INDEFINITE QUADRATIC PROGRAMMING AND TRANSPORTATION TECHNIQUE

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A technique, similar to the transportation technique in linear programming, has been described to minimize a locally indefinite quadratic function. The problem is attacked directly starting from a basic feasible solution and the conditions under which the solution can be improved have been indicated. Conditions for local optimality have been obtained.

For non-linear programming problems which may have local optima different from a global optimum, such as the problem of the minimization of a concave function under linear constraints, etc., most of the known computational techniques do not do better than find a point which yields a local optimum of the objective function. They do not give, in general, any indication as to whether or not the solution so obtained provides the global optimum. None the less, methods for finding local optima are often very useful in practice; the determination of even a local optimum gives useful information.

This paper takes into account the special structure of the problem considered. However, we seek only a local optimum. The problems considered are of far-reaching importance from a theoretical point of view. The computational procedure developed for such problems is an example of the simplifications that result if we take into account the structure within the problem matrix. The present paper is the outcome of the main result that for such problems absolute minimum occurs at a basic feasible solution (Martos 1964, Swarup 1966).

Kanti Swarup has published a number of papers related to the same problem. Chandra (1967) described a method for solving the capacitated transportation problem in linear fractional functional programming.

Let us consider the following programming problem:

$$\min Z = \left(\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} c_{ijk}\, x_{ijk} \right) \left(\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} d_{ijk}\, x_{ijk} \right)$$

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subject to

$$\sum_{j=1}^{m} \sum_{k=1}^{p} x_{ijk} = a_i, \qquad i = 1, 2, \ldots, m$$

$$\sum_{j=1}^{m} \sum_{k=1}^{p} x_{ijk} = b_j, \qquad j = 1, 2, \ldots, n$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} = c_k, \qquad k = 1, 2, \ldots, p$$

$$x_{ijk} \geqslant 0$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \sum_{k=1}^{p} c_k$$

$$a_i, b_j, c_k > 0.$$

We assume that the objective function Z is positive for all feasible solutions of the problem considered.

With the notations from Corban (1964), we consider now the dual variables, u_i, v_j, w_k and u_i^*, v_j^*, w_k^* defined such that

$$c_{ijk} = u_i + v_j + w_k$$

$$d_{ijk} = u_i^* + v_j^* + w_k^* \text{ for basic variables.}$$

We shall denote

$$c_{ijk}^1 = c_{ijk} - (u_i + v_j + w_k)$$

$$d_{ijk}^1 = d_{ijk} - (u_i^* + v_j^* + k_k^*)$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} c_{ijk} x_{ijk}$$

$$= \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} c_{ijk} x_{ijk} + \sum_{i=1}^{m} (a_i - \sum_{j=1}^{n} \sum_{k=1}^{p} x_{ijk}) u_i$$

$$+ \sum_{j=1}^{n} (b_j - \sum_{i=1}^{m} \sum_{k=1}^{p} x_{ijk}) v_j + \sum_{k=1}^{p} (c_k - \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk}) w_k$$

$$= \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} (c_{iik} - u_i - v_j - w_k) x_{ijk}$$

$$+ \sum_{i=1}^{m} a_i u_i \sum_{j=1}^{n} b_j v_j + \sum_{k=1}^{p} c_k w_k$$

$$= \sum_{(i, j, k) \in S} c_{ijk}^1 x_{ijk} + Z_1$$

where

$$Z_1 = \sum_{i=1}^{m} a_i u_i + \sum_{j=1}^{n} b_j v_j + \sum_{k=1}^{p} c_k w_k$$

and S is the set of non-basic variables.

We shall denote

$$Z_2 = \sum_{i=1}^{m} a_i u_i^* + \sum_{j=1}^{n} b_j v_j^*$$

$$+ \sum_{k=1}^{p} c_k w_k^* \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} d_{ijk} x_{ijk} = \sum_{(i,j,k)\in S} d_{ijk}^1 x_{ijk} + Z_2$$

$$Z = \Big(\sum_{(i,j,k)\in S} c_{ijk}^1 x_{ijk} + Z_1\Big)\Big(\sum_{(i,j,k)\in S} d_{ijk}^1 x_{ijk} + Z_2\Big).$$

The value of the objective function at this basic feasible solution is $Z = Z_1 \cdot Z_2$.

According to Swarup (1970), the solution is optimal (local) if $R_{ijk} = Z_2 c_{rst}^1 + Z_1 d_{rst}^1 + \theta\ c_{rst}^1 \cdot d_{rst}^1 \geqslant 0$ for all non-basic cells (r, s, t).

If one of this value is not non-negative, we shall consider $R_{rst} = \min \{R_{ijk};\ R_{ijk} < 0\}$ and we pick out the most negative R_{ijk} to determine X_{rst} (i.e., variable for introduction into the basis). We improve the value of Z.

The method for finding the initial basic feasible solution, the values of the dual variables, treating degeneracy and improvement of the transportation plan remain the same as in Corban (1964).

Acknowledgement

We are thankful to Dr. S. C. Agrawal, D. N. College, Meerut, for discussion and co-operation and to C.S.I.R. for the financial assistance.

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