CREEP TRANSITION IN TRANSVERSELY ISOTROPIC SHELLS UNDER UNIFORM PRESSURE

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The transition theory of creep developed by Seth (1962) has been used to derive the creep stresses in shells of orthotropic compressible materials under uniform internal pressure. It has been shown that for vanishing anisotropy, the creep stresses are the same as those given by Hulsurker (1966).

1. INTRODUCTION

Many authors use the incompressibility of the material in the theory of creep as the starting point for calculating stresses. The condition of incompressibility in the problems of creep deformations is one of the most important assumptions which simplifies the problem. Moreover, in some cases, it is impossible to find the closed form solutions without this assumption. It is well known that there are many materials which show compressibility effect in creep deformation. The classical theory does not account for these effects. Seth (1962) has developed the transition heory of creep, which does not require assumptions of incompressibility condition, creep strain laws and yield conditions. It utilizes the concept of generalized strain measure and asymptotic transition through the critical points of the differential system defining the deformation field. A number of problems has been solved using this measure (Seth 1963*a*, *b*, 1972, 1974, Hulsurker 1966).

In this paper, creep stresses in shells of a transversely isotropic material under uniform internal pressure have been calculated using the concept of generalized strain measure.

Seth (1972) has defined the generalized strain measure for uni-axial case as

$$e = \left[\frac{1}{n} \left\{ 1 - \left(\frac{l_0}{l} \right)^n \right\} \right]^m \qquad ...(1.1)$$

where n is the measure; m, the irreversibility index; and l_0, l are the initial and strained lengths respectively.

2. GOVERNING EQUATIONS

Consider a spherical shell of constant thickness under uniform internal pressure. Due to the symmetry of the structure and the loading about the centre of the spherical shell, the displacements in spherical co-ordinates (r, θ, ϕ) can be taken as (Seth 1963a)

$$u = r(1-\beta), \quad v = 0, \quad \omega = 0. \qquad \ldots(2.1)$$

where

$$\beta = \beta(r).$$

The generalized components of strain from eqn. (1.1) are

$$\left.\begin{aligned} e_{rr} &= \frac{1}{n^m}[1-(r\beta'+\beta)^n]^m, \quad e_{\theta\theta} = \frac{1}{n^m}[1-\beta^n]^m = e_{\phi\phi} \\ e_{r\theta} &= 0, \quad e_{\theta\phi} = 0, \quad e_{\phi r} = 0. \end{aligned}\right\} \qquad \ldots(2.2)$$

The stress-strain relations for transversely isotropic materials are (Sokolnikoff 1956).

$$\left.\begin{aligned} \tau_{rr} &= Fe_{rr} + Ae_{\theta\theta} + Be_{\phi\phi}, \\ \tau_{\theta\theta} &= Ae_{rr} + Ge_{\theta\theta} + Ce_{\phi\phi}, \\ \tau_{\phi\phi} &= Be_{rr} + Ce_{\theta\theta} + He_{\phi\phi}, \\ \tau_{r\theta} &= Le_{r\theta}, \quad \tau_{\theta\phi} = Me_{\theta\phi}, \quad \tau_{\phi r} = Ne_{\phi r}. \end{aligned}\right\} \qquad \ldots(2.3)$$

where $A, B, C, F, G, H, L, M, N$ are the constants of the material.

Using eqns. (2.2) in eqns. (2.3), the non-vanishing components of the stress are

$$\left.\begin{aligned} \tau_{rr} &= Fn^{-m}[1-(r\beta'+\beta)^n]^m + (A+B)n^{-m}[1-\beta^n]^m, \\ \tau_{\theta\theta} &= An^{-m}[1-(r\beta'+\beta)^n]^m + (G+C)n^{-m}[1-\beta^n]^m, \\ \tau_{\phi\phi} &= Bn^{-m}[1-(r\beta'+\beta)^n]^m + (C+H)n^{-m}[1-\beta^n]^m. \end{aligned}\right\} \qquad \ldots(2.4)$$

The remaining equations of equilibrium, which are still to be satisfied, are

$$\frac{\partial \tau_{rr}}{\partial r} + \frac{(2\tau_{rr} - \tau_{\theta\theta} - \tau_{\phi\phi})}{r} = 0, \qquad \ldots(2.5)$$

and

$$\frac{(\tau_{\theta\theta} - \tau_{\phi\phi})}{r}\cot\theta = 0. \qquad \ldots(2.6)$$

From eqn. (2.6), the only case of interest is

$$\tau_{\theta\theta} - \tau_{\phi\phi} = 0. \qquad \ldots(2.7)$$

Clearly, eqn. (2.7) is satisfied by $\tau_{\theta\theta}$ and $\tau_{\phi\phi}$, as given by eqn. (2.3), if

$$A = B \text{ and } G = H. \qquad \ldots(2.8)$$

Eqn. (2.5) then becomes

$$\frac{\partial \tau_{rr}}{\partial r} + \frac{2(\tau_{rr} - \tau_{\theta\theta})}{r} = 0. \qquad \ldots(2.9)$$

Using eqns. (2.8) and (2.4) in eqn. (2.9), we have a nonlinear equation satisfied by β as

$$\beta \frac{dP}{d\beta} = -\frac{2A}{F}(P+1)^{1-n}[1-\beta^n(P+1)^n]^{1-m}[1-\beta^n]^{m-1}$$

$$+\frac{2(F-A)[1-\beta^n(P+1)^n]}{mn\,F\beta^n\,P(P+1)^{n-1}}$$

$$+\frac{2(2A-G-C)\,(1-\beta^n)^m[1-\beta^n(P+1)^n]^{1-m}}{Fmn\,\beta^n(P+1)^{n-1}.P} - (P+1) \quad \ldots(2.10)$$

where

$$\gamma\beta' = \beta P.$$

If $m = 1$, which holds for secondary creep, eqn. (2.10) reduces to

$$\beta\frac{dP}{d\beta} = -\frac{2A}{F}(P+1)^{1-n} + \frac{2(F-A)}{nF}\frac{[1-\beta^n(P+1)^n]}{\beta^n P(P+1)^{n-1}}$$

$$+\frac{2(2A-G-C)(1-\beta^n)}{Fn\beta^n P(P+1)^{n-1}} - (P+1). \quad \ldots(2.11)$$

The only critical point of interest is $P \to -1$, and asymptotic transition through $P \to -1$ gives the creep stresses.

3. Creep Stresses in Shells under Internal Pressure

For finding the creep stresses, the transition function is taken through the principal stress-difference at the transition point $P \to 1$. The transition function R_2 is given as

$$R_2 = \tau_{rr} - \tau_{\theta\theta} = \frac{F-A}{n^m}[1-\beta^n(P+1)^n]^m$$

$$+\frac{2A-G-C}{n^m}[1-\beta^n]^m. \quad \ldots(3.1)$$

Taking logarithmic differentiation of (3.1) with respect to β, we have

$$\frac{1}{mn}\frac{d}{d\beta}[\log R_2]$$

$$= \frac{(F-A)[1-\beta^n(P+1)^n]^{m-1}\left[-\beta^{n-1}(P+1)^n - \beta^n(P+1)^{n-1}\frac{dP}{d\beta}\right] - (2A-G-C)(1-\beta^n)^{m-1}\beta^{n-1}}{(F-A)[1-\beta^n(P+1)^n]^m + (2A-G-C)[1-\beta^n]^m}.$$

$$\ldots(3.2)$$

Substituting the value of $dP/d\beta$ from eqn. (2.10) and using the asymptotic value $P \to -1$, we get

$$\frac{d}{d\beta}[\log R_2] = \frac{(F-A)\left\{\frac{2A\,mn}{F}\beta^{n-1}(1-\beta^n)^{m-1} + \frac{2(F-A)}{F\beta} + \frac{2(2A-G-C)}{F\beta}(1-\beta^n)^m\right\}}{(F-A)+(2A-G-C)[1-\beta^n]^m} - \frac{(2A-G-C)\,mn\,[1-\beta^n]^{m-1}\beta^{n-1}}{(F-A)+(2A-G-C)[1-\beta^n]^m}. \qquad ...(3.3)$$

Integrating eqn. (3.3), we have

$$R_2 = \tau_{rr} - \tau_{\theta\theta} = A_0 . r^{-2\frac{(F-A)}{F}} . [(F-A) + (2A-G-C)(1-\beta^n)^m]^{1-2A\frac{(F-A)}{F(2A-G-C)}} \qquad ...(3.4)$$

where A_0 is a constant of integration.

Substituting eqn. (3.4) in eqn. (2·9) and integrating, we get

$$\tau_{rr} = -2A_0 \int [(F-A) + (2A-G-C)(1-\beta^n)^m]^{1-\frac{2A(F-A)}{F(2A-G-C)}} . r^{\frac{2A-3F}{F}} \times dr + A_1 \qquad ...(3.5)$$

where A_1 is a constant of integration and the asymptotic value of β as $P \to -1$ is D/r, D being a constant.

The boundary conditions for shell under uniform internal pressure are

$$\tau_{rr} = \begin{cases} -p, & \text{at } r = a, \\ 0, & \text{at } r = b, \end{cases} \qquad ...(3.6)$$

where a, b are the internal and external radii of the shell and p is the internal pressure.

Putting the boundary conditions (3.6) in eqn. (3.5), we get the transitional creep stresses as

$$\left.\begin{aligned} \tau_{rr} &= 2A_0 \int_r^b [(F-A) + (2A-G-C) \\ &\quad \times (1 - D^n r^{-n})^m]^{1-\frac{2A(F-A)}{F(2A-G-C)}} r^{\frac{2A-3F}{F}} . dr \\ \tau_{\theta\theta} &= \tau_{\phi\phi} = \tau_{rr} - A_0 [(F-A) + (2A-G-C) \\ &\quad \times (1 - D^n r^{-n})^m]^{1-\frac{2A(F-A)}{F(2A-G-C)}} . r^{-2\frac{(F-A)}{F}} \end{aligned}\right\} \qquad ...(3.7)$$

where

$$A_0 = -p/2 \int_a^b [(F-A) + (2A-G-C) \times (1 - D^n r^{-n})^m]^{1-\frac{2A(F-A)}{F(2A-G-C)}} . r^{\frac{2A-3F}{F}} . dr.$$

Eqn. (3.4) corresponds to only one stage of creep. If all the three stages are to be aken into account, we shall add the incremental values (Seth 1972) of $\tau_{rr} - \tau_{\theta\theta}$.

Thus, from eqn. (3.4), we have

$$\tau_{rr} - \tau_{\theta\theta} = A_0 r^{-\frac{6(F-A)}{F}} \prod_{m,n} [(F-A) + (2A-G-C) \times (1 - D^n r^{-n})]^{1-\frac{2A(F-A)}{F(2A-G-C)}} \quad \ldots(3.8)$$

where m, n having three different sets of values each corresponding to one stage of creep and the transitional creep stresses are given by

$$\left.\begin{aligned} \tau_{rr} &= 2A_0 \int_r^b r^{\frac{6A-7F}{F}} \prod_{m,n} [(F-A) + (2A-G-C) \times (1 - D^n r^{-n})^m]^{1-\frac{2A(F-A)}{F(2A-G-C)}} \, . \, dr \\ \tau_{\theta\theta} &= \tau_{\phi\phi} = \tau_{rr} - A_0 r^{-\frac{6(F-A)}{F}} \prod_{m,n} [(F-A) + (2A-G-C) \times (1 - D^n r^{-n})^m]^{1-\frac{2A(F-A)}{F(2A-G-C)}} \end{aligned}\right\} \quad \ldots(3.9)$$

where

$$A_0 = -p/2 \int_a^b r^{\frac{6A-7F}{F}} \prod_{m,n} [(F-A) + (2A-G-C) \times (1 - D^n r^{-n})^m]^{1-\frac{2A(F-A)}{F(2A-G-C)}} \, . \, dr$$

It is to be noted that the factor, $r^{-6(F-A)/F}$ corresponds to Hencky's or logarithmic measure given by putting $n \to 0$ in eqn. (3.8).

4. Isotropic Steady State Creep

Transitional creep stresses for stationary state are obtained by putting $m = 1$ in eqn. (3.7)

$$\left.\begin{aligned} \tau_{rr} &= 2A_0 \int_r^b r^{\frac{2A-3F}{F}} [(F-A) + (2A-G-C) \times (1 - D^n r^{-n})]^{1-\frac{2A(F-A)}{F(2A-G-C)}} \, . \, dr \\ \tau_{\theta\theta} &= \tau_{\phi\phi} = \tau_{rr} - A_0 [(F-A) + (2A-G-C) \times (1 - D^n r^{-n})]^{1-\frac{2A(F-A)}{F(2A-G-C)}} \, . \, r^{-\frac{2(F-A)}{F}} \end{aligned}\right\} \quad \ldots(4.1)$$

where

$$A_0 = -p/2 \int_a^b [(F-A) + (2A-G-C) \times (1 - D^n r^{-n})]^{1-\frac{2A(F-A)}{F(2A-G-C)}} \, . \, r^{\frac{2A-3F}{F}} \, . \, dr$$

For isotropic materials, the constants of the material reduce to two only (Sokolnikoff 1956), i.e.,

$$F = G = H; \quad A = B = C.$$

in term of constants λ and μ, we have the relations

$$\lambda \equiv A; \qquad \mu \equiv \tfrac{1}{2}(F - A).$$

Eqns. (4.1) then reduce to

$$\left.\begin{aligned} \tau_{rr} &= -p\,\frac{[(b/r)^{3n-2c(n-1)} - 1]}{[(b/a)^{3n-2c(n-1)} - 1]} \\ \tau_{\theta\theta} = \tau_{\phi\phi} &= p\,\frac{[\tfrac{1}{2}\{n(3-2c) - 2(1-c)\}(b/r)^{3n-2c(n-1)} + 1]}{(b/a)^{3n-2c(n-1)}} \end{aligned}\right\} \quad \ldots(4.2)$$

where

$$c = \frac{F-A}{F} = \frac{2\mu}{\lambda + 2\mu}$$

It is to be noted that the results obtained in eqn. (4.2) are exactly those given by Hulsurker (1966) for isotropic compressible materials.

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