AN APPLICATION OF NON-LINEAR PROGRAMMING IN POWER PLANT LOCATION PROBLEM

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The present paper is an attempt to make use of principles of non-linear programming to solve the problem of location of power plant. Some conclusive decisions have been arrived in regard to capital outlays for future growth of power plant installations.

Introduction

Ours is a country that has been hit by power crisis. Sincere efforts are being made to tide over the crisis. While the country can discipline its consumption, consolidate the existing capacity, generation of power still continues to be a dominant inoperative of the country.

Another way of looking at the problem is to strengthen the capacity of existing power plants and to instal new ones in keeping with growing demands. One can systematise the situation by describing it in the language and nuance of mathematical programming. The present note is an attempt of this sort and it seeks to apply principles of non-linear programming to a simple problem concerning, in essence, the location of the existing power plants, keeping in mind demand conditions. The objective function has been formulated and solution has been achieved by indicating the extremes attained by the objective function in the lines of Arrow (1958), Kuhn and Tucker (1951), Saaty (1959) and Sasieni *et al.* (1959). In the final section of this paper some conclusive remarks in regard to minimum capital outlays on future power plant installations, vis-a-vis distribution and generation of power have been arrived at.

Objective Function

As is customary in a problem of mathematical programming, we define the objective function of the problem.

Let B_1, B_2 ... B_n be n input centres or demand centres of power and d_j ($j = 1$, 2 ... n) be the demands of these centres for a specific time.

Let A_1, A_2 ... A_m be the profile of existing power generation system, whose respective capacities of generation are

$$\bar{a}_1, \bar{a}_2, \ldots \bar{a}_m \text{ and } \bar{a}_i = 0 \text{ when } i > m. \quad \ldots(1)$$

Let us consider A_{m+1}, A_{m+2}, ... A_{m+K} be the proposed power generation centres having capacities a_i ($i = m + 1, m + 2, \ldots m + K$) which may be set up in different locations to satisfy the growing demand for power in future.

Let x_{ij} be the quantity of power supplied by the generating station i to the demand centre j so that we have

$$\sum_{i=1}^{m+K} x_{ij} = d_j \qquad \ldots(1)$$

where d_j is the variable demand at j.

We assume in every case

$$a_i \leqslant D_i \; (i = 1, 2, \ldots, m + k) \qquad \ldots(2)$$

where D_i represents the limitation of the capacity of power generation. The quantity of power supplied to all the centres from each source i is then expressed as

$$\sum_j x_{ij} = a_i \leqslant D_i. \qquad \ldots(3)$$

Let the expenditures be denoted by

$$\pi_{ij} = c_i + K_i + t_{ij} \qquad \ldots(4)$$

where (i) $c_i = \alpha_i + c_i(a_i)$ is the cost of unit of power produced at the power generation centre i, α_i be the fixed cost at the ith centre for per unit of generation and $c_i(a_i)$ be the variable cost due to the generation a_i unit. (ii) t_{ij} be the expenditure on transmission for power from ith source to jth station. (iii) K_i be the additional capital outlays to expand generation capacities at ith centre and it will be defined as $K_i = k_i(a_i - \bar{a}_i)$ where $\bar{a}_i = 0$ for $i > m$.

Then the overall outlays on supplier A_i satisfying the demand for power by the centre B_j will be $\pi_{ij}\, x_{ij}$. The total outlay z (for producing and supplying the power) is given by

$$z = \sum_{i=1}^{m+K} \sum_{j=1}^{n} \pi_{ij}\, x_{ij} \qquad \ldots(5)$$

This is the objective function which is to be minimized subject to the above conditions (1) to (3).

The problem will be solved if we indicate the number

$$a_i \leqslant D_i \; (i = 1, 2, \ldots, m + K)$$

and X^*_{ij} $(i = 1, 2, ..., m + K;\ j = 1, 2, ..., n)$.

Now substituting the value of π_{ij} in the objective function (5) we get the following expression

$$Z = \sum_{i=1}^{m+K} \sum_{j=1}^{n} (c_i + K_i + t_{ij})\, x_{ij}$$

$$= \sum_{i=1}^{m+K} \sum_{j=1}^{n} [\alpha_i + c_i(a_i) + k_i(a_i - \bar{a}_i) + t_{ij}]\, x_{ij}$$

$$= \sum_{i=1}^{m+K} [\alpha_i + c_i(a_i)] \left(\sum_{j=1}^{n} x_{ij}\right) + \sum_{i=1}^{m+K} k_i(a_i - \bar{a}_i) \left(\sum_{j=1}^{n} x_{ij}\right) + \sum_{i=1}^{m+K} \sum_{j=1}^{n} t_{ij} x_{ij}$$

$$= \sum_{i=1}^{m+K} [\{\alpha_i + c_i(a_i)\}\, a_i + k_i(a_i - \bar{a}_i)\,(a_i - \bar{a}_i)] + \sum_{i=1}^{m+K} \sum_{j=1}^{n} t_{ij} x_{ij}. \qquad ...(6)$$

This is a non-linear programming problem.

The Optimality Conditions

Let the set of numbers $X(x_{ij})$ $(i = 1, 2, ..., m + K)$ satisfying limitations (1) to (3) of the problem and $X^*(x^*_{ij})$ be such that it minimizes the objective function.

Solution

On the basis of the initial solution $X_1 = x^1_{ij}$ we should transfer to the optimum solution $X^* = x^*_{ij}$ after a finite number of steps. It follows from the form of restrictions (1) to (3) that the number of variables x_{ij} is not equal to zero as well as will not be less than zero.

Let $X_1 = (x^1_{ij})$ be the initial solution of the problem, then the objective function becomes

$$Z_1 = \sum_{i=1}^{m+K} [\{\alpha_i + c_i(a_i)\}\, a_i + k_i(a_i - \bar{a}_i)\,(a_i - \bar{a}_i)] + \sum_{i=1}^{m+K} \sum_{j=1}^{n} t_{ij} x^1_{ij} \qquad ...(7)$$

Now for the solution $X^1 = x^1_{ij}$ we consider two components. Then a transfer from the solution $X^1 = x^1_{ij}$ to the solution $X^2 = x^2_{ij}$ will be accomplished by reducing the component x^1_{ij} and increasing x^2_{ij} accordingly by an amount θ, where $0 \leqslant \theta \leqslant x^1_{ij}$. It is necessary here that

$$\Sigma\, x_{i_2 j} = a_{i_2} \leqslant D_{i_2} \qquad \ldots(8)$$

The value of the objective function z in solution X_2 is obtained as follows

$$Z_2 = \sum_{i=1}^{m+K} [\{\alpha_i + c_i(a_i)\}\, a_i + k_i(a_i - \bar{a}_i)\,(a_i - \bar{a}_i)] + \sum_{i=1}^{m+K} \sum_{j=1}^{n} t_{ij} x^1_{ij}$$
$$+ [\alpha_{i_1}(a_{i_1} - \theta) + c_{i_1}(a_{i_1} - \theta)\,(a_{i_1} - \theta)$$
$$+ k_{i_1}(a_{i_1} - \bar{a}_{i_1} - \theta)\,(a_{i_1} - \bar{a}_{i_1} - \theta)]$$
$$+ [\alpha_{i_2}(a_{i_2} + \theta) + c_{i_2}(a_{i_2} + \theta)\,(a_{i_2} + \theta)$$
$$+ k_{i_2}(a_{i_2} - \bar{a}_{i_2} + \theta)\,(a_{i_2} - \bar{a}_{i_2} + \theta)] + (t_{i_2 j} - t_{i_1 j})\,\theta. \qquad \ldots(9)$$

Let us assume

$$\alpha_{ij}^{(\pm\theta)} = \{\alpha_i + c_i(a_i \pm \theta)\}\,(a_i \pm \theta) + k_i(a_i - \bar{a}_i \pm \theta)\,(a_i - \bar{a}_i \pm \theta) \qquad \ldots(10)$$

where $\quad i = 1, 2, \ldots m + K; \quad j = 1, 2 \ldots n.$

Now we have defined the increment in objective function

$$\triangle\, Z^j_{i_1 i_2} = Z_2 - Z_1. \qquad \ldots(11)$$

Similarly we can find Z_3 from Z_2 by iteration and it will be

$$Z_3 = \sum_i [\{\alpha_i + c_i(a_i)\}\, d_i + k_i(a_i - \bar{a}_i)\,(a_i - \bar{a}_i)] + \sum_i \sum_j t_{ij} x^2_{ij}$$
$$+ [\alpha_{i_2}(a_{i_2} - \phi) + c_{i_2}(a_{i_2} - \phi)\,(a_{i_2} - \phi)$$
$$+ k_{i_2}(a_{i_2} - \bar{a}_{i_2} - \phi)\,(a_{i_2} - \bar{a}_{i_2} - \phi)]$$
$$+ [\alpha_{i_3}(a_{i_3} + \phi) + c_{i_3}(a_{i_3} + \phi)\,(a_{i_3} + \phi)$$
$$+ k_{i_3}(a_{i_3} - \bar{a}_{i_3} + \phi)(a_{i_3} - \bar{a}_{i_3} + \phi)] + (t_{i_3 j} - t_{i_2 j})\,\phi$$

where $\quad 0 \leqslant \phi \leqslant x^2_{ij} \quad$ and $\quad \triangle\, Z^j_{i_3 i_2} = Z_3 - Z_2$

if the value of $\triangle\, Z^j_{i_1 i_2}(\theta) \geqslant 0$ for all values of i_1 and i_2

where $\quad i_1, i_2 = 1, 2, \ldots, m + K; \; j = 1, 2, \ldots, n \;$ and $\; i_1 \neq i_2$

then the solution will be optimal in respect of the capacity of the plant A_2.

Then we can find the increment in the objective function for different values of i and the optimal solution can be derived from these increment values. Now if we consider the objective function z to be convex in the boundary of the region defined by the inequalities (1) to (3) then we have

$$\text{grad } Z = \frac{\partial Z}{\partial x_{ij}} = \left\{ \frac{\partial c_i(a_i)}{\partial x_{ij}} a_i + c_i(a_i) + \alpha_i + \frac{\partial k_i(a_i - \bar{a}_i)}{\partial x_{ij}} (a_i - \bar{a}_i) + k_i(a_i - \bar{a}_i) + t_{ij} \right\} \geqslant 0 \quad \ldots(12)$$

if $\frac{\partial c_i(a_i)}{\partial x_{ij}} \geqslant 0$ and $\frac{\partial k_i(a_i - \bar{a}_i)}{\partial x_{ij}} \geqslant 0$

The objective function z then reaches its extremum at the boundary of the region.

Concluding Remarks

While calculating K_i, we have three possible cases :

(i) If $a_i \leqslant \bar{a}_i$ with $i \leqslant m$ then $k_i = 0$.

It means the capital outlays for existing power plants will not increase and their capacity is sufficient to satisfy the demand.

(ii) If $a_i > \bar{a}_i$ with $i \leqslant m$ then $K_i = k_i(a_i - \bar{a}_i)$.

That is for existing power plant whose capacity is increasing, the capital outlay has to provide for this growth.

(iii) If $a_i > \bar{a}_i$ with $m < i \leqslant m + K$ then $K_i = k_i(a_i)$

It means the power plant that does not at present exist ($\bar{a}_i = 0$), but which is to be built, the capital outlay must be sufficient to provide the future power plants with their full capacity.

The solution to the problem gives the optimum plans for increase in the capacity of the power generating stations, for a particular period of time and also most economic system of distribution of power from the generation station to the demand centres. This also gives the minimum expenditure on production and distribution of power.

It should be remembered that the results of the above calculations are based on generalized figure of production cost, transportation cost and capital outlay and these results being also generalized in nature, they would be of effective use in integrated computation in regard to forecast planning.

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