WEIERSTRASS TRANSFORM OF GENERALIZED FUNCTIONS

by V. S. SALUNKHE, *Lecturer in Mathematics, Government Polytechnic, 3, Mahapalika Marg, Dhobitalao, Bombay* 400001

(*Received* 8 *September* 1975)

The conventional Weierstrass transform has been discussed by Hirschman and Widder (1955). The above-said transform is entered for generalized functions following Jones (1966).

1. INTRODUCTION

he conventional Weierstrass transform is defined by

$$F(S) = \frac{1}{\sqrt{4\pi}} \int_{-\infty}^{\infty} f(T) \exp \left[\frac{-(S-T)}{4}\right]^2 dT \qquad \ldots(1 \cdot 1)$$

where $f(T)$ is a suitably restricted function on $-\infty < T < \infty$ and S is a complex variable. This transform has been discussed by Hirschman and Widder (1955). This is known by other names *i.e.* Gauss transform, the Gauss Weierstrass transform and the Hille transform.

The purpose of this note is to give generalization of (1·1) following Jones (1966). We shall make use of the definition given by Jones (1966).

2. THE GENERALIZATION OF WEIERSTRASS TRANSFORM

The Kernel in (1·1) is a good function. Let $f_n(T)$ be a regular sequence of good functions such that the generalized limit of which gives a generalized function $f(T)$. We assume that the following conditions are satisfied by $f(T)$

$$f(T) = O\left(\exp\left(\frac{T^2}{4} - \frac{C_1}{2}\right)\right) \text{ as } T \to -\infty \qquad \ldots(2 \cdot 1)$$

$$f(T) = O\left(\exp\left(\frac{T^2}{4} - \frac{C_2}{4}\right)\right) \text{ as } T \to \infty \qquad \ldots(2 \cdot 2)$$

c_1, c_2 are fixed numbers in $(-\infty, \infty)$.

Let $f_n(T)$ be a regular sequence of good functions then the generalised Weierstrass Transform will be given as follows,

$$F(S) = \lim_{n \to \infty} \frac{1}{\sqrt{4\pi}} \int_{-\infty}^{\infty} f_n(T) \exp\left[\frac{-(S-T)^2}{4}\right] dT$$

$$= \frac{1}{\sqrt{4\pi}} \int_{-\infty}^{\infty} f(T) \exp\left[\frac{-(S-T)^2}{4}\right] dT \qquad \ldots(2 \cdot 3)$$

where $c_1 < \text{Re } S < c_2$. The integral on the right-hand side of (2·3) will exists on the account of the Definition 3·11 of Jones (1966, p. 68).

It may be observed that (2·3) may be related to two sided Laplace transform of the generalized function $e^{-T^2/4} f(T)$.

ACKNOWLEDGEMENT

The author is indebted to Dr. B. R. Bhonsle, D.SC., F.N.A.Sc., Professor and Head of the Department of Mathematics, Shivaji University, Kolhapur (India), for the help and guidance during the preparation of this short note.

REFERENCES

Hirschman, J. J., and Widder, D. V. (1955). The Convolution Transform. Princeton University, Press, Princeton, N.J.

Jones, D. S. (1966). Generalised Functions. McGraw-Hill Book Co., Inc., London.