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SELECTION PROCEDURES FOR HAZARD RATES BASED ON TWO-SAMPLE STATISTICS

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Let $\pi_1, \pi_2, \cdots, \pi_k$ be k independent populations and F_i be the absolutely continuous cdf (cumulative distribution function) with continuous pdf (probability density function) as f_i of the life time of individuals in the ith population, $i = 1, 2, \cdots k$. We assume that the hazard rate $r_i(x) = f_i(x)/\bar{F}_i(x)$, $x \geqslant 0$ exists for each $i = 1, 2, \cdots, k$. Here $\bar{F}_i(x) = 1 - F_i(x)$ is the survival function. In this paper we consider the problem of selecting a subset of k populations containing the one associated with uniformly smaller hazard rate. The proposed selection procedures are based on two sample statistics which are functions of ordered ranks of the two samples uuder consideration. The procedure in general has may desirable properties.

1. INTRODUCTION

Suppose we have k independent populations of life lengths of their individuals which are being ranked in terms of hazard (failure) rates. We propose in this paper subset selection procedures to select a subset containing the population associated with uniformly smaller hazard rate based on two sample statistics given by Cheng[1]. Earlier, in an abstract, Patel[2] suggested a procedure for the problem of selecting a subset containing the population with the largest failure rate average out of several IFRA populations based on the number of failures observed by some fixed common time T. To the best of our knowledge no such paper appeared in the literature thereafter.

In section 2 of this paper we have formulated the problem. Proposed selection procedures are given in section 3. Section 4 deals with the probability of a correct selection, expected subset size and infimum of a probability of correct selection. In section 5 some desirable properties of the proposed procedure are discussed.

2. FORMULATION OF THE PROBLEM

Let $\pi_1, \pi_2, \ldots, \pi_k$ be k independent populations and F_i be the absolutely continuous cdf (cumulative distribution function) with continuous pdf (probability density function) as f_i of the life time of individuals in the ith population, $i = 1, \ldots, k$. Let the hazard rate $r_i(x) = f_i(x)/\bar{F}_i(x)$, $x > 0$ exist for each i, $i = 1, 2, \ldots, k$. Here $\bar{F}_i(x) = 1 - F_i(x)$ is the survival function of ith population, $i = 1, 2, \ldots, k$. Let

$\Omega = \{\mathbf{r} : \mathbf{r} = (r_1, r_2, \ldots, r_k)^t, r_i(x) < \infty, x \geq 0$ for all $i = 1, 2, \ldots, k\}$ be the space of hazard rates of the life times of individuals of k populations. For any two population π_i and π_j, π_i is considered to be better than π_j if $r_i(x) \leqslant r_j(x) \;\forall\; x \geqslant 0$. We assume that there is a best population, that is, for each $\mathbf{r} \in \Omega$, there is an index i such that $r_i(x) \leqslant r_j(x)$, $x \geqslant 0$ and $\forall\; j, j \neq i$. If more than one populations are tied for the best then arbitrarily one of them is labelled as the best.

For any $\mathbf{r} = (r_1, r_2, \ldots, r_k)^t$, we shall denote by $r_{[1]}$ the unique component of $\mathbf{r}$ corresponding to the best population. The goal is to select a subset of k populations containing the best population, the one with uniformly smaller hazard rate $r_{[1]}$. Any such selection will be called as CS (correct selection). Then the problem is to find a rule R such that for a pre-assigned probability P^* $(\frac{1}{k} < P^* < 1)$, this satisfies the probability requirement :

$$P_{\mathbf{r}}[CS \mid R] \geqslant P^* \;\forall\; \mathbf{r} \in \Omega. \qquad \ldots(2.1)$$

Let A be the action space of the subset selection problem which is the set of all nonempty subsets of $\{1, 2, \ldots, k\}$, where taking action $a \in A$ means the selection of those populations whose indices are in a. For any $a \in A$, let

$$CS(\mathbf{r}, a) = \begin{cases} 1 \text{ if } r_{[1]} \in \{r_i \,;\, i \in a\} \\ 0 \text{ otherwise} \end{cases} \qquad \ldots(2.2)$$

and $|a|$ = number of elements in a.

Let $X_{i1}, X_{i2}, \ldots, X_{in_i}$ be a random sample of size n_i from the ith population, $i = 1, 2, \ldots, k$. Let $\mathbf{X}_i = (X_{i1}, X_{i2}, \ldots, X_{in_i})^t$ be the vector of observations from the ith population and let $\mathbf{X} = (X_{11}, X_{12}, \ldots, X_{1n_1}, X_{21}, \ldots, X_{2n_2}, \ldots, X_{k1}, \ldots, X_{kn_k})^t$ be the vector of all the observations. For any subset selection procedure R, let $Z_R(\mathbf{X}, a)$ be the probability assigned to a by R having observed $\mathbf{X}$.

3. Proposed Selection Procedure

We shall now develop procedures for the above problem on the basis of the estimators of the parameters

$$\Delta(F_i, F_j) = \frac{\int_0^\infty \{\bar{F}_i(x)/[\bar{F}_i(x) + \bar{F}_j(x)]^{1/2}\}\, d\bar{F}_j(x)}{\int_0^\infty \{\bar{F}_j(x)/[\bar{F}_i(x) + \bar{F}_j(x)]^{1/2}\}\, d\bar{F}_i(x)}, \quad i \neq j.$$

Such parameters have been proposed by Cheng[1].

For the ith and the jth population

$$r_i(x) \leqslant r_j(x) \ \forall\ x \geqslant 0$$

$$\Leftrightarrow f_i(x)/\bar{F}_i(x) \leqslant f_j(x)/\bar{F}_j(x) \ \forall\ x \geqslant 0,$$

$$\Leftrightarrow \bar{F}_j(x)\, f_i(x) \leqslant \bar{F}_i(x)\, f_j(x) \ \forall\ x \geqslant 0,$$

$$\Leftrightarrow \{\bar{F}_j(x)/[\bar{F}_i(x) + \bar{F}_j(x)]^{1/2}\}\, f_i(x) \leqslant \{\bar{F}_i(x)/[\bar{F}_i(x) + \bar{F}_j(x)]^{1/2}\}$$

$$f_j(x) \ \forall\ x \geqslant 0,$$

$$\Rightarrow \int_0^\infty \{\bar{F}_i(x)/[\bar{F}_i(x) + \bar{F}_j(x)]^{1/2}\}\, f_j(x)\, dx$$

$$\geqslant \int_0^\infty \{\bar{F}_j(x)/[\bar{F}_i(x) + \bar{F}_j(x)]^{1/2}\}\, f_i(x)\, dx$$

$$\Leftrightarrow \Delta(F_i, F_j) \geqslant 1. \qquad \ldots(3.1)$$

Equality in (3.1) holds if $F_i(x) = F_j(x)$ for all $x \geqslant 0$.

To obtain the estimator of $\Delta(F_i, F_j)$, we replace $\bar{F}_i$ and $\bar{F}_j$ by the respective emperical survival functions. Let $n_i + n_j = n$ and $S^i_{(1)} \leqslant S^i_{(2)} \leqslant \ldots \leqslant S^i_{(n_i)}$ and $R^j_{(1)} \leqslant \ldots \leqslant R^j_{(n_j)}$ be respectively the ordered ith and jth sample ranks in the combined sample. The estimator of $\Delta(F_i, F_j)$ is $T_{ij} = T_{jn}/T_{in}$, where

$$T_{jn} = \sum_{\alpha=1}^{n_j-1} \left[\left\{ n_i + \alpha - R^j_{(\alpha)} \right\} \Big/ \left\{ 2n_j^2\, n_i^2 + n_j^2\, n_i\, \alpha - n_j\, n_i^2\, \alpha - n_j^2\, n_i\, R^j_{(\alpha)} \right\}^{1/2} \right] + \left\{ n^{3/2} - n^{1/2}\, R^j_{(n_j)} \right\} \Big/ \left\{ n_i^2\, n_j^2 + n_j^2\, n_i\, n^2 - n_j^2\, n_i\, n\, R^j_{(n_j)} \right\}^{1/2}$$

and

$$T_{in} = \sum_{\beta-1}^{n_i-1} \left[\left\{ n_j + \beta - S^i_{(\beta)} \right\} \Big/ \left\{ 2n_i^2\, n_j^2 + n_j\, n_i^2\, \beta - n_j^2\, n_i\, \beta - n_i^2\, n_j\, S^i_{(\beta)} \right\}^{1/2} \right] + \left\{ n^{3/2} - n^{1/2}\, S^i_{(n_i)} \right\} \Big/ \left\{ n_i^2\, n_j^2 + n_j\, n_i^2\, n^2 - n_j\, n_i^2\, n\, S^i_{(n_i)} \right\}^{1/2}.$$

The statistics T_{ij}, proposed by Cheng[1] for testing $r_i(x) = r_j(x) \; \forall \; x \geqslant 0$ against $r_i(x) \leqslant r_j(x) \; \forall \; x \geqslant 0$, depends only on the ordered ranks, which are uniquely defined, since F_i and F_j are both continuous. For tied observations average ranks can be used. Our selection procedure is based on T_{ij} and is defined as follows :

R_a : Select π_i in the subset iff

$$T_{ij} \geqslant C_j^{(i)}(\mathbf{n}, P^*) \; \forall \; j, j \neq i. \qquad ...(3.2)$$

The constants $C_j^{(i)}(\mathbf{n}, P^*) \geqslant 0$ are chosen such that

$$P_0 \left[T_{ij} \geqslant C_j^{(i)}(\mathbf{n}, P^*) \; \forall \; j, j \neq i\right] \geqslant P^*. \qquad ...(3.3)$$

Here P_0 indicates that the probability is computed under $r_1 \equiv r_2 \equiv \ldots \equiv r_k$ and $\mathbf{n} = (n_1, n_2, \ldots, n_k)^t$.

The procedure (3.2) can be modified to select a subset of k populations better than the unknown control population. Let $r_0(x) = f_0(x)/\bar{F}_0(x)$ be the hazard rate of the control population π_0 and $r_i(x) = f_i(x)/\bar{F}_i(x)$ be the hazard rate of the population π_i, $i = 1, 2, \ldots, k$. Population π_i is considered to be better than π_0 if $r_i(x) \leqslant r_0(x)$ for all $x > 0$. Let n_j be the number of observations taken from population π_j, $j = 0, 1, 2, \ldots, k$ and let $\mathbf{n}^* = (n_0, n_1, \ldots, n_k)^t$. The proposed selection procedure based on the statistic T_{i0}, the estimator of parameter $\Delta(F_i, F_0)$, is

R_{a_1} : Select π_i in the subset if and only if

$$T_{i0} \geqslant C_0^{(i)}(\mathbf{n}^*, P^*). \qquad ...(3.4)$$

The nonnegative constants $C_0^{(i)}(\mathbf{n}^*, P^*)$ are chosen such that

$$P_0 \left[T_{i0} \geqslant C_0^{(i)}(\mathbf{n}^*, P^*), i = 1, 2, \ldots, k\right] \geqslant P^*.$$

Here P_0 indicates that the probability is computed under $r_0(x) = r_1(x) = \ldots = r_k(x)$ for all $x \geqslant 0$.

The procedure (3.4) can approximately be used with the help of existing tables for the case of large and moderate equal sample sizes from all the $(k + 1)$ populations as follows :

Let $n_0 = n_1 = \ldots = n_k = n$. By Theorem 2.1 of Cheng[1], it follows that as $N \to \infty$ such that $n/N \to p$ (here $N = n(k + 1)$), the limiting distribution of

$$Z_{i0} = 2\sqrt{n}\,(T_{i0} - 1)/3$$

is normal with mean zero and variance 1 if $r_i(x) = r_0(x)$ for all $x \geq 0$. It is a known fact that the joint distribution of standardized correlated variables often tends asymptotically to multivariate normal distribution (c.f. Gupta *et al.*[3]). Thus the limiting distribution of the random vector $\mathbf{Z} = (Z_{10}, \ldots, Z_{k0})^t$ under $r_0(x) = r_1(x) = \ldots = r_k(x)$ for all $x \geq 0$ will be asymptotically multivariate normal of equally correlated standard variables when $n_0 = n_1 = \ldots = n_k = n$. Now the constants $C_0^{(i)}(\mathbf{n}^*, P^*)$ are determined such that

$$\begin{aligned} P^* &= P_0 [Z_{i0} \geq C_0 \text{ for all } i = 1, 2, \ldots, k] \\ &= P_0 [\min_i Z_{i0} \geq C_0] \\ &= P_0 [\max_i Z_{i0} \leqslant -C_0]. \end{aligned}$$

Here

$$C_0 = 2\sqrt{n}\,[C_0^{(i)}(\mathbf{n}^*, P^*) - 1]/3.$$

Now we can use the Table I of Gupta *et al.*[3] to read the constant $-C_0$ and thereby get the values of constants $C_0^{(i)}(\mathbf{n}^*, P^*)$, $i = 1, 2, \ldots, k$.

4. Probability of Correct Selection and Expected Subset Size

Let us define, $\tilde{A} = \{a \in A \mid CS(\mathbf{r}, a) = 1\}$, the probability of a correct selection is then

$$\begin{aligned} P_r[CS \mid R_a] &= P\,[r_{[1]} \text{ is in the selected subset} \mid R_a] \\ &= P_r[\bigcup_{a \in \tilde{A}} (\mathbf{X} \text{ is observed and action } a \text{ is taken}) \mid R_a] \\ &= \int_0^\infty P_r[\bigcup_{a \in \tilde{A}} \{\text{action } a \text{ is taken} \mid \mathbf{X} = \mathbf{x}, R_a\}]\, dF(\mathbf{x}) \end{aligned}$$

There may be different subsets in A which contain the best population. All these subsets form $\tilde{A}$. However, on the basis of available observations one and only one subset from A can be chosen, that is, only one action at a time is possible. Thus the above integral may be written as

$$E_r[\sum_{a \in \tilde{A}} P_r(\text{action } a \text{ is taken} \mid \mathbf{X}, R_a)]$$

(equation continued on p. 778)

$$= E_{\mathbf{r}} [\sum_{a \in A} CS (\mathbf{r}, a) Z_{R_a} (\mathbf{X}, a)].$$

The expected subset size is

$$E_{\mathbf{r}}[S \mid R_a] = E_{\mathbf{r}} [\sum_{a \in A} \mid a \mid Z_{R_a}(\mathbf{X}, a)].$$

For fixed P^*, $0 \leqslant P^* \leqslant 1$, a procedure R is said to satisfy the P^* condition if

$$\inf_{\mathbf{r} \in \Omega} P_{\mathbf{r}} [CS \mid R] \geqslant P^*.$$

Let $P^* > \frac{1}{k}$ be specified. The following lemma due to Cheng[1] will be used to show that the procedure R_a satisfies the P^* condition.

Lemma 4.1—Assume that the statistic $S (x_{j1}, \ldots, x_{jn_j}, x_{i1} \ldots, x_{in_i})$ is symmetric in arguments $\{x_{j\alpha}, \alpha = 1, \ldots, n_j\}$ and $S (x_j^{(1)}, \ldots x_j^{(n_j)}; x_{i1}, \ldots, x_{in_i}) \leqslant S (\dot{x}_j^{(1)}, \ldots, \dot{x}_j^{(n_j)}; x_{i1}, \ldots, x_{in_i})$ for every $\dot{x}_j^{(\alpha)} \leqslant x_j^{(\alpha)}$, $\alpha = 1, 2, \ldots, n_j$ and $x_{i\beta}$'s, $\beta = 1, 2, \ldots, n_i$. Here $x_j^{(\alpha)}$ denotes the αth order statistic from the jth sample. If $\bar{F}_i (x) \geqslant \bar{F}_j (x)$ for all $x \geqslant 0$ and F_i and F_j are both continuous distributions then for the random samples $\{X_{i\beta}, \beta = 1, \ldots, n_i\}$ and $\{X_{j\alpha}, \alpha = 1, 2, \ldots, n_j\}$ and every constant C^*,

$$P [S (X_{j1}, \ldots, X_{jn_j}; X_{i1}, \ldots X_{in_i}) \geqslant C^* \mid X_{i\beta} \sim F_i; X_{j\alpha} \sim F_i]$$

$$\leqslant P [S (X_{j1}, \ldots, X_{jn_j}; X_{i1}, \ldots, X_{in_i}) \geqslant C^* \mid X_{i\beta} \sim F_i, X_{j\alpha} \sim F_j].$$

Theorem 4.2—For the procedure R_a, P^* condition is satisfied.

PROOF : Assume without loss of generality, that π_i is the best population i.e., $r_i (x) \leqslant r_j (x)$ for all j $(j \neq i)$ and $x \geqslant 0$. This implies $\bar{F}_i (x) \geqslant \bar{F}_j (x)$ for all j $(j \neq i)$ and $x \geqslant 0$. The statistic T_{ij} satisfies the conditions of the above lemma. Hence

$$P^* \leqslant P_0 [T_{ij} \geqslant C_j^{(i)} (\mathbf{n}, P^*) \; \forall \; j, j \neq i]$$

$$\leqslant P_{\mathbf{r}} [T_{ij} \geqslant C_j^{(i)} (\mathbf{n}, P^*) \text{ for all } j, j \neq i].$$

5. PROPERTIES OF PROCEDURE R_a

Gupta and Nagel[4] and Santner[5] have defined some desirable properties of a selection procedure, viz. unbiasedness, monotonicity, strong monotonicity while proposing selection procedures for parametrtc families. We see below that with necessary modifications in the definitions, these properties also hold for our procedure. In what follows we define and prove the strong monotonicity of R_a, which implies its monotonicity and hence the unbiasedness. For any $\mathbf{r} \in \Omega$, let

$$p_{\mathbf{r}}^{\mathbf{n}}(i) = P_{\mathbf{r}}[R_a \text{ selects } \pi_i] \text{ for } i = 1, 2, \ldots, k.$$

Definition 5.1—The procedure R_a is strongly monotone in π_i means

$$p_{\mathbf{r}}^{\mathbf{n}}(i) \text{ is } \downarrow \text{ in } r_i(x) = f_i(x)/\bar{F}_i(x) \text{ when all other components of } \mathbf{r} \text{ are fixed;}$$
$$\text{is } \uparrow \text{ in } r_j(x)\ (j \neq i) \text{ when all other components of } \mathbf{r} \text{ are fixed.}$$

To show the strong monotonicity of R_a we prove the following lemmas :

Lemma 5.1—The family of distributions of T_{ij} is stochastically increasing.

PROOF : Let $r_i(x) < r_0(x) < r_j(x)\ \forall\ x \geqslant 0$ which implies $\Delta(F_i, F_j) > \Delta(F_0, F_j)$ and let $G(x; \Delta(F_i, F_j)) = P_{\Delta(F_i,\ F_j)}[T_{ij} \leqslant x]$ be the cdf of T_{ij}. We want to show that $G(x; \Delta(F_i, F_j)) \leqslant G(x; \Delta(F_0, F_j))\ \forall\ x > 0$. Now $r_i(x) < r_0(x) < r_j(x) \Rightarrow F_i(x) \leqslant F_0(x) \leqslant F_j(x)\ \forall\ x \geqslant 0$. The result follows from Theorem 4.3.3 of Randles and Wolfe[6] by taking $H \equiv F_i$, $G \equiv F_0$ and $F \equiv F_j$.

Lemma 5.2—Let $G(x)$ and $H(x)$ be two absolutely continuous distributions with $G(0) = H(0) = 0$ and let $r_G(x)$ and $r_H(x)$ be the corresponding hazard rates. If $r_H(x) \leqslant r_G(x)\ \forall\ x \geqslant 0$ and $S(x)$ is a nondecreasing function of x, then

$$E_H[S(X)] \geqslant E_G[S(X)].$$

PROOF : Since, $r_H(x) \leqslant r_G(x) \Rightarrow H(x) \leqslant G(x)\ \forall\ x \geqslant 0$, the lemma follows.

Lemma 5.3—Let $X_1, X_2, \ldots, X_k$ be k independent random variables where distribution of X_i is F_i with hazard rate $r_i(x)$, $x \geqslant 0$, $i = 1, 2, \ldots, k$. For any fixed $i\ (1 \leqslant i \leqslant k)$ if $S(x_1, x_2, \ldots, x_k)$ is a nondecreasing function of x_i when all x_j, $j \neq i$, are held fixed and $r_G(x)$ be the hazard rate corresponding to a continuous distribution $G(x)$ such that $r_i(x) \leqslant r_G(x)$, $x \geqslant 0$, then

$$E_{r_1, r_2, \ldots, r_{i-1}, r_i, r_{i+1}, \ldots, r_k}[S(X_1, \ldots, X_{i-1}, X_i, X_{i+1}, \ldots, X_k)]$$
$$\geqslant E_{r_1, r_2, \ldots, r_{i-1}, r_G\, r_{i+1}, \ldots, r_k}[S(X_1, \ldots, X_{i-1}, X_i, X_{i+1}, \ldots, X_k)].$$

PROOF : $r_i(x) \leqslant r_G(x)\ \forall\ x \geqslant 0 \Rightarrow F_i(x) \leqslant G(x)\ \forall\ x \geqslant 0$.

Now

$$E_{r_1, \ldots, r_i, \ldots, r_k} [S(X_1, \ldots, X_i, \ldots, X_k)]$$

$$= E_{r_1, \ldots, r_{i-1}, r_{i+1}, \ldots, r_k} [E_{r_i} \{S(x_1, x_2, \ldots, x_{i-1}, X_i, x_{i+1}, \ldots, x_k)\}]$$

and $S(x_1, x_2, \ldots, x_i, \ldots, x_k)$ is a nondecreasing function of x_i. Hence by Lemma 5.2 it follows that

$$E_{r_i} \{S(x_1, \ldots, x_{i-1}, X_{i+1}, \ldots\ldots, x_k)\}$$

$$\geqslant E_{r_G} \{S(x_1, x_2, \ldots, x_{i-1}, X_i, x_{i+1}, \ldots, x_k)\}.$$

Since this is true for each fixed i, the lemma follows.

Theorem 5.1—The procedure R_a defined in (3.2) is strongly monotone in π_i, for any $i = 1, 2, \ldots, k$.

PROOF : The family of distribution of T_{ij} is stochastically increasing and for each fixed i, $\min_{j \neq i} T_{ij}$ is nondecreasing in X_i, when other components of $\mathbf{X}$ are held fixed. For fixed i $(1 \leqslant i \leqslant k)$ define.

$$Q(T_{ij}) = \begin{cases} 1 \text{ if } T_{ij} \geqslant C_j^{(i)} (\mathrm{n}, P^*) \text{ for all } j \neq i \\ 0 \text{ otherwise.} \end{cases}$$

If $r_i(x) \leqslant r_i^*(x)$ and r_j $(j \neq i)$ are held fixed, it follows from Lemma 5.3, that

$$E_{r_1, \ldots, r_i, \ldots, r_k} [Q(T_{ji})] \geqslant E_{r_1, \ldots r_i^*, \ldots, r_k} [Q(T_{ij})]$$

$$\Rightarrow P_{\mathbf{r}} [R_a \text{ selects } \pi_i] \geqslant P_{\mathbf{r}^*} [R_a \text{ selects } \pi_i]$$

i.e.

$$p_{\mathbf{r}}^{\mathrm{n}}(i) > p_{\mathbf{r}^*}^{\mathrm{n}}(i) \qquad \ldots(5.1)$$

where $\mathbf{r} = (r_1, r_2, \ldots, r_i, \ldots, r_k)^t$ and

$$\mathbf{r}^* = (r_1, r_2, \ldots, r_i^*, \ldots, r_k)^t.$$

(5.1) implies that the selection procedure R_a is strongly monotone.

Since a selection procedure which is strongly monotone is also monotone (see, for example Santner[5]), and hence unbiased, we have the following corollary.

Corollary 5.1—The selection procedure R_a is monotone and unbiased.

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