure, kinematic viscosity and density of the double diffusive fluid; ϕ and k are the porosity and permeability of the medium; \vec{g} is the acceleration due to gravity; T and C are the temperature and concentration; ρ_0 is the density at temperature T_0 and concentration C_0 ; K and D are the thermal and solutal diffusivities; D_1 is the Soret coefficient; α_T and α_c are the coefficients of thermal and mass expansions which are both positive; $M = (\rho_0 c_p)_f / (\rho_0 c_p)_m$, c_p is the heat capacity, suffixes f and m stand for fluid and fluid-solid mixture. But M has been dropped in the subsequent analysis because liquid-saturated porous media is considered for this work.

Small perturbations are imposed on the basic state given by

$$\begin{aligned} \vec{q}_b &= 0; T_b(x, z) = \bar{T}_x x + \bar{T}_z z + T_0; C_b(x, z) = \bar{C}_x x + \bar{C}_z z + C_0; \\ \frac{\partial \rho_b}{\partial x} &= 0 \text{ (by assumption) i. e., } \alpha_T \bar{T}_x - \alpha_c \bar{C}_x = 0 \end{aligned}$$

where $\vec{q}_b, T_b(x, z), C_b(x, z)$ are the velocity, temperature and concentration in the basic state; \bar{T}_x, \bar{T}_z are the horizontal and vertical temperature gradients; \bar{C}_x, \bar{C}_z are the horizontal and vertical salinity gradients. The perturbed state is given by

$$\begin{aligned} T_b(x, z) + T(x, z, t); C_b(x, z) + C(x, z, t); \\ p_b(z) + p(x, z, t); \vec{q}(x, z, t) \end{aligned}$$

where p_b is independent of x from the basic state momentum equation.

$$\text{Let the stream function } \psi \text{ be defined by } u = -\frac{\partial \psi}{\partial z} \text{ and } w = \frac{\partial \psi}{\partial x}.$$

The linearised equations in T, C and ψ are given by

$$\left(\frac{\partial}{\partial t} - K \nabla^2 \right) T = \frac{\partial \psi}{\partial z} \bar{T}_x - \frac{\partial \psi}{\partial x} \bar{T}_z \quad \dots(6)$$

$$\left(\frac{\partial}{\partial t} - D \nabla^2 \right) C = D_1 \nabla^2 T + \frac{\partial \psi}{\partial z} \bar{C}_x - \frac{\partial \psi}{\partial x} \bar{C}_z \quad \dots(7)$$

$$\left(\frac{1}{\phi} \frac{\partial}{\partial t} + \frac{\nu}{k} \right) \nabla^2 \psi = g \left(\alpha_T \frac{\partial T}{\partial x} - \alpha_c \frac{\partial C}{\partial x} \right). \quad \dots(8)$$

On eliminating T and C (6) – (8) lead to

$$\left(\frac{\partial}{\partial t} - K \nabla^2 \right) \left(\frac{\partial}{\partial t} - D \nabla^2 \right) \left(\frac{1}{\phi} \frac{\partial}{\partial t} + \frac{\nu}{k} \right) \nabla^2 \psi$$

(equation continued on p. 719)

$$\begin{aligned}
 &= -g \alpha_T \bar{T}_z \left(\frac{\partial}{\partial t} - D \nabla^2 \right) \frac{\partial^2 \psi}{\partial x^2} + g \alpha_c \bar{C}_z \left(\frac{\partial}{\partial t} - K \nabla^2 \right) \frac{\partial^2}{\partial x^2} \\
 &\quad + g \alpha_c \bar{C}_x (K - D) \nabla^2 \psi_{zx} \\
 &\quad + g \alpha_c D_1 \left(\bar{T}_z \frac{\partial^2}{\partial x^2} - \bar{T}_x \frac{\partial^2}{\partial z \partial x} \right) \nabla^2 \psi. \quad \dots(9)
 \end{aligned}$$

The solution to (9) is assumed in the form

$$\psi = \psi_0 \exp (i a_x x + i a_z z + \sigma t).$$

On substitution for ψ in (9), the equation in σ which decides the stability of the system is given by

$$\begin{aligned}
 &(\sigma + K a^2) (\sigma + D a^2) \left(\sigma \frac{a^2}{\phi} + \frac{\nu a^2}{k} \right) \\
 &\quad + \sigma g a_x^2 (\alpha_T \bar{T}_z - \alpha_c \bar{C}_z) + g a^2 \alpha_c \bar{C}_x [K - D (1 + S)] a_x a_z \\
 &\quad - g a^2 [K \alpha_c \bar{C}_z - D (1 + S) \alpha_T \bar{T}_z] a_x^2 = 0
 \end{aligned}$$

where a_x and a_z are the horizontal and vertical wave numbers and $a^2 = a_x^2 + a_z^2$; $S = \alpha_c D_1 / \alpha_T D$ is the Soret parameter.

For $a^2/\phi \ll 1$ (i. e., when the wave number is small compared to ϕ or, the wavelength is large when compared to ϕ . This does not imply that ϕ is large. It is small enough to justify Darcy model with ' $a' \ll \phi$), the above equation becomes

$$\begin{aligned}
 &(\sigma + K a^2) (\sigma + D a^2) \frac{\nu a^2}{k} + \sigma g a_x^2 (\alpha_T \bar{T}_z - \alpha_c \bar{C}_z) \\
 &\quad + g a^2 \alpha_c \bar{C}_x [K - D (1 + S)] a_x a_z \\
 &\quad - g a^2 [K \alpha_c \bar{C}_z - D (1 + S) \alpha_T \bar{T}_z] a_x^2 = 0. \quad \dots(10)
 \end{aligned}$$

For neutral stability through stationary mode (i. e., $\sigma = 0$) (10) gives

$$\begin{aligned}
 \frac{K D \nu a^4}{k} &= -g \alpha_c \bar{C}_x [K - D (1 + S)] a_x a_z \\
 &\quad + g [K \alpha_c \bar{C}_z - D (1 + S) \alpha_T \bar{T}_z] a_x^2. \quad \dots(11)
 \end{aligned}$$

From (11) it is seen that ' a ' will be real when

$$\alpha_c \bar{C}_x [1 - \tau (1 + S)] \frac{a_z}{a_x} < \alpha_c \bar{C}_z - \tau (1 + S) \alpha_T \bar{T}_z$$

where $\tau = \frac{D}{K}$ (12)

The inequality (12) is the same for a Newtonian fluid layer (i. e., without porous medium) and hence the values of a_z/a_x for which convection sets in through stationary mode in a Newtonian fluid layer are the same even in the presence of a porous medium.

For neutral stability through oscillatory mode ($\sigma = i\omega$, where ω real is the frequency) the equation (10) gives

$$\begin{aligned} & - \frac{\alpha^2 a^2 v}{k} + \frac{K D v a^6}{k} + g a^2 \alpha_c \bar{C}_x [K - D(1 + S)] a_x a_z \\ & - g a^2 [K \alpha_c \bar{C}_z - D(1 + S) \alpha_T \bar{T}_z] a_x^2 + i\omega \left[\frac{a^4}{k} (K + D) v \right. \\ & \left. + g a_x^2 (\alpha_T \bar{T}_z - \alpha_c \bar{C}_z) \right] = 0. \end{aligned} \quad \dots(13)$$

The imaginary part of (13) equated to zero gives

$$\omega \left[\frac{a^4}{k} (K + D) v + g a_x^2 (\alpha_T \bar{T}_z - \alpha_c \bar{C}_z) \right] = 0.$$

implying $\omega = 0$ for a stable basic state ($\alpha_T \bar{T}_z - \alpha_c \bar{C}_z > 0$).

Hence, when the wavelength is large compared to ϕ convection does not set in through oscillatory mode. Under this assumption, only stationary convection is discussed in what follows. Since the Soret parameter S can have any value^{3,4,18} the discussion is based on different ranges of S .

The basic state in which both vertical thermal and salinity gradients are stable ($\alpha_T \bar{T}_z > 0$, $\alpha_c \bar{C}_z < 0$) is considered in the discussion given below. It is seen from (12) that the inequality cannot be satisfied when $\bar{C}_x = 0$ for $S \geq -1$. This shows that \bar{C}_x derives the instability and if, in addition, $S < \tau^{-1} - 1$, the inequality (12) gives $\frac{a_z}{a_x} < 0$ implying that hot salty solution overlies cold fresh solution which is usually the set up for stationary mode. But for $S < -1$, convection can set even in the absence of \bar{C}_x provided $\alpha_c \bar{C}_z - \tau(1 + S) \alpha_T \bar{T}_z > 0$. A difference in the set up with $\bar{C}_x \neq 0$ is noticed for $S > \tau^{-1} - 1$, when the stationary convection sets in with $\frac{a_z}{a_x} > 0$ for which cold fresh solution is above hot salty solution which is a diffusive regime. This tilting has been reported by McDougall¹⁹ for a Newtonian fluid layer. It is also seen that (12) is not satisfied for any \bar{C}_x when $S = \tau^{-1} - 1$, i. e., a system which is stable in the basic state remains stable for small perturbations, implying that \bar{C}_x is compensated by this value of the Soret parameter. A similar analysis can be carried out for other directions of the gradients subject to a stable basic state condition. The regions of stationary instability in the $\frac{a_z}{a_x} - R_c$ plane are shown in Fig. 1 (a) — (c) for different values of S and H for given R_c

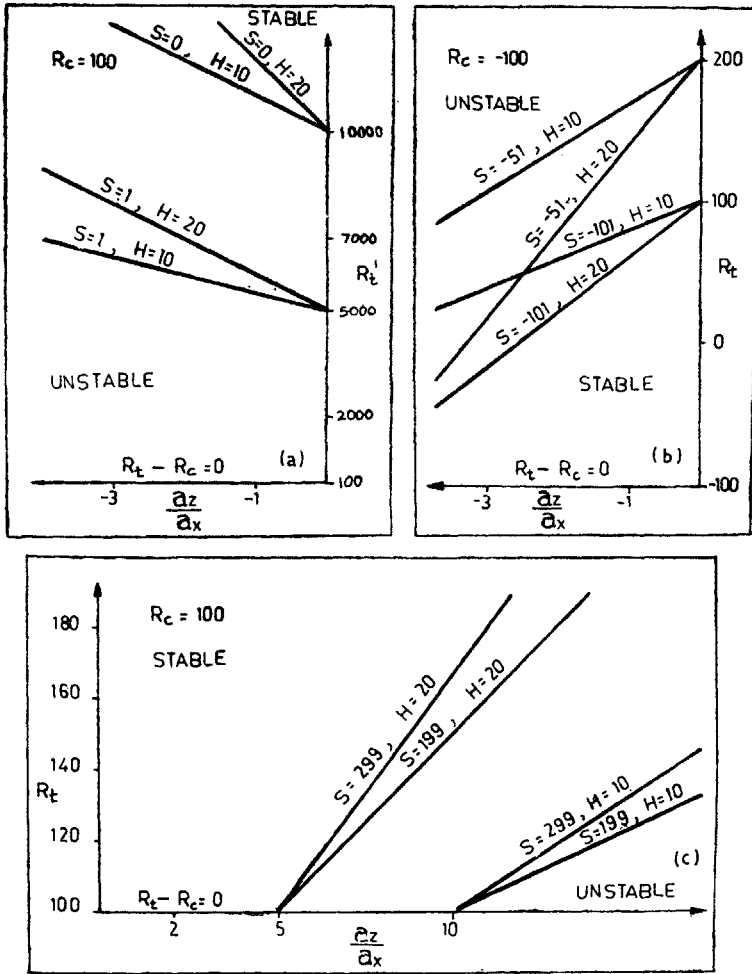


Fig. 1. $\frac{a_s}{a_x} - R_t$ plots for $\tau = 0.01$ and different values of H .

- (a) $-1 < S < \tau^{-1} - 1$. The region of instability lies between the line considered and $R_t - R_c = 0$.
- (b) $S < -1$. The region of instability is above the line considered.
- (c) $S > \tau^{-1} - 1$. The region of instability lies between the line considered and $R_t - R_c = 0$.

where the horizontal Rayleigh number $H = \frac{g \alpha_c \bar{C}_x k}{\nu K a^2}$;

the vertical thermal Rayleigh number $R_t = \frac{g \alpha_T \bar{T}_z k}{\nu K a^2}$;

the vertical salinity Rayleigh number $R_c = \frac{g \alpha_c \bar{C}_z k}{\nu K a^2}$.

It can be seen from these figures that an increase in H increases the region of instability. Figure 1 (a) shows that when S increases, the region of instability decreases. Figures 1(b)–(c) also reveal that the Soret parameter affects the regions of instability.

3. MAXIMUM GROWTH RATE OF INSTABILITY

The growth rate of unstable mode is given by σ which has to be real for the instability to grow via stationary mode. The maximum growth is obtained from

$$\frac{\partial \sigma}{\partial a_x} = 0 = \frac{\partial \sigma}{\partial a_z}. \quad \dots(14)$$

Proceeding on the same lines as Holyer¹⁶, the Rayleigh numbers for the maximum growth rate are given by

$$H = -2 \frac{a_z}{a_x} \frac{(\lambda + 1)(\lambda + \tau)}{[1 - \tau(1 + S)]} \quad \dots(15a)$$

$$R_t = \frac{(\lambda + 1)[a^2 \tau (\lambda + 1) - 2a_z^2 \lambda (\lambda + \tau)]}{\lambda a_x^2 [1 - \tau(1 + S)]} \quad \dots(15b)$$

$$R_c = \frac{1}{\lambda a_x^2 [1 - \tau(1 + S)]} \{a^2 [\lambda^2 (1 - \tau S) \dots(15c)$$

$$+ 2 \lambda \tau + \tau^2 (1 + S)] - 2a_z^2 \lambda (\lambda + 1)(\lambda + \tau)\}$$

where

$$\lambda = \frac{\sigma}{K a^2}.$$

It can be seen that S affects all quantities though $\frac{H}{R_t}$ is independent of S . To find the maximum dimensionless growth rate λ and the corresponding horizontal and vertical wave numbers a_x , a_z , equations (15) are to be simultaneously solved for given values of the gradients and τ . But, following Baines and Gill²⁰ and Holyer¹⁶, it is easier to compute H , R_t , R_c assigning values for $\frac{a_z}{a_x}$ and λ for given τ . $\frac{H}{R_t} - \frac{R_c}{R_t}$ plots are displayed in Fig. 2 (a), (b) and 3 for different values of S for given values of λ and $\frac{a_z}{a_x}$. These figures are drawn for positive values of R_t subject to this condition $R_t - R_c > 0$ (stable basic state) and for values of S in the range $-1 \leq S < \tau^{-1} - 1$ with $\tau = 0.01$ for a heat-salt pair. It is seen from Figures 2(a), (b) and 3 that when S increases $\frac{R_c}{R_t}$ increases. The effect of S becomes less for higher growth rates.

For the growth rate $\lambda = \tau^{1/2}$, it is found that $\frac{R_c}{R_t} = 1$ for all $\frac{a_z}{a_x}$ and S , i. e., $\frac{H}{R_t}$

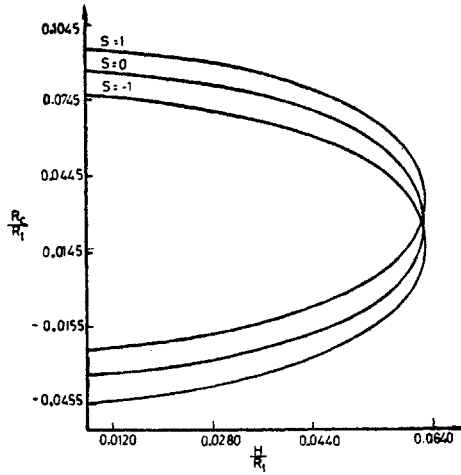


FIG. 2. (a) $\frac{H}{R_t} - \frac{R_c}{R_t}$ plots for $R_t > \theta$ subject to $R_t - R_c > 0$ for $\tau = 0.01$, $\lambda = 0.02$, $-1 \leq S < \tau^{-1} - 1$.

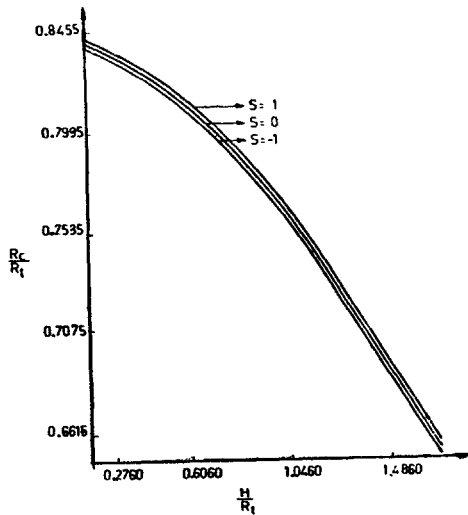


FIG. 2 (b) $\frac{H}{R_t} - \frac{R_c}{R_t}$ plots for $R_t > 0$ subject to $R_t - R_c > 0$ for $\tau = 0.01$, $\lambda = 0.09$, $-1 \leq S < \tau^{-1} - 1$.

$-\frac{R_c}{R_t}$ plot is the same straight line (not shown in the figure). Figure 3 shows that $\frac{R_c}{R_t}$ decreases when $|\frac{a_z}{a_x}|$ increases and the effect of S is very nearly the same for different $\frac{a_z}{a_x}$.

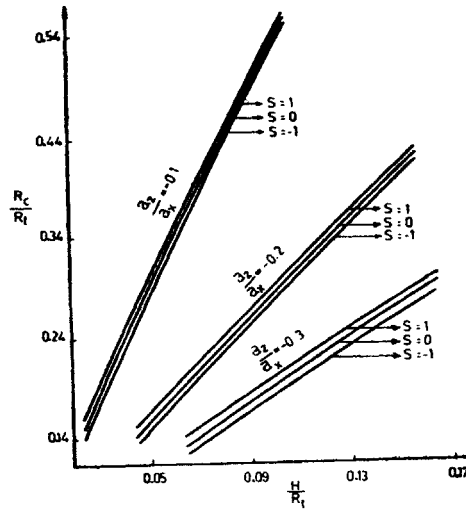


FIG. 3. $\frac{H}{R_t} - \frac{R_c}{R_t}$ plots for $R_t > 0$ subject to $R_t - R_c > 0$ for $\tau = 0.01$, $\frac{a_z}{a_x} = -0.1, -0.2, -0.3$ and $-\infty < S < \tau^{-1} - 1$.

For marginal stability for which $\lambda \rightarrow 0$, it is easily seen from (15) that

$$\frac{H}{R_t} \rightarrow 0, \frac{R_c}{R_t} \rightarrow \tau (1 + S) \text{ with } R_t \rightarrow \infty$$

hence $a \rightarrow 0$. When $\lambda \rightarrow \infty$, it is seen that $\frac{H}{R_t} \rightarrow \frac{a_x}{a_z}$ and $\frac{R_c}{R_t} \rightarrow 1$. These results hold good in the absence of porous medium also.

4. FLUXES

For a propagating mode,

$$(\psi, T, C) = \text{Re} [(\psi_0, T_0, C_0) \exp (i (a_x x + i a_z z + \sigma t))] \quad \dots (16)$$

where ψ_0 is assumed and real and σ is positive for a stationary mode. Rewriting equations (3) (for $M = 1$ and (4) in the form

$$\begin{aligned} \frac{\partial T}{\partial t} + \psi_x \bar{T}_z - \psi_z \bar{T}_x &= K \nabla^2 T \\ \frac{\partial C}{\partial t} + \psi_x \bar{C}_z - \psi_z \bar{C}_x &= D \nabla^2 C + D_1 \nabla^2 T \end{aligned}$$

and substituting (16), T_0 and C_0 are given by

$$T_0 = \frac{i \psi_0 (a_z \bar{T}_x - a_x \bar{T}_z)}{\sigma + K a^2} \quad \dots(17a)$$

$$C_0 = i \psi_0 \left[\frac{a_z \bar{C}_x - a_x \bar{C}_z}{\sigma + D a^2} \right]$$

(equation continued on p. 725)

$$- D_1 a^2 \frac{(a_z \bar{T}_x - a_x \bar{T}_z)}{\sigma + K a^2} \Big]. \quad \dots(17b)$$

The vertical heat flux F_T and vertical salt flux F_C are given by

$$F_T = \frac{1}{2} \frac{a_z \bar{T}_x - a_x \bar{T}_z}{\sigma + K a^2} a_x \psi_0^2$$

$$F_C = \frac{1}{2} \left[\frac{a_z \bar{C}_x - a_x \bar{C}_z}{\sigma + D a^2} - \frac{D_1 a^2 (a_z \bar{T}_x - a_x \bar{T}_z)}{(\sigma + D a^2)(\sigma + K a^2)} \right] a_x \psi_0^2$$

where $F_T = \langle \omega T \rangle$, $F_C = \langle \omega C \rangle$ and $\langle \rangle$ denotes an average over a wavelength.

The flux ratio is given by

$$\frac{\alpha_T F_T}{\alpha_C F_C} = \frac{(R_t - \frac{a_z}{a_x} H) (\lambda + \tau)}{(R_c - \frac{a_z}{a_x} H) (\lambda + 1) - S \tau (R_t - \frac{a_z}{a_x} H)}$$

Substituting for R_t , R_c and H from (15), this becomes

$$\frac{\alpha_T F_T}{\alpha_C F_C} = \frac{(\lambda + 1) \tau}{\lambda (1 - \tau S) + \tau} \quad \dots(18)$$

For $S < \tau^{-1} - 1$, it is clear that the flux ratio given by (18) is less than unity for all values of λ and hence salt is transported faster than heat. The flux ratio is

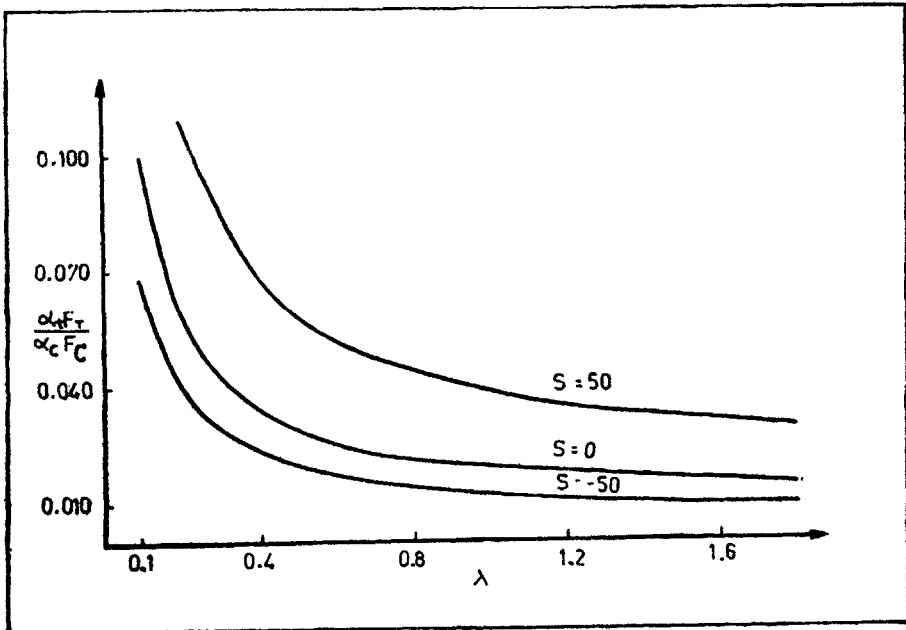


FIG. 4. (a) $\lambda - \frac{\alpha_T F_T}{\alpha_C F_C}$ plots for $\tau = 0.01$ for $S < \tau^{-1} - 1$.

greater than unity (i) for all values of λ when $\tau^{-1} - 1 < S \leq \tau^{-1}$ (ii) for $\lambda < \frac{\tau}{\tau S - 1}$ when $S > \tau^{-1}$ implying that the heat gets transported faster than salt. It is further noticed that for $S > \tau^{-1}$ with $\lambda > \frac{\tau}{\tau S - 1}$, transports of heat and salt are in opposite directions. At $\lambda = \frac{\tau}{\tau S - 1}$, the flux ratio tends to infinity, i. e., the heat flux is very large when compared to the salt flux. Figures (4) a — (c) are $\lambda - \frac{a_T F_T}{a_c F_C}$ plots in which the above results are shown. Figure 4 (a) shows that the flux ratio increases when S increases, but decreases when λ increases where $S < \tau^{-1} - 1$. For $S = \tau^{-1} - 1$ there is no convection. It is seen from Fig. 4(b) that the flux ratio is greater than unity for all λ and that it increases with both S and λ when $\tau^{-1} - 1$

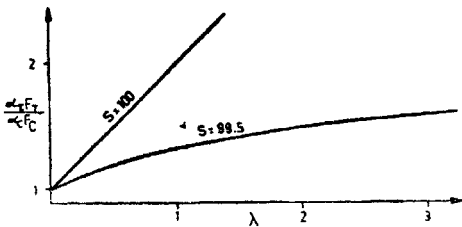
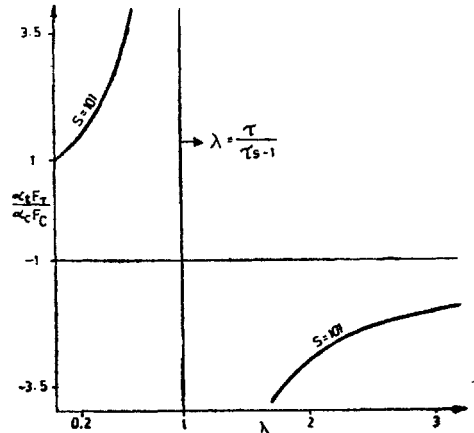


FIG. 4. (b) $\lambda - \frac{a_T F_T}{a_c F_C}$ plots for $\tau = 0.01$ for $\tau^{-1} - 1 < S \leq \tau^{-1}$.



(c) $\lambda - \frac{a_T F_T}{a_b F_C}$ plot for $\tau = 0.01$ for $S > \tau^{-1}$.

$< S \leq \tau^{-1}$. The discontinuity of the graph at $\lambda = \frac{\tau}{\tau S - 1}$ for $S > \tau^{-1}$ can be noticed from Fig. 4 (c).

5. CONCLUSION

This paper describes in detail the results which are due to the presence of the horizontal gradient and thermal diffusion in a fluid saturated porous medium. It is shown that when the vertical gradients of both temperature and salinity are stable (i) for $S \geq -1$, there is no convection in the absence of \bar{C}_x , but for $S < -1$, conditional convection sets in. (ii) for $-1 \leq S < \tau^{-1} - 1$, when \bar{C}_x derives convection, $\frac{a_z}{a_x} < 0$ i. e. hot salty solution is above cold fresh solution. (iii) for $S > \tau^{-1} - 1$, with

\bar{C}_x driving stationary convection $\frac{a_z}{a_x}$ is positive i. e. cold fresh solution overlies hot salty solution which is usually a diffusive regime. It is also shown that for $S = \tau^{-1} - 1$, no convection is possible for all \bar{C}_x . Also salt gets transported faster than heat when $S < \tau^{-1} - 1$ while transport of heat is more for $S > \tau^{-1}$ for some values of the growth rate.

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