TETRAD FORMALISM OF MAXWELL EQUATIONS WITH EFFECT OF GRAVITATION—II

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In this paper the physical significance of the twelve differential identities, obtained by introducing an orthogonal tetrad formalism of the Maxwell's equations and stress-energy tensor, have been discussed. Frenet formulae have been derived to discuss the mechanism of these identities. In case of constant curvature matrix the world-line of gravitating electromagnetic fluid particles is a helix. It has been shown that the electric field intensity or the magnetic field intensity becomes constant according as the rotation of the fluid flow and electric field lines or fluid flow and magnetic field lines happen to be zero. The known results have been recovered in particular cases of the present analysis.

NOMENCLATURE

ρ = proper matter density

ϵ = internal energy density

p = hydrodynamical pressure

S^{α} = Poynting vector

μ = magnetic permeability of the fluid

k = electric permitivity of the fluid

q = proper density of electric charge

σ = electric conductivity of the fluid

$|e|$ = electric field intensity

$|h|$ = magnetic field intensity

$S^{\alpha} = |e| \; |h| \; \eta^{\alpha\beta\gamma\delta} \, b_{\beta} a_{\gamma} u_{\delta}$

$\theta = u^{\alpha}_{;\alpha}$

$\omega_{\alpha\beta} = u_{[\alpha;\beta]} - \dot{u}_{[\alpha} u_{\beta]}$

$\sigma_{\alpha\beta} = u_{(\alpha;\beta)} - \dot{u}_{(\alpha} u_{\beta)} - \frac{1}{3}\theta h_{\alpha\beta}$

$$\overset{*}{\theta} = a^{\alpha}{}_{;\alpha}$$

$$\overset{*}{\omega}_{\alpha\beta} = a_{[\alpha;\beta]} + \overset{*}{D}a_{[\alpha}a_{\beta]}$$

$$\overset{*}{\sigma}_{\alpha\beta} = a_{(\alpha;\beta)} + \overset{*}{D}a_{(\alpha}a_{\beta)} - \tfrac{1}{3}\,\overset{*}{\theta}\,\overset{*}{h}_{\alpha\beta}$$

$$\overset{\vee}{\theta} = b^{\alpha}{}_{;\alpha}$$

$$\overset{\vee}{\omega}_{\alpha\beta} = b_{[\alpha;\beta]} + \overset{\vee}{D}b_{[\alpha}b_{\beta]}$$

$$\overset{\vee}{\sigma}_{\alpha\beta} = b_{(\alpha;\beta)} + \overset{\vee}{D}b_{(\alpha}b_{\beta)} - \tfrac{1}{3}\,\overset{\vee}{\theta}\overset{\vee}{h}_{\alpha\beta}$$

$$\underset{\vee}{\theta} = n^{\alpha}{}_{;\alpha}$$

$$\underset{\vee}{\omega}_{\alpha\beta} = n_{[\alpha;\beta]} + \underset{\vee}{D}n_{[\alpha}n_{\beta]}$$

$$\underset{\vee}{\sigma}_{\alpha\beta} = n_{(\alpha;\beta)} + \underset{\vee}{D}n_{(\alpha}n_{\beta)} - \tfrac{1}{3}\,\underset{\vee}{\theta}\,\underset{\vee}{h}_{\alpha\beta}$$

$$\omega^{\alpha} = \tfrac{1}{2}\,\eta^{\alpha\beta\gamma\delta}\,u_{\beta}u_{\gamma;\delta}$$

$$\overset{*}{\omega}^{\alpha} = \tfrac{1}{2}\,\eta^{\alpha\beta\gamma\delta}\,a_{\beta}a_{\gamma;\delta}$$

$$\overset{\vee}{\omega}^{\alpha} = \tfrac{1}{2}\,\eta^{\alpha\beta\gamma\delta}\,b_{\beta}b_{\gamma;\delta}$$

( ) = denotes symmetrization

[ ] = denotes anti-symmetrization

$\omega_{\alpha\beta}$, ω^{α} = rotation of fluid flow

$\overset{*}{\omega}_{\alpha\beta}$, $\overset{*}{\omega}^{\alpha}$ = rotation of magnetic field lines

$\overset{\vee}{\omega}_{\alpha\beta}$, $\overset{\vee}{\omega}^{\alpha}$ = rotation of electric field lines

1. Introduction

In first part of this paper Prasad (1977) has derived certain differential identities elucidating the behaviour of gravitating electromagnetic fluid by introducing an orthogonal tetrad formalism of the Maxwell's equations and stress energy tensor. These identities have been derived under the assumption that the time like and space like congruences associated with the gravitating electromagnetic fluid particles, are geodesics for a free observer. Since the world-line of an observer is time like curve in space-time, geodesic in presence of gravitational force and curve in non-gravitational forces, the congruences associated with electromagnetic phenomena are assumed to be geodesics. This assumption is not violating the laws governing the electromagnetic phenomena. In fact, the space like congruences are associated with magnetic and electric field lines and unit Poynting vector and the time like congruence is associated with unit velocity vector field of the fluid.

Our aim here is to discuss the physical significance of differential identities, derived in first part, by considering different physical conditions. In course of discussion we have derived Frenet's formulae to discuss the full mechanism of differential identities and we have seen that the world-line of gravitating electromagnetic fluid particles is a helix when the curvature matrix is constant. In particular cases we have recovered known results in discussions of the differential identities.

For further reference, we shall write here differential identities directly derived in first part. Consider an orthogonal tetrad (u^α, a^α, b^α, n^α), at a point on the observer's world-line, be introduced in such a way that u^α is time like and a^α, b^α, n^α are space like unit vectors satisfying the conditions

$$u^\alpha u_\alpha = 1; \; a^\alpha a_\alpha = b^\alpha b_\alpha = n^\alpha n_\alpha = -1$$
$$u^\alpha a_\alpha = u^\alpha b_\alpha = u^\alpha n_\alpha = a^\alpha b_\alpha = a^\alpha n_\alpha = b^\alpha n_\alpha = 0 \qquad \ldots(1.1)$$

where the Greek indices run from 1 to 4.

Ehlers (1962) defined the kinematical parameters associated with time like congruence. The velocity field (fluid flow) was considered as a superposition of three parts known as kinematical parameters. These are interpreted as expansion, rotation and shear of the fluid element, denoted by θ, $\omega_{\alpha\beta}$, $\sigma_{\alpha\beta}$ respectively and are given by

$$u_{\alpha;\beta} = \omega_{\alpha\beta} + \sigma_{\alpha\beta} + \tfrac{1}{3}\,\theta h_{\alpha\beta} + \dot{u}_\alpha u_\beta \qquad \ldots(1.2)$$

where

$$h_{\alpha\beta} = g_{\alpha\beta} - u_\alpha u_\beta; \; \dot{u}_\alpha = u_{\alpha;\beta} u^\beta$$

and semicolon denotes covariant derivative.

In analogy with kinematical parameters, associated with flow vector, Date (1972) defined the kinematical parameters associated with space like unit magnetic field vector $a^\alpha = \dfrac{h^\alpha}{|h|}$ as follows :

$$a_{\alpha;\beta} = \overset{*}{\omega}_{\alpha\beta} + \overset{*}{\sigma}_{\alpha\beta} + \tfrac{1}{3}\,\overset{*}{\theta}\,\overset{*}{h}_{\alpha\beta} - \overset{*}{D}a_\alpha a_\beta \qquad \ldots(1.3)$$

where

$$\overset{*}{h}_{\alpha\beta} = g_{\alpha\beta} + a_\alpha a_\beta; \; \overset{*}{D}a_\alpha = a_{\alpha;\beta}\, a^\beta \text{ and } \overset{*}{\omega}_{\alpha\beta},\ \overset{*}{\sigma}_{\alpha\beta},\ \overset{*}{\theta}$$

are called rotation, shear, expansion of the magnetic field respectively.

Prasad (1977) has defined the kinematical parameters associated with unit electric field $b^\alpha = \dfrac{e^\alpha}{|e|}$ and unit Poynting vector $n^\alpha = \dfrac{S^\alpha}{|S|}$ as follows :

$$b_{\alpha;\beta} = \overset{\vee}{\omega}_{\alpha\beta} + \overset{\vee}{\sigma}_{\alpha\beta} + \tfrac{1}{3}\,\overset{\vee}{\theta}\,\overset{\vee}{h}_{\alpha\beta} - \overset{\vee}{D}b_\alpha b_\beta \qquad \ldots(1.4)$$

where

$$\check{h}_{\alpha\beta} = g_{\alpha\beta} + b_\alpha b_\beta;\ \check{D} b_\alpha = b_{\alpha;\beta}\, b^\beta \text{ and } \check{\omega}_{\alpha\beta},\ \check{\sigma}_{\alpha\beta},\ \check{\theta}$$

are called rotation, shear, expansion of the electric field respectively.

$$n_{\alpha;\beta} = \underset{\vee}{\omega}_{\alpha\beta} + \underset{\vee}{\sigma}_{\alpha\beta} + \tfrac{1}{3}\, \underset{\vee}{\theta}\, \underset{\vee}{h}_{\alpha\beta} - \underset{\vee}{D} n_\alpha n_\beta \qquad \ldots(1.5)$$

where

$$\underset{\vee}{h}_{\alpha\beta} = g_{\alpha\beta} + n_\alpha n_\beta;\ \underset{\vee}{D} n_\alpha = n_{\alpha;\beta}\, n^\beta \text{ and } \underset{\vee}{\omega}_{\alpha\beta},\ \underset{\vee}{\sigma}_{\alpha\beta},\ \underset{\vee}{\theta}$$

are called rotation, shear, expansion of unit Poynting vector field.

With the help of stress energy tensor and the Maxwell equations the following identities have been derived in the first part of this paper :

$$\frac{dX}{du} + X\theta = \frac{d\,|\,S\,|}{dn} + |\,S\,|\,(\underset{\vee}{\theta} + u^\beta u^\alpha\, n_{\alpha;\beta}) \qquad \ldots(1.6)$$

$$\frac{dY}{da} + Y\overset{*}{\theta} = -\,|\,S\,|\,(u_{\alpha;\beta}\, a^\alpha n^\beta + u^\beta a^\alpha\, n_{\alpha;\beta}) \qquad \ldots(1.7)$$

$$\frac{dZ}{db} + Z\check{\theta} = -\,|\,S\,|\,(u_{\alpha;\beta}\, b^\alpha n^\beta + n_{\alpha;\beta}\, b^\alpha u^\beta) \qquad \ldots(1.8)$$

$$\frac{dL}{dn} + L\underset{\vee}{\theta} = \frac{d\,|\,S\,|}{du} - |\,S\,|\,(u_{\alpha;\beta}\, n^\alpha n^\beta - \theta) \qquad \ldots(1.9)$$

where

$$X = \rho + \rho\epsilon + (k\,|\,e\,|^2 + \mu\,|\,h\,|^2) \qquad \ldots(1.10)$$

$$Y = (p + \tfrac{1}{2}\, k\,|\,e\,|^2) - \tfrac{1}{2}\,\mu\,|\,h\,|^2 \qquad \ldots(1.11)$$

$$Z = (p + \tfrac{1}{2}\,\mu\,|\,h\,|^2) - \tfrac{1}{2}\, k\,|\,e\,|^2 \qquad \ldots(1.12)$$

$$L = p + \tfrac{1}{2}\,(\mu\,|\,h\,|^2 + k\,|\,e\,|^2) \qquad \ldots(1.13)$$

and again we have

$$\frac{d}{da}(\mu\,|\,h\,|) + \mu\,|\,h\,|\,(\overset{*}{\theta} - u^\beta u^\alpha\, a_{\alpha;\beta}) + 2\,|\,e\,|\,\omega^\alpha b_\alpha = 0 \qquad \ldots(1.14)$$

$$\frac{d}{du}(\mu\,|\,h\,|) + \mu\,|\,h\,|\,(u_{\alpha;\beta}\, a^\alpha a^\beta + \theta) - \frac{d\,|\,e\,|}{dn} + |\,e\,|\,\eta^{\alpha\beta\gamma\delta}\, a_\alpha u_{\gamma;\beta}\, b_\delta$$
$$+\,|\,e\,|\,\eta^{\alpha\beta\gamma\delta}\, a_\alpha u_\gamma\, b_{\delta;\beta} = 0 \qquad \ldots(1.15)$$

$$\mu\,|\,h\,|\,(u_{\alpha;\beta}\, b^\alpha a^\beta - a_{\alpha;\beta}\, b^\alpha u^\beta) - 2\,|\,e\,|\,\check{\omega}^\alpha u_\alpha = 0 \qquad \ldots(1.16)$$

$$\mu\,|\,h\,|\,(u_{\alpha;\beta}\, n^\alpha a^\beta - a_{\alpha;\beta} n^\alpha u^\beta) + \frac{d\,|\,e\,|}{da} + |\,e\,|\,\eta^{\alpha\beta\gamma\delta}$$
$$\times\, n_\alpha u_{\gamma;\beta}\, b_\delta + |\,e\,|\,\eta^{\alpha\beta\gamma\delta}\, n_\alpha u_\gamma\, b_{\delta;\beta} = 0 \qquad \ldots(1.17)$$

$$\frac{d}{db}(k \mid e \mid) + k \mid e \mid (\overset{\vee}{\theta} - b_{\alpha;\beta}\, u^{\alpha}u^{\beta}) - 2 \mid h \mid \omega^{\alpha} a_{\alpha} = - q \quad ...(1.18)$$

$$k \mid e \mid (u_{\alpha;\beta}\, a^{\alpha} b^{\beta} - b_{\alpha;\beta}\, a^{\alpha} u^{\beta}) + 2 \mid h \mid \overset{*}{\omega}{}^{\alpha} u_{\alpha} = 0 \quad ...(1.19)$$

$$\frac{d \mid h \mid}{dn} + \mid h \mid \eta^{\alpha\beta\gamma\delta} (u_{\gamma;\beta}\, b_{\alpha} a_{\delta} + a_{\delta;\beta}\, b_{\alpha} u_{\gamma})$$

$$- \frac{d}{du}(k \mid e \mid) - k \mid e \mid (u_{\alpha;\beta}\, b^{\alpha} b^{\beta} + \theta) = \sigma \mid e \mid \quad ...(1.20)$$

$$k \mid e \mid (u_{\alpha;\beta}\, n^{\alpha} b^{\beta} - b_{\alpha;\beta}\, n^{\alpha} u^{\beta}) - \frac{d \mid h \mid}{db}$$

$$- \mid h \mid \eta^{\alpha\beta\gamma\delta}\, n_{\alpha} u_{\gamma;\beta} a_{\delta} - \mid h \mid \eta^{\alpha\beta\gamma\delta}\, n_{\alpha} u_{\gamma}\, a_{\delta;\beta} = 0 \quad ...(1.21)$$

where $\frac{dX}{du} = X_{,\beta} u^{\beta}$; $\frac{dY}{da} = Y_{,\beta} a^{\beta}$ ... and so on. Comma denotes partial differentiation w.r.t. x. The time like and space like congruences are geodesics, i.e.

$$\dot{u}^{\alpha} = \overset{*}{D} a^{\alpha} = \overset{\vee}{D} b^{\alpha} = \underset{\vee}{D} n^{\alpha} = 0.$$

2. The Frenet Formulae

To discuss the full mechanism of our differential identities, we will derive the Frenet formulae according to our purpose. Following Synge (1967) let x^{α} be the coordinates with $X_4 = it$, such that the metric is of the form $g_{\alpha\beta}\, dx^{\alpha}\, dx^{\beta}$. Let Γ be any time like curve with equations $x^{\alpha} = x^{\alpha}(s)$. Where s is proper time on Γ. Denoting by D, the operator $\frac{d}{ds}$, the Frenent-Serret formulae read

$$\left.\begin{aligned} Du^{\alpha} &= k_1 a^{\alpha} + k_2 b^{\alpha} + k_3 n^{\alpha} \\ Da^{\alpha} &= k_1 u^{\alpha} + k_4 b^{\alpha} + k_5 n^{\alpha} \\ Db^{\alpha} &= k_2 u^{\alpha} - k_4 a^{\alpha} + k_6 n^{\alpha} \\ Dn^{\alpha} &= k_3 u^{\alpha} - k_5 a^{\alpha} - k_6 b^{\alpha}. \end{aligned}\right\} \quad ...(2.1)$$

Let us define 4×4 matrix

$$T = (\mathbf{u}, \mathbf{a}, \mathbf{b}, \mathbf{n}) \quad ...(2.2)$$

To represent the orthonormal tetrad, each vector symbol standing for a column of four elements; by orthogonality we have

$$T^{\tilde{t}} . T = \text{diag}\,(1, -1, -1, -1) \quad ...(2.3)$$

with $\tilde{t}$ for transpose. Let us define the curvature matrix.

$$K = \begin{bmatrix} 0 & k_1 & k_2 & k_3 \\ k_1 & 0 & k_4 & k_5 \\ k_2 & -k_4 & 0 & k_6 \\ k_3 & -k_5 & -k_6 & 0 \end{bmatrix} \qquad \ldots(2.4)$$

Then the whole set of equations (1.1), (2.1) are summed up in (2.3) and

$$DT = TK \qquad \ldots(2.5)$$

of which the general solution is

$$T = T_0 \exp(sk) \qquad \ldots(2.6)$$

where T_0 is a constant matrix.

If we suppose that the curvature matrix K is constant then using equations (2.1) the following relations may be obtained.

$$\left.\begin{aligned} Du_\alpha Du^\alpha &= \text{constant} \\ Da^\alpha Da_\alpha &= \text{constant} \\ Db^\alpha Db_\alpha &= \text{constant} \\ Dn^\alpha Dn_\alpha &= \text{constant} \end{aligned}\right\} \qquad \ldots(2.7)$$

In view of eqns. (2.4) and (2.7) we conclude that the world-line of gravitating electromagnetic fluid particles is a helix when the curvature matrix K is constant.

3. Discussions of the Differential Identities

In this section we shall conclude some physical significance compatible with our differential identities which are formulated in terms of time like and space like congruences. We shall discuss all the differential identities by dividing them into three sections.

(a) In first section the equations (1.6) – (1.9) can be simplified with the aid of (1.2), (2.1) and we have the following equations :

$$\frac{dX}{du} + X\theta = \frac{d\,|\,S\,|}{dn} + |\,S\,|\,(\underset{\vee}{\theta} + k_3) \qquad \ldots(3.1a)$$

$$\frac{dY}{da} + Y\overset{*}{\theta} = -\,|\,S\,|\,[(\omega_{\alpha\beta} + \sigma_{\alpha\beta})\,a^\alpha n^\beta + k_5] \qquad \ldots(3.2a)$$

$$\frac{dZ}{db} + Z\overset{\vee}{\theta} = -\,|\,S\,|\,[(\omega_{\alpha\beta} + \sigma_{\alpha\beta})\,b^\alpha n^\beta + k_6] \qquad \ldots(3.3a)$$

$$\frac{dL}{dn} + L\overset{\vee}{\theta} = \frac{d\,|\,S\,|}{du} + |\,S\,|\,\theta. \qquad \text{...(3.4}a\text{)}$$

In case of null electric field, eqns. (3.1*a*) – (3.4*a*) with the aid of eqns. (1.10) – (1.13) reduce to

$$\frac{d}{du}(\hat{\sigma} + \tfrac{1}{2}\,\mu\,|\,h\,|^{2}) + (\hat{\sigma} + \tfrac{1}{2}\,\mu\,|\,h\,|^{2})\,\theta = 0 \qquad \text{...(3.5}a\text{)}$$

where

$$\hat{\sigma} = \rho + \rho\epsilon$$

$$\frac{d}{da}(p - \tfrac{1}{2}\,\mu\,|\,h\,|^{2}) + (p - \tfrac{1}{2}\,\mu\,|\,h\,|^{2})\,\overset{*}{\theta} = 0 \qquad \text{...(3.6}a\text{)}$$

$$\frac{d}{db}(p + \tfrac{1}{2}\,\mu\,|\,h\,|^{2}) = 0 \qquad \text{...(3.7}a\text{)}$$

$$\frac{d}{dn}(p + \tfrac{1}{2}\,\mu\,|\,h\,|^{2}) = 0. \qquad \text{...(3.8}a\text{)}$$

The equation (3.5*a*) is conservation equation for magnetofluid and its further importance will be discussed in the next paper. Equation (3.6*a*) reveals that the difference in variation of hydrodynamic and magnetic pressure depends on the expansion of the magnetic field. Further, if the magnetic field is free from expansion the difference in these two pressures remains constant along the magnetic field. The equations (3.7*a*) and (3.8*a*) show that the total (hydrodynamic and magnetic) pressure is constant along the electric field and Poynting vector respectively, are similar to the equations obtained by Date (1972).

In case of null magnetic field, eqns. (3.1*a*) – (3.4*a*) with the aid of eqns. (1.10) – (1.13) reduce to the equation

$$\frac{d}{du}(\hat{\sigma} + \tfrac{1}{2}\,k\,|\,e\,|^{2}) + (\hat{\sigma} + \tfrac{1}{2}\,k\,|\,e\,|^{2})\,\theta = 0 \qquad \text{...(3.9}a\text{)}$$

$$\frac{d}{da}(p + \tfrac{1}{2}\,k\,|\,e\,|^{2}) = 0 \qquad \text{...(3.10}a\text{)}$$

$$\frac{d}{db}(p - \tfrac{1}{2}\,k\,|\,e\,|^{2}) + (p - \tfrac{1}{2}\,k\,|\,e\,|^{2})\,\overset{\vee}{\theta} = 0 \qquad \text{...(3.11}a\text{)}$$

$$\frac{d}{dn}(p + \tfrac{1}{2}\,k\,|\,e\,|^{2}) = 0. \qquad \text{...(3.12}a\text{)}$$

The eqn. (3.9*a*) is conservation eqn. for the fluid with electric field. Equations (3.10*a*) and (3.12*a*) show that the total (hydrodynamic and electric) pressure is constant along the magnetic field and the Poynting vector respectively. The eqn. (3.11*a*) elucidates that the difference in variation of hydrodynamic and electric pressure depends on the expansion of the electric field.

In case of null electromagnetic field and with the consequence of eqns. (1.10) – (1.13), eqns. (3.1*a*) – (3.4*a*) reduce to

$$\frac{d\hat{\sigma}}{du} + \hat{\sigma}\theta = 0 \qquad \text{...(3.13}a\text{)}$$

$$\frac{dp}{da} = 0 \qquad \text{...(3.14}a\text{)}$$

$$\frac{dp}{db} = 0 \qquad \text{...(3.15}a\text{)}$$

$$\frac{dp}{dn} = 0. \qquad \text{...(3.16}a\text{)}$$

These equations are similar to the equations obtained and discussed by Suryanarayan (1961) for pure hydrodynamic case.

Thus we conclude that eqns. (3.1*a*) – (3.4*a*) are governing the general phenomena of hydrodynamic fluid with electromagnetic field.

(b) In second section we present the result in simple form suitable to kinematical interpretation. Let us write out the differential identities (1.14) – (1.17) with the help of eqns. (1.2), (1.4) and (2.1) as follows :

$$\frac{d}{da}(\mu \mid h \mid) + \mu \mid h \mid (\overset{*}{\theta} - k_1) + 2 \mid e \mid \omega^{\alpha} b_{\alpha} = 0 \qquad \text{...(3.1}b\text{)}$$

$$\frac{d}{du}(\mu \mid h \mid) + \mu \mid h \mid (\sigma_{\alpha\beta} a^{\alpha} a^{\beta} + \tfrac{2}{3}\theta) - \frac{d \mid e \mid}{dn}$$
$$- 2 \mid e \mid a_{\alpha} b_{\delta} \underset{\sim}{\omega}^{\alpha\delta} + 2 \mid e \mid a_{\alpha} u_{\gamma} \underset{\sim}{\overset{\vee}{\omega}}{}^{\alpha\gamma} = 0 \qquad \text{...(3.2}b\text{)}$$

$$\mu \mid h \mid [(\omega_{\alpha\beta} + \sigma_{\alpha\beta}) b^{\alpha} a^{\beta} + k_4] - 2 \mid e \mid \overset{\vee}{\omega}{}^{\alpha} u_{\alpha} = 0 \qquad \text{...(3.3}b\text{)}$$

$$\mu \mid h \mid [(\omega_{\alpha\beta} + \sigma_{\alpha\beta}) n^{\alpha} a^{\beta} + k_5] + \frac{d \mid e \mid}{da}$$
$$- 2 \mid e \mid n_{\alpha} b_{\delta} \underset{\sim}{\omega}^{\alpha\delta} + 2 \mid e \mid n_{\alpha} u_{\gamma} \underset{\sim}{\overset{\vee}{\omega}}{}^{\alpha\gamma} = 0 \qquad \text{...(3.4}b\text{)}$$

where

$$\underset{\sim}{\omega}^{\alpha\delta} = \tfrac{1}{2}\eta^{\alpha\delta\gamma\beta}\omega_{\gamma\beta}; \quad \underset{\sim}{\overset{\vee}{\omega}}{}^{\alpha\delta} = \tfrac{1}{2}\eta^{\alpha\delta\gamma\beta}\overset{\vee}{\omega}_{\gamma\beta}$$

and are called dual tensor to $\omega_{\gamma\beta}$, $\overset{\vee}{\omega}_{\gamma\beta}$ respectively.

These equations are simply the intrinsic form of the Maxwell first equations.

In case of null electric field, the eqns. (3.1*b*) – (3.4*b*) reduce to

$$\frac{d}{da}(\log \mu \mid h \mid) + (\overset{*}{\theta} - k_1) = 0 \qquad \ldots(3.5b)$$

$$\frac{d}{du}(\log \mu \mid h \mid) + (\sigma_{\alpha\beta}\, a^{\alpha} a^{\beta} + \tfrac{2}{3}\,\theta) = 0 \qquad \ldots(3.6b)$$

$$k_4 = -\,(\omega_{\alpha\beta} + \sigma_{\alpha\beta})\, b^{\alpha} a^{\beta} \qquad \ldots(3.7b)$$

$$k_5 = -\,(\omega_{\alpha\beta} + \sigma_{\alpha\beta})\, n^{\alpha} a^{\beta}. \qquad \ldots(3.8b)$$

The eqns. (3.5*b*) and (3.6*b*) govern the variation of magnetic induction along the magnetic field and fluid flow respectively. The eqns. (3.7*b*) and (3.8*b*) are giving the values of scalar curvatures in terms of rotation and shear of the fluid.

In case of null magnetic field, eqns. (3.1*b*) and (3.3*b*) give the same results as discussed in the first part of this paper. From eqns. (3.2*b*) and (3.4*b*), we have

$$\frac{d}{dn}(\log \mid e \mid) + 2a_{\alpha} b_{\delta}\, \underset{\sim}{\omega}^{\alpha\delta} - 2a_{\alpha} u_{\gamma}\, \underset{-}{\overset{\vee}{\omega}}{}^{\alpha\gamma} = 0 \qquad \ldots(3.9b)$$

$$\frac{d}{da}(\log \mid e \mid) + 2n_{\alpha} u_{\gamma}\, \underset{-}{\overset{\vee}{\omega}}{}^{\alpha\gamma} - 2n_{\alpha} b_{\delta}\, \underset{-}{\omega}^{\alpha\delta} = 0. \qquad \ldots(3.10b)$$

The eqns. (3.9*b*) and (3.10*b*) show that the variation of the electric field intensity depends on the rotation of the fluid and electric field lines. The electric field intensity becomes constant if the rotation of the fluid flow and electric field lines happen to be zero.

In case of null rotation (fluid, magnetic field and electric field), eqns. (3.2*b*) – (3.4*b*) reduce to

$$\frac{d}{du}(\mu \mid h \mid) + \mu \mid h \mid (\sigma_{\alpha\beta}\, a^{\alpha} a^{\beta} + \tfrac{2}{3}\,\theta) - \frac{d \mid e \mid}{dn} = 0 \qquad \ldots(3.11b)$$

$$k_4 = -\,\sigma_{\alpha\beta}\, b^{\alpha} a^{\beta} \qquad \ldots(3.12b)$$

$$\mu \mid h \mid (\sigma_{\alpha\beta}\, n^{\alpha} a^{\beta} + k_5) + \frac{d \mid e \mid}{da} = 0. \qquad \ldots(3.13b)$$

These equations have their clear physical meanings.

In case of null rotation and shear free, eqns. (3.11*b*) to (3.13*b*) assume the form

$$\frac{d}{du}(\mu \mid h \mid) + \tfrac{2}{3}\,\mu \mid h \mid \theta - \frac{d \mid e \mid}{dn} = 0 \qquad \ldots(3.14b)$$

$$k_4 = 0 \qquad \ldots(3.15b)$$

$$\frac{d\mid e\mid}{da}+k_5\,\mu\mid h\mid=0. \qquad ...(3.16b)$$

These equations are excellent and self-explanatory. The eqns. (3.14*b*) and (3.16*b*) will be discussed in next part since they have good relations with change of entropy

(*c*) In third section, we write the differential identities (1.18) – (1.21) with the aid of eqns. (1.2), (1.3) and (2.1) as follows :

$$\frac{d}{db}(k\mid e\mid)+k\mid e\mid(\overset{\vee}{\theta}-k_2)-2\mid h\mid\omega^{\alpha}a_{\alpha}=-q \qquad ...(3.1c)$$

$$k\mid e\mid[(\omega_{\alpha\beta}+\sigma_{\alpha\beta})a^{\alpha}b^{\beta}-k_4]+2\mid h\mid\overset{*}{\omega}{}^{\alpha}u_{\alpha}=0 \qquad ...(3.2c)$$

$$\frac{d\mid h\mid}{dn}-\frac{d}{du}(k\mid e\mid)-k\mid e\mid(\sigma_{\alpha\beta}\,b^{\alpha}b^{\beta}+\tfrac{2}{3}\,\theta)$$
$$-2\mid h\mid b_{\alpha}a_{\delta}\,\underset{\sim}{\omega}{}^{\alpha\delta}+2\mid h\mid b_{\alpha}u_{\gamma}\,\underset{\sim}{\overset{*}{\omega}}{}^{\alpha\gamma}=-\sigma\mid e\mid \qquad ...(3.3c)$$

$$k\mid e\mid[(\omega_{\alpha\beta}+\sigma_{\alpha\beta})n^{\alpha}b^{\beta}+k_6]-\frac{d\mid h\mid}{db}+2\mid h\mid n_{\alpha}a_{\delta}\,\underset{\sim}{\omega}{}^{\alpha\delta}$$
$$-2\mid h\mid n_{\alpha}u_{\gamma}\,\underset{\sim}{\overset{*}{\omega}}{}^{\alpha\gamma}=0 \qquad ...(3.4c)$$

where $\underset{\sim}{\overset{*}{\omega}}{}^{\alpha\gamma}=\frac{1}{2}\,\eta^{\alpha\gamma\beta\delta}\,\overset{*}{\omega}_{\beta\delta}$, called dual tensor to $\overset{*}{\omega}_{\beta\delta}$ and $\underset{\sim}{\omega}{}^{\alpha\gamma}$ is already defined in section (b).

These equations are intrinsic form of the second Maxwell's equations.

In case of null electric field, eqns. (3.1*c*) and (3.2*c*) give the same result as discussed in first part of this paper. We can write eqns. (3.3*c*) and (3.4*c*) in the forms

$$\frac{d}{dn}(\log\mid h\mid)+2b_{\alpha}u_{\gamma}\,\underset{\sim}{\overset{*}{\omega}}{}^{\alpha\gamma}-2b_{\alpha}a_{\delta}\,\underset{\sim}{\omega}{}^{\alpha\delta}=0 \qquad ...(3.5c)$$

$$\frac{d}{db}(\log\mid h\mid)+2n_{\alpha}u_{\gamma}\,\underset{\sim}{\overset{*}{\omega}}{}^{\alpha\gamma}-2n_{\alpha}a_{\delta}\,\underset{\sim}{\omega}{}^{\alpha\delta}=0. \qquad ...(3.6c)$$

These equations show that the variation of magnetic field intensity depends on the rotation of the fluid flow and magnetic field lines along the Poynting vector and electric field respectively. The magnetic field intensity becomes constant if the rotation of the fluid flow and magnetic field lines happen to be zero.

In case of null magnetic field, the equations (3.1*c*) – (3.4*c*) assume the forms

$$\frac{d}{db}(k\mid e\mid)+k\mid e\mid(\overset{\vee}{\theta}-k_2)=-q \qquad ...(3.7c)$$

$$k_4 = (\omega_{\alpha\beta} + \sigma_{\alpha\beta})\, a^\alpha b^\beta \qquad \text{...(3.8}c\text{)}$$

$$\frac{d}{du}(\log k \mid e \mid) + \left[\sigma_{\alpha\beta}\, b^\alpha b^\beta + \tfrac{2}{3}\,\theta - \frac{\sigma}{k}\right] = 0 \qquad \text{...(3.9}c\text{)}$$

$$k_6 = -\,(\omega_{\alpha\beta} + \sigma_{\alpha\beta})\, n^\alpha b^\beta. \qquad \text{...(3.10}c\text{)}$$

The eqns. (3.7*c*) and (3.9*c*) govern the variation of electric induction (electric displacement) along the electric field and fluid flow respectively. The variation of electric induction along the electric field depends on the expansion of the electric field, charge and the second curvature. The variation of the electric induction along the fluid flow depends on the shear, expansion, conductivity and electric permitivity of the fluid. The equations (3.8*c*) and (3.10*c*) give the value of scalar curvature.

In case of null rotation (as in section '*b*'), the eqns. (3.3*c*) and (3.4*c*) become

$$\frac{d \mid h \mid}{dn} - \frac{d}{du}(k \mid e \mid) - k \mid e \mid \left[\sigma_{\alpha\beta}\, b^\alpha b^\beta + \tfrac{2}{3}\,\theta - \frac{\sigma}{k}\right] = 0 \qquad \text{...(3.11}c\text{)}$$

$$\frac{d \mid h \mid}{db} - k \mid e \mid (\sigma_{\alpha\beta}\, n^\alpha b^\beta + k_6) = 0. \qquad \text{...(3.12}c\text{)}$$

These equations have their clear physical meanings

In case of null rotation and shear free, eqns. (3.2*c*) – (3.4*c*) reduce to

$$k_4 = 0 \qquad \text{...(3.13}c\text{)}$$

$$\frac{d \mid h \mid}{dn} - \frac{d}{du}(k \mid e \mid) - k \mid e \mid \left(\tfrac{2}{3}\,\theta - \frac{\sigma}{k}\right) = 0 \qquad \text{...(3.14}c\text{)}$$

$$\frac{d \mid h \mid}{db} - k \mid e \mid k_6 = 0. \qquad \text{...(3.15}c\text{)}$$

These equations, which will be discussed with entropy relations, have their own physical properties.

Further development will be given in next paper.

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