ON SOME BRANS-DICKE FIELDS GENERATED FROM EINSTEIN'S VACUUM FIELDS II

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In the earlier paper (Singh 1975) applying the technique due to Janis *et al.* (1969), the author obtained the solutions of the Brans-Dicke theory for some well-known solutions of Einstein's equations in plane and cylindrical symmetry. In the present work the same technique has been applied to the 'rod solution' and the corresponding solution of the Brans-Dicke theory has been investigated.

This is in continuation to our previous paper (Singh 1975). In the earlier paper, applying the technique developed by Janis *et al.* (1969), we have obtained the solutions of the Brans-Dicke theory for some well-known exact solutions of Einstein's equations in plane and cylindrical symmetry. Recently Datta and Rao (1975) have obtained the generalized 'rod solution' of Einstein-Maxwell scalar field equations. Here, for completeness of our earlier investigation, we have applied the technique due to Janis *et al.* (1969) to the rod solution obtained by Datta and Rao (1975) and have obtained the corresponding solution in the Brans-Dicke theory (1961).

The static axially symmetric metric is

$$ds^2 = -e^{2\nu-2\lambda}\ (dr^2 + dz^2) - r^2\ e^{-2\lambda} d\phi^2 + e^{2\lambda} dt^2 \qquad \ldots(1)$$

where $\lambda = \lambda(r, z)$, $\nu = \nu(r, z)$.

The solution for the Newtonian potential of a rod of length $2a$ and mass per unit length $\varepsilon/2$ is given by

$$\lambda = \frac{\varepsilon}{2} \log \left(\frac{l_1 + l_2 - 2a}{l_1 + l_2 + 2a}\right), \nu = \frac{\varepsilon^2}{2} \log \left[\frac{(l_1 + l_2)^2 - 4a^2}{4 l_1 l_2}\right] \qquad \ldots(2)$$

where $l_1^2 = r^2 + (z - a)^2$, $l_2^2 = r^2 + (z + a)^2$.

Following the technique of Janis *et al.* (1969), Datta and Rao (1975) have obtained the solution of the coupled Einstein-Maxwell scalar field equations which is given by

$$\Psi = \left(\frac{AA'\varepsilon}{2}\right) \log \left(\frac{l_1 + l_2 - 2a}{l_1 + l_2 + 2a}\right)$$

$$\lambda = \log \operatorname{cosec} \mathrm{h}\left(\frac{A'\varepsilon}{2} \log \left(\frac{l_1 + l_2 - 2a}{l_1 + l_2 + 2a}\right)\right)$$

$$\nu = \frac{\varepsilon^2}{2}\left[\frac{(l_1 + l_2)^2 - 4a^2}{4l_1 l_2}\right]$$

$$F_{14} = -\left(\frac{8\pi}{k}\right)^{1/2}\left(\frac{A'\varepsilon}{2}\right) \operatorname{cosech}^2\left(\frac{A'\varepsilon}{2} \log \left(\frac{l_1 + l_2 - 2a}{l_1 + l_2 + 2a}\right)\right) \times \left[\frac{(l_1 + l_2)4ar}{l_1 l_2 (l_1 + l_2 + 2a)(l_1 + l_2 - 2a)}\right]$$

$$F_{24} = -\left(\frac{8\pi}{k}\right)^{\frac{1}{2}}\left(\frac{A'\varepsilon}{2}\right) \operatorname{cosech}^2\left(\frac{A'\varepsilon}{2} \log \left(\frac{l_1 + l_2 - 2a}{l_1 + l_2 + 2a}\right)\right) \times \left[\frac{(l_1 + l_2)4az - 4a^2(l_2 - l_1)}{l_1 l_2 (l_1 + l_2 + 2a)(l_1 + l_2 - 2a)}\right] \quad \ldots (3)$$

where $A' = [1 + (kA^2/8\pi)^{1/2}]$ is a constant and other symbols have the meaning as in our previous paper (Singh 1975).

The corresponding solution of Brans–Dicke theory is given by

$$\Phi = \exp\left(\frac{AA'\varepsilon}{2p}\right)\left(\frac{l_1 + l_2 - 2a}{l_1 + l_2 + 2a}\right), p = \left(2\omega + \frac{6\pi}{K}\right)^{\frac{1}{2}}$$

$$\bar{F}_{14} = -\left(\frac{8\pi G_0}{k}\right)^{\frac{1}{2}}\left(\frac{A'\varepsilon}{2}\right) \operatorname{cosech}^2\left(\frac{A'\varepsilon}{2} \log \left(\frac{l_1 + l_2 - 2a}{l_1 + l_2 + 2a}\right)\right) \times \left[\frac{(l_1 + l_2)4ar}{l_1 l_2 (l_1 + l_2 + 2a)(l_1 + l_2 - 2a)}\right]$$

$$\bar{F}_{24} = -\left(\frac{8\pi G_0}{k}\right)^{\frac{1}{2}}\left(\frac{A'\varepsilon}{2}\right) \operatorname{cosech}^2\left(\frac{A'\varepsilon}{2} \log \left(\frac{l_1 + l_2 - 2a}{l_1 + l_2 + 2a}\right)\right) \times \left[\frac{(l_1 + l_2)4az - 4a^2(l_2 - l_1)}{l_1 l_2 (l_1 + l_2 + 2a)(l_1 + l_2 - 2a)}\right]$$

$$\bar{g}_{44} = \exp\left(\frac{-AA'\varepsilon}{2p}\right)\left(\frac{l_1 + l_2 + 2a}{l_1 + l_2 - 2a}\right) \operatorname{cosech}^2\left(\frac{A'\varepsilon}{2} \log \left(\frac{l_1 + l_2 - 2a}{l_1 + l_2 + 2a}\right)\right)$$

$$\bar{g}_{22} = -r^2 \exp\left(\frac{-AA'\varepsilon}{2p}\right)\left[\frac{l_1 + l_2 + 2a}{l_1 + l_2 - 2a}\right] \sinh^2\left(\frac{A'\varepsilon}{2} \log \left(\frac{l_1 + l_2 - 2a}{l_1 + l_2 + 2a}\right)\right)$$

$$\bar{g}_{11} = \bar{g}_{33} = -\exp\left(\frac{-AA'\varepsilon}{2p}\right)\left[\frac{l_1 + l_2 + 2a}{l_1 + l_2 - 2a}\right]\left[\frac{(l_1 + l_2)^2 - 4a^2}{4l_1 l_2}\right]\varepsilon^2$$

$$\times \sinh^2\left(\frac{A'\varepsilon}{2}\log\left[\frac{l_1 + l_2 - 2a}{l_1 + l_2 + 2a}\right]\right) \qquad \ldots(4)$$

It may be noted that the Curzon particle solution is a special case of the 'rod solution' when the rod shrinks to a point (Gautreau and Anderson 1967) and as such the Brans–Dicke field corresponding to a Curzon particle can be obtained under the condition $A'\varepsilon < 1$ from our solution in this paper (Datta and Rao 1975).

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