UNSTEADY FLOW OF AN ELECTRICALLY CONDUCTING DUSTY VISCOUS LIQUID THROUGH A CHANNEL

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The unsteady two-dimensional laminar flow of an electrically conducting dusty viscous liquid through a channel in the presence of a transverse magnetic field has been discussed. Initially, the liquid and dust particles are at rest and a constant pressure gradient is imposed on the system. The changes in velocity profiles for liquid and dust particles have been determined and the effect of the strength of magnetic field on velocity profiles at fixed time has been depicted graphically.

INTRODUCTION

Studies on the influence of dust particles on viscous fluid flows are of importance in petroleum industry and in the purification of crude oils. Other important applications involving dust particles in boundary layers include soil solvation by natural winds, lunar surface erosion by the exhaust of a landing vehicle and dust entrainment in a cloud formed during a nuclear explosion. Saffman (1962) studied the stability of the laminar flow of a dusty gas with uniform distribution of dust particles. Michael (1965) considered the Kelvin–Helmholtz instability of the dusty gas. Michael and Miller (1966) have discussed the motion of the dusty gas enclosed in the semi-infinite space above a rigid plane boundary. We have studied the unsteady two-dimensional flow of an electrically conducting dusty viscous liquid in a channel bounded by two parallel plates in the presence of a transverse magnetic field. The change in velocity profiles for dust and liquid particles has been depicted graphically.

GOVERNING EQUATIONS

The x-axis is taken along the plate and the y-axis normal to it. The basic equations of hydromagnetic flow are

$$\frac{\partial \vec{u}_1'}{\partial t'} + (\vec{u}_1' \cdot \nabla)\,\vec{u}_1' = -\frac{1}{\rho'}\nabla p' + \nu' \nabla^2 \vec{u}_1' + \frac{KN_0}{\rho'}(\vec{u}_2' - \vec{u}_1') + \frac{\vec{J} \times \vec{B}}{\rho'}, \quad \ldots(1)$$

$$\frac{\partial \vec{u}_2'}{\partial t'} + (\vec{u}_2' \cdot \nabla)\,\vec{u}_2' = \frac{K}{m}(\vec{u}_1' - \vec{u}_2'), \quad \ldots(2)$$

$$\operatorname{div} \vec{u}_1' = 0, \quad \ldots(3)$$

$$\operatorname{div} \vec{u}_2' = 0, \quad \ldots(4)$$

where the last term on the right hand side of (1) represents the force on the fluid due to the interaction of magnetic induction $\vec{B}$ and the electric current $\vec{J}$ in the fluid; $\vec{u}_1'$, $\vec{u}_2'$ denote the velocity vectors of liquid and dust particles respectively; p', the pressure; ρ', the density of the fluid; ν', the kinematic coefficient of viscosity; t', the time; m, the mass of the dust particle; N_0 the number density of dust particles; K, the Stokes resistance coefficient which for spherical particles of radius a is $6\pi\mu' a$; and μ', the coefficient of viscosity of fluid particles.

In the present analysis, the following important assumptions are made:

(1) The dust particles are spherical in shape and are uniformly distributed.

(2) Chemical reaction, mass transfer and radiation between the particles and fluid are not considered.

(3) The temperature is uniform within a particle.

(4) Interaction between particles themselves is not considered.

(5) The flow is fully developed.

(6) The buoyancy force is neglected.

(7) The number density of dust particles is constant throughout the motion.

(8) The displacement currents are zero, since the flow velocity is small relative to the speed of light.

(9) The flow induced magnetic field is neglected.

(10) There is no external applied electric field.

(11) The magnetic field is considered fixed relative to the channel.

(12) The Hall effects are negligible.

(13) The fluid is electrically neutral, i.e., no surplus electrical charge distribution is present in the fluid.

(14) Only the electromagnetic body forces are present.

(15) Fluid properties are invariable.

(16) Viscous dissipation is negligible.

Maxwell's equations, together with Ohm's law and the law of electromagnetic conservation, are written in the case of zero-displacement and Hall currents as:

$$\nabla \times \vec{B} = \vec{J} \qquad \text{...(5)}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \text{...(6)}$$

$$\vec{J} = \sigma_1 (\vec{E} + \vec{V} \times \vec{B}) \quad ...(7)$$

$$\nabla . \vec{B} = 0 \quad ...(8)$$

$$\nabla . \vec{E} = 0. \quad ...(9)$$

The usual Prandtl boundary layer assumptions along with assumptions (5)–(9) leads to the following reductions of the previous equations:

$$\frac{\partial u_1'}{\partial t'} = - \frac{1}{\rho'} \frac{\partial p'}{\partial x'} + \nu' \frac{\partial^2 u_1'}{\partial y'^2} + \frac{KN_0}{\rho'} (u_2' - u_1') - \frac{\sigma_1 B_0^2}{\rho'} u_1' \quad ...(10)$$

$$m \frac{\partial u_2'}{\partial t'} = K (u_1' - u_2'). \quad ...(11)$$

which are to be solved subject to the boundary conditions

$$\left. \begin{array}{l} t' = 0, \quad u_1' = u_2' = 0 \\ t' > 0, \quad - \dfrac{1}{\rho'} \dfrac{\partial p'}{\partial x'} = c \text{ (constant)} \\ y' = \pm h, \quad u_1' = 0, \quad u_2' = 0. \end{array} \right\} \quad ...(12)$$

Changing it into non-dimensional form by putting

$$y = \frac{y'}{h}, \quad t = \frac{\nu' t'}{h^2}, \quad u = \frac{u_1' h}{\nu'},$$

$$v = \frac{u_2' h}{\nu'}, \quad p = \frac{p' h^2}{\rho' \nu'^2}$$

we have

$$\frac{\partial u}{\partial t} = - \frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \frac{l}{\sigma} (v - u) - M^2 u, \quad ...(13)$$

$$\sigma \frac{\partial v}{\partial t} = u - v, \quad ...(14)$$

where

$$\sigma = \frac{m\nu'}{kh^2}, \quad l = \frac{mN_0}{\rho'} \quad \text{and} \quad M = \sqrt{\frac{\sigma_1}{\mu'}} B_0 h \text{ (Hartmann number).}$$

The boundary conditions are

$$\left. \begin{array}{lll} t = 0; & u = 0, & v = 0 \\ t > 0; & u = 0 \text{ at } & y = -1, \\ & u = 0 \text{ at } & y = 1 \end{array} \right\} \quad ...(15)$$

where

$$- \frac{\partial p}{\partial x} = c \text{ (constant) for } t > 0.$$

Applying the Laplace transform, we have from (13) and (14)

$$s\bar{u} = \frac{c}{s} + \frac{d^2\bar{u}}{dy^2} + \frac{l}{\sigma}(\bar{v} - \bar{u}) - M^2\bar{u}, \qquad \text{...(16)}$$

$$\sigma s\bar{v} = \bar{u} - \bar{v}, \qquad \text{...(17)}$$

where

$$\bar{u} = \int_0^\infty e^{-st}\, u dt$$

$$\bar{v} = \int_0^\infty e^{-st}\, v dt.$$

The boundary conditions (15) are transformed to

$$\bar{u} = 0, \qquad \bar{v} = 0$$

$$\bar{u} = 0, \quad \bar{v} = 0, \quad y = \pm 1. \qquad \text{...(18)}$$

Solving eqns. (16) and (17) subject to the boundary conditions (18), we have

$$\bar{u} = \frac{c(1 + \sigma s)}{s[sl + (s + M^2)(1 + \sigma s)]} - \frac{c(1 + \sigma s)}{s[sl + (s + M^2)(1 + \sigma s)]} \times \frac{\cosh py}{\cosh p}, \qquad \text{...(19)}$$

$$\bar{v} = \frac{c}{s[sl + (s + M^2)(1 + \sigma s)]} - \frac{c \cosh py}{s[sl + (s + M^2)(1 + \sigma s) \cosh p}, \qquad \text{...(20)}$$

where

$$p^2 = \frac{s(1 + l + \sigma s)}{(1 + \sigma s)} + M^2.$$

On inversion, we get

$$\frac{2u}{c} = \frac{1}{M^2}\left[1 - \frac{\cosh My}{\cosh M}\right] + \frac{8}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)\alpha_1} e^{-\alpha_1 t}\,\frac{(1 - \sigma\alpha_1)}{X} P - \frac{8}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)\alpha_2} e^{-\alpha_2 t}\,\frac{(1 - \sigma\alpha_2)}{X} P,$$

$$\frac{2v}{c} = \frac{1}{M^2}\left[1 - \frac{\cosh My}{\cosh M}\right] + \frac{8}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)\alpha_1} e^{-\alpha_1 t}\,\frac{1}{X} P - \frac{8}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)\alpha_2} e^{-\alpha_2 t}\,\frac{1}{X} P,$$

where

$$X = \sqrt{\left[\left\{(l+1) + \sigma M^2 + \frac{(2n+1)^2 \pi^2}{4}\sigma\right\}^2 - \left\{4\sigma M^2 + (2n+1)^2 \pi^2 \sigma\right\}\right]}$$

$$2\sigma\alpha_1 = (l+1) + \sigma M^2 + \frac{(2n+1)^2 \pi^2}{4}\sigma + X$$

$$2\sigma\alpha_2 = (l+1) + \sigma M^2 + \frac{(2n+1)^2 \pi^2}{4}\sigma - X$$

and

$$P = \cos\left[\frac{(2n+1)\pi y}{2}\right].$$

Conclusion

Figures 1 and 2 show the velocity profiles for the liquid and dust particles at a fixed Hartmann number. It is evident that the maximum velocities of liquid and dust are within 1% of their steady state values at $t = 2$ and $t = 3$ respectively. It means the liquid particles will reach the steady state earlier than the dust particles. Figures 3 and 4 show the effect of the strength of the magnetic field on velocity profiles at fixed time. It is seen that for a constant value of t, the velocity profiles for liquid and dust particles decrease as the Hartmann number increases.

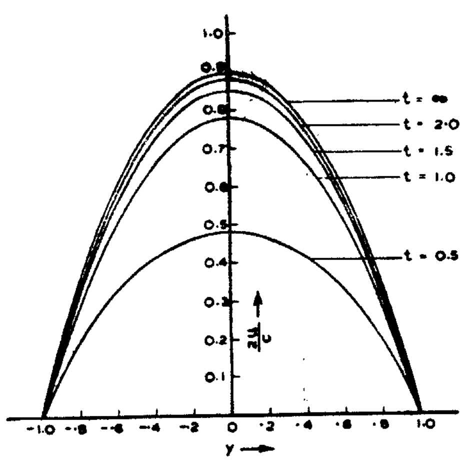

FIG. 1. Velocity profiles for liquid particles at different times and at a fixed Hartmann number ($l = 0{\cdot}2$; $\sigma = 0{\cdot}8$; and $M = 0{\cdot}5$).

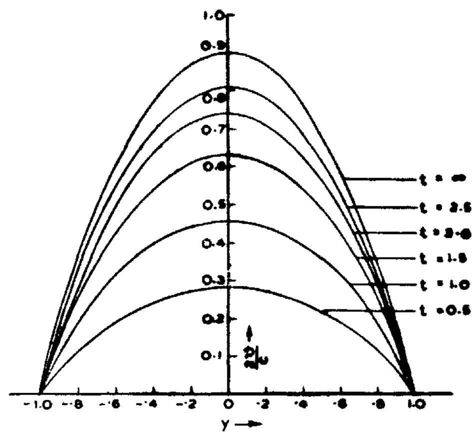

FIG. 2. Velocity profiles for dust particles at different times and at a fixed Hartmann number ($l = 0{\cdot}2$; $\sigma = 0{\cdot}8$; and $M = 0{\cdot}5$).

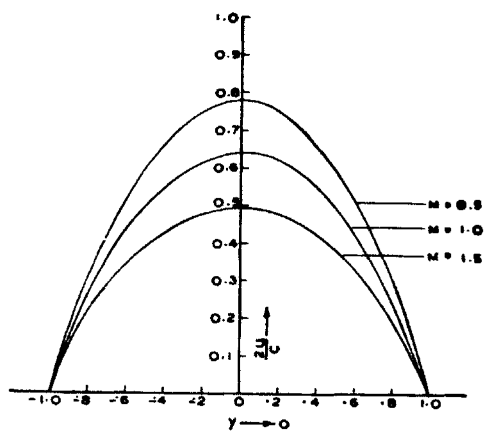

FIG. 3. Velocity profiles for liquid particles at fixed time and different values of Hartmann number ($l = 0{\cdot}2$; $\sigma = 0{\cdot}8$; and $t = 1{\cdot}0$).

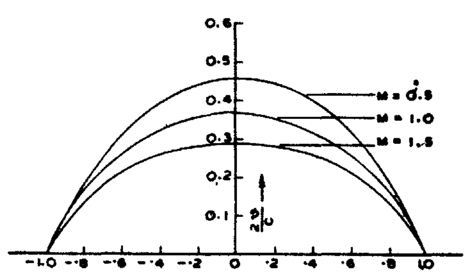

FIG. 4. Velocity profiles for dust particles at fixed time and different values of Hartmann number ($l = 0{\cdot}2$; $\sigma = 0{\cdot}8$; and $t = 1{\cdot}0$).

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