ON THE EXISTENCE OF AFFINE MOTION IN A RECURRENT FINSLER SPACE

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In this paper the author has studied the affine motion in a recurrent Finsler space and obtained many important results.

1. INTRODUCTION

Let us consider an n-dimensional affinely connected Finsler space Fn (Rund 1959) with a positively homogeneous metric function $F(x^i, \dot{x}^i)$, $(i = 1, 2, ..., n)$ of degree one in its directional arguments $\dot{x}^i$'s. The fundamental metric tensor $g_{ij}(x, \dot{x})$ of the space is given by

$$g_{ij}(x, \dot{x}) \overset{def}{=} \tfrac{1}{2}\, \dot{\partial}_i \dot{\partial}_j\, F^2(x, \dot{x}). \qquad ...(1.1)$$

Let $T^i_j(x, \dot{x})$ be any tensor field depending upon both the positional and directional arguments. The covariant derivative of T^i_j with respect to x^k in the sense of Cartan is given by

$$T^i_{j\,|\,k} = \partial_k T^i_j - \dot{\partial}_m T^i_j\, G^m_k + T^h_j\, \Gamma^{*i}_{hk} - T^i_h\, \Gamma^{*h}_{jk} \qquad ...(1.2)$$

where $\Gamma^{*i}_{hk}(x, \dot{x})$ are called the Cartan's connection coefficients and satisfy the following relations :

$$\text{(a)} \quad \dot{\partial}_h \Gamma^{*i}_{jk}\, \dot{x}^h = 0, \qquad \text{(b)} \quad \Gamma^{*i}_{jk} = \Gamma^{*i}_{kj}\,. \qquad ...(1.3)$$

Involving the above covariant derivative, we have the following commutation formulae :

$$\dot{\partial}_h\, T^i_{j\,|\,k} - (\dot{\partial}_h T^i_j)_{\,|\,k} = T^s_j\, \dot{\partial}_s \Gamma^{*i}_{hk} - T^i_s\, \dot{\partial}_j \Gamma^{*s}_{hk} \qquad ...(1.4)$$

and

$$2T^i_{j\,|\,[hk]} = -\, \dot{\partial}_r T^i_j\, K^r_{shk}\, \dot{x}^s + T^s_j\, K^i_{shk} - T^i_s\, K^s_{jhk} \qquad ...(1.5)$$

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$2A_{[hk]} = A_{hk} - A_{kh}$.

where

$$K^i_{jhk}(x, \dot{x}) \overset{def}{=} 2\{\partial_{[k}\Gamma^{*i}_{h]j} - \dot{\partial}_s\Gamma^{*i}_{j[h}\; G^s_{k]} + \Gamma^{*i}_{s[h}\;\Gamma^{*s}_{h]j}\} \qquad \ldots(1.6)$$

is a curvature tensor and satisfies the following identities (Rund 1959)

$$K^i_{jhk} = -K^i_{jkh} \qquad \ldots(1.7)$$

and

$$K^i_{hjk|l} + K^i_{hkl|j} + K^i_{hlj|k} = -\dot{x}^s\{(\dot{\partial}_r\Gamma^{*i}_{hj})K^r_{skl} + (\dot{\partial}_r\Gamma^{*i}_{hk})K^r_{slj} + (\dot{\partial}_r\Gamma^{*i}_{hl})K^r_{sjk}\}. \qquad \ldots(1.8)$$

If the above curvature tensor $K^i_{hjk}(x, \dot{x})$ satisfies the relation

$$K^i_{hjk|s} = \gamma_s K^i_{hjk} \qquad \ldots(1.9)$$

where $\gamma_s(x)$ means a non-zero covariant recurrence vector, then the space is called a recurrent Finsler space.

Let us consider an infinitesimal point transformation

$$\bar{x}^i = x^i + v^i(x)\, dt \qquad \ldots(1.10)$$

where $v^i(x)$ is any vector field and dt an infinitesimal point constant. The above transformation considered at each point in the space is called an affine motion, when and only when, we have

$$\mathcal{L}v\, \Gamma^{*i}_{jk} = 0 \qquad \ldots(1.11)$$

where $\mathcal{L}v$ denotes the well-known Lie-derivative with respect to the infinitesimal transformation given above. In view of (1.10) and Cartan's covariant derivative the Lie-derivatives of $T^i_j(x, \dot{x})$ and connection coefficient $\Gamma^{*i}_{jk}(x, \dot{x})$ are given by (Yano 1957).

$$\mathcal{L}v\, T^i_j(x, \dot{x}) = T^i_{j|h} v^h + T^i_h v^h_{|j} - T^h_j v^i_{|h} + \dot{\partial}_h T^i_j\, v^h_{|r}\, \dot{x}^r \qquad \ldots(1.12)$$

and

$$\mathcal{L}v\, \Gamma^{*i}_{jk}(x, \dot{x}) = v^i_{|jk} + K^i_{jkh} v^h + \dot{\partial}_r \Gamma^{*i}_{jk}\, v^r_{|s}\, \dot{x}^s \qquad \ldots(1.13)$$

respectively.

We have the following commutation formulae :

$$\dot{\partial}_h(\mathcal{L}v\, T^i_j) - \mathcal{L}v\, \dot{\partial}_h T^i_j = 0 \qquad \ldots(1.14)$$

$$(\mathcal{L}v\, \Gamma^{*i}_{jk})_{|k} - (\mathcal{L}v\, \Gamma^{*i}_{kh})_{|j} = \mathcal{L}v\, K^{i}_{hjk} + 2\dot{x}^{s}\dot{\partial}_{r}\Gamma^{*i}_{h[j}\, \Gamma^{*r}_{k]s} \qquad \ldots(1.15)$$

and

$$(\mathcal{L}v\, T^{i}_{jk|l}) - (\mathcal{L}v\, T^{i}_{jk})_{|l} = \overset{s}{T}_{jk}\, \mathcal{L}v\, \Gamma^{*i}_{sl} - T^{i}_{sk}\, \mathcal{L}v\, \Gamma^{*s}_{jl} - T^{i}_{js}\, \mathcal{L}v\, \Gamma^{*s}_{kl}\,. \qquad \ldots(1.16)$$

Hence, for an infinitesimal affine motion (1.11), the two operators $\mathcal{L}v$ and $|\,k$ are commutative with each other. Remembering the equations (1.9) and (1.15), we get

$$\mathcal{L}v\, K^{i}_{hjk} = 0. \qquad \ldots(1.17)$$

Taking the Lie-derivative of the both sides of (1.9) and noting the equations (1.11), (1.16) and (1.17), we obtain

$$(\mathcal{L}v\, \gamma_s)\, K^{i}_{hjk} = 0. \qquad \ldots(1.18)$$

Consequently, if the space is non flat one, (i.e. $K^{i}_{hjk} \neq 0$), we have

$$\mathcal{L}v\, \gamma_s = 0 \qquad \ldots(1.19)$$

i.e. the recurrence vector γ_s of the space must be a Lie-invariant one.

In what follows, we shall study a recurrent Finsler space admitting an infinitesimal transformation $\bar{x}^i = x^i + v^i(x)\, dt$ which satisfies (1.19). We shall call such a restricted space, for brevity, an *SRFn*-space.

2. The Vanishing of $\mathcal{L}v\, K^{i}_{hjk}(x, \dot{x})$

First of all, let us prove the following :

Lemma 2.1 — In an *SRFn*-space, if the recurrence vector γ_s is a gradient one, we have $\gamma_s v^s$ = constant.

Proof : For brevity, let us put

$$\gamma_s v^s = \rho(x) \qquad \ldots(2.1)$$

then with the help of eqns. (1.12) and (1.19), we get

$$\mathcal{L}v\, \gamma_s = \gamma_{s|m} v^m + \gamma_m v^m_{\ |s}. \qquad \ldots(2.2)$$

In view of the assumption $\gamma_{s|m} = \gamma_{m|s}$, we have $\gamma_{|m} = 0$. This completes the proof.

By virtue of eqn. (1.12), the Lie-derivative of the curvature tensor field $K^{i}_{hjk}(x, \dot{x})$ is given by

$$\mathcal{L}v\, K^i_{hjk} = K^i_{hjk|s}\, v^s + K^i_{sjk}\, v^s_{|h} + K^i_{hsk}\, v^s_{|j} + K^i_{hjs}\, v^s_{|k}$$
$$- K^s_{hjk}\, v^i_{|s} + \dot{\partial}_s K^i_{hjk}\, v^s_{|r}\, \dot{x}^r \quad \text{...(2.3)}$$

which in view of (1.9) and (2.1) reduces to

$$\mathcal{L}v\, K^i_{hjk} = \rho K^i_{hjk} + K^i_{sjk}\, v^s_{|h} + K^i_{hsk}\, v^s_{|j} + K^i_{hjs}\, v^s_{|k}$$
$$- K^s_{hjk}\, v^i_{|s} + \dot{\partial}_s K^i_{hjk}\, v^s_{|r}\, \dot{x}^r. \quad \text{...(2.4)}$$

Applying the commutation formula (1.5) to the curvature tensor field $K^i_{hjk}(x, \dot{x})$, we get

$$2K^i_{hjk|[mn]} = -\dot{\partial}_r K^i_{hjk}\, K^r_{smn}\, \dot{x}^s + K^r_{hjk}\, K^i_{rmn}$$
$$- K^i_{rjk}\, K^r_{hmn} - K^i_{hrk}\, K^r_{jmn} - K^i_{hjr}\, K^r_{kmn}. \quad \text{...(2.5)}$$

Remembering the definition (1.9), the above relation yields

$$(\gamma_{m|n} - \gamma_{n|m})\, K^i_{hjk} = -\dot{\partial}_r K^i_{hjk}\, K^r_{smn}\, \dot{x}^s$$
$$+ K^r_{hjk}\, K^i_{rmn} - K^i_{rjk}\, K^r_{hmn} - K^i_{hrk}\, K^r_{jmn} - K^i_{hjr}\, K^r_{kmn}. \quad \text{...(2.6)}$$

Next, let us assume that ρ is not a constant. Then, from the Lemma (2.1), we can see

$$M_{mn}(x) \overset{def}{=} (\gamma_{m|n} - \gamma_{n|m}) \neq 0. \quad \text{...(2.7)}$$

If we take a suitable non-symmetric tensor t^{mn}, which satisfies the relation

$$K^i_{hmn}\, t^{mn} = v^i_{|h} \quad \text{...(2.8)}$$

then multiplying (2.6) by t^{mn} and summing over m and n, we obtain

$$K^i_{hjk}\, M_{mn} t^{mn} = -\dot{\partial}_r K^i_{hjk}\, v^r_{|s}\, \dot{x}^s - K^i_{sjk}\, v^s_{|h} - K^i_{hsk}\, v^s_{|j}$$
$$- K^i_{hjs}\, v^s_{|k} + K^s_{hjk}\, v^i_{|s}\,. \quad \text{...(2.9)}$$

Comparing the last equation with (2.4), we get

$$\mathcal{L}v\, K^i_{hjk} = (\rho - t^{mn} M_{mn})\, K^i_{hjk} \quad \text{...(2.10)}$$

which vanishes when and only when $\rho = t^{mn} M_{mn}$.

For $\rho =$ constant and $M_{mn} \neq 0$, from (2.4) and (2.6), we can construct the following indentity :

$$M_{mn}\mathcal{L}v K^i_{hjk} = K^s_{hjk}(\rho K^i_{smn} - M_{mn}v^i_{|s}) - K^i_{sjk}(\rho K^s_{hmn} - M_{mn}v^s_{|h}) - K^i_{hsk}(\rho K^s_{jmn} - M_{mn}v^s_{|j}) - K^i_{hjs}(\rho K^s_{kmn} - M_{mn}v^s_{|k}) - \dot{\partial}_r K^i_{hjk}(\rho K^r_{smn} - M_{mn}v^r_{|s})\dot{x}^s. \quad ...(2.11)$$

Thus, for $\mathcal{L}vk^i_{hjk} = 0$, from the above equation we can easily obtain (Takano 1966)

$$\rho K^i_{hjk} = M_{jk}v^i_{|h} \quad ...(2.12)$$

where v^i does not mean a parallel vector.

Thus, we put the

Definition 2.1 — An *SRFn*-space satisfying $\gamma_m v^m \neq$ const. is called a special one of the first kind.

Next, let us back again to the $\gamma_m v^m =$ constant of the foregoing Lemma (2.1). Then (2.6) is replaced by

$$-\dot{\partial}_r K^i_{hjk} K^r_{smn}\dot{x}^s + K^s_{hjk} K^i_{smn} - K^i_{sjk} K^s_{hmn} - K^i_{hsk} K^s_{jmn} - K^i_{hjs} K^s_{kmn} = 0. \quad ...(2.13)$$

Multiplying the last equation by t^{mn} and summing over m and n, we get

$$-\dot{\partial}_r K^i_{hjk} v^r_{|s}\dot{x}^s + K^s_{hjk} v^i_{|s} - K^i_{sjk} v^s_{|h} - K^j_{hsk} v^s_{|j} - K^i_{hjs} v^s_{|k} + K^s_{hjk} v^i_{|s} = 0 \quad ...(2.14)$$

where we have used (2.8).

Introducing (2.14) into the right-hand side of (2.4), we obtain

$$\mathcal{L}v K^i_{hjk} = \rho K^i_{hjk} \cdot \quad ...(2.15)$$

Hence, when the arbitrary constant ρ vanishes, we have $\mathcal{L}v K^i_{hjk} = 0$. We put the

Definition 2.2 — When $v^m\gamma_m =$ constant holds good, an *SRFn*-space is called a special one of the second kind.

Then, summarizing all the above results, we have the following theorems :

Theorem 2.1 — In a special *SRFn*-space of the first kind, if the space has the resolved curvature tensor K^{i}_{hjk} of the form (2.12), $\mathcal{L}v\, K^{i}_{hjk} = 0$ holds good.

Theorem 2.2 — In a special S_R *Fn* space of the second kind, if the arbitrary constant $\rho = \gamma_m v^m$ vanishes, we have $\mathcal{L}v\, K^{i}_{hjk} = 0$.

From the last theorem when λ_m vanishes i.e. $\lambda_m = 0$, we can also deduce the

Corollary 2.1 — In a symmetric Finsler space (i.e. $K^{i}_{hjk\,|\,r} = 0$), $\mathcal{L}v K^{i}_{hjk} = 0$ holds identically.

3. Complete Condition

We shall find a necessary and sufficient condition for (2.12). From the assumption (1.19), we have

$$\mathcal{L}v\, \gamma_m = \gamma_{m\,|\,s} v^s + (\gamma_s v^s)_{\,|\,m} - \gamma_{s\,|\,m} v^s = 0 \qquad \text{...(3.1)}$$

which by virtue of (2.1) and (2.7) reduces to

$$\rho_{\,|\,m} + M_{ms} v^s = 0. \qquad \text{...(3.2)}$$

In view of eqn. (1.12), the Lie-derivative of M_{mn} is given by

$$\mathcal{L}v\, M_{mn} = M_{mn\,|\,s} v^s + M_{sn} v^s_{\,|\,m} + M_{ms} v^s_{\,|\,n}\,. \qquad \text{...(3.3)}$$

Remembering the commutation formula (1.16), we get

$$\mathcal{L}v(\gamma_{m\,|\,n}) - (\mathcal{L}v\gamma_m)_{\,|\,n} = -\gamma_s \mathcal{L}v\, \Gamma^{*s}_{mn} \qquad \text{...(3.4)}$$

which by virtue of the equations (1.3*b*), (1.19) and (2.7), reduces to

$$\mathcal{L}v\, M_{mn} = 0. \qquad \text{...(3.5)}$$

Differentiating (2.6) covariantly with respect to x^s and using the equations (1.4), (1.9), (2.6) and (2.7), we obtain

$$M_{mn\,|\,s} K^{i}_{hjk} = \gamma_s M_{mn} K^{i}_{hjk} + K^{r}_{pmn}\, \dot{x}^p \{ K^{p}_{hjk}\, \dot{\partial}_p \Gamma^{*i}_{rs}$$
$$- K^{i}_{pjk}\, \dot{\partial}_r\, \Gamma^{*p}_{hs} - K^{i}_{hpk}\, \dot{\partial}_r \Gamma^{*p}_{js} - K^{i}_{hjp}\, \dot{\partial}_r \Gamma^{*p}_{ks} \}. \qquad \text{...(3.6)}$$

Transvecting the above equation by $\dot{x}^s$ and using (1.3*a*), we get after a little simplification

$$M_{mn\,|\,s} = \gamma_s M_{mn}. \qquad \text{...(3.7)}$$

Thus, with the help of the eqns. (3.3), (3.5) and (3.7), we obtain

$$\rho M_{mn} + M_{sn} v^s_{|m} + M_{ms} v^s_{|n} = 0. \qquad \ldots(3.8)$$

Next, from (3.2), we get

$$\rho_{|mn} - \rho_{|nm} = -(M_{ms} v^s)_{|n} + (M_{ns} v^s)_{|m} \qquad \ldots(3.9)$$

ρ being a non-constant scalar function, this becomes

$$M_{ms} v^s_{|n} + M_{sn} v^s_{|m} = -\gamma_n M_{ms} v^s + \gamma_m M_{ns} v^s \qquad \ldots(3.10)$$

where we have used (3.7) and $M_{ns} = -M_{sn}$. Introducing last relation into the left hand side of (3.8) and using eqn. (3.2), we get

$$\rho M_{mn} = -\gamma_n \rho_{|m} + \gamma_m \rho_{|n}. \qquad \ldots(3.11)$$

In view of equations (1.7) and (1.9), the identity (1.8) in an affinely connected Finsler space reduces to

$$\rho K^i_{hjk} = \gamma_k K^i_{hjl} v^l - \gamma_j K^i_{hkl} v^l. \qquad \ldots(3.12)$$

Hence, from (3.11) and (3.12), we can make the following identity :

$$\rho(\rho K^i_{hjk} - M_{jk} v^i_{|h}) = \gamma_k(\rho K^i_{hjl} v^l - \rho_{|j} v^i_{|h})$$

$$- \gamma_j(\rho K^i_{hkl} v^l - \rho_{|k} v^i_{|h}). \qquad \ldots(3.13)$$

Consequently (2.12) follows when and only when

$$\rho K^i_{hjl} v^l - \rho_{|j} v^i_{|h} = \gamma_j C^i_h \qquad \ldots(3.14)$$

where C^i_h means a suitable tensor. Multiplying the last relation by v^j and summing over j by virtue of $K^i_{hje} v^j v^l = 0$, $\rho_{|j} v^j = 0$ derived from (3.2) and (2.1), we get

$$\rho C^i_h = 0. \qquad \ldots(3.15)$$

Since $\rho \neq 0$, therefore, the above relation yields

$$C^i_h = 0. \qquad \ldots(3.16)$$

Therefore in view of (3.16), the equation (3.14) reduces to

$$K^i_{hjl} v^l + \rho_j v^i_{|h} = 0, \; (\rho_j = \rho_{|j} \mid \rho). \qquad \ldots(3.17)$$

In this way we have

Theorem 3.1 — In order that we have (2.12), (3.17) is necessary and sufficient.

Now, the last condition (3.17) suggests the concrete form of the tensor $t^{lm}(x, \dot{x})$ used in the first half of §2.

In fact, being $\rho_j \neq 0$, there exists a suitable vector ξ^m such that

$$\rho_m \xi^m = 1. \quad \ldots(3.18)$$

Then, by virtue of the above relation transvecting (3.17) by ξ^j, we get

$$v^i_{\mid h} = K^i_{hlj}\, v^l \xi^j. \quad \ldots(3.19)$$

If, we introduce t^{mn} by

$$t^{mn} = v^m \xi^n \quad \ldots(3.20)$$

then

$$M_{mn} t^{mn} = M_{mn} v^m \xi^n = \rho_{\mid n} \xi^n = \rho \cdot \rho_n \xi = \rho.^n$$

That is from (3.17) and (2.12), we obtain

$$\rho = M_{mn} t^{mn} \quad \ldots(3.21)$$

straightway. Therefore, (3.20) can be taken as concretely. Hence in order to have the concrete form t^{mn}, (3.17) should be taken as a basic condition in our theory. If this is done, we are able to have (2.12) always, so $\mathcal{L}v\, K^i_{hjk} = 0$ holds good. Thus, we have the next

Theorem 3.2 — If $v^i_{\mid h}$ will be introduced by (3.17), $\mathcal{L}v\, K^i_{hjk} = 0$ is satisfied identically.

4. Appendices

In a *SRFn*-space of the first kind, we shall show the existence of affine motion. For this purpose, let us take up (2.12) being equivalent to (3.17) which has been introduced for the purpose of getting the form of $v^i_{\mid h}$. In this case according to Theorem (2.1) or (3.2), we have $\mathcal{L}v\, K^i_{hjk} = 0$ identically, so $\mathcal{L}v\, \Gamma^{*i}_{jk} = 0$ ought to be considered. However, in what follows, we shall study this fact in detail.

In view of the equations (1.9) and (3.7) differentiating (2.12) covariantly with respect to x^m, we get

$$\rho_{\mid m} K^i_{hjk} = M_{jk} v^i_{\mid hm}. \quad \ldots(4.1)$$

Multiplying the above relation by v^k and using (3.2), we obtain

$$\rho_{|m} K^i_{hjk} v^k = -\rho_{|j} v^i_{|hm} \qquad \ldots(4.2)$$

which in view of the eqn. (3.17), reduces to

$$\rho_{|m} \rho_j v^i_{|h} = \rho_{|j} v^i_{|hm}. \qquad \ldots(4.3)$$

Since $\rho \neq$ const., the above relation yields

$$\rho_m v^i_{|h} = v^i_{|hm}.$$

Hence, with the help of eqns. (3.17) and (4.4), we obtain

$$v^i_{|hj} + K^i_{hjs} v^s = \rho_j v^i_{|h} - \rho_j v^i_{|h} = 0. \qquad \ldots(4.5)$$

Introducing the above relation into (1.13), we get

$$\mathcal{L}v\, \Gamma^{*i}_{jk} = \dot{\partial}_r \Gamma^{*i}_{jh} v^r_{|s} \dot{x}^s. \qquad \ldots(4.6)$$

Thus, we have

Theorem 4.1 — An *SRFn*-space satisfying $\mathcal{L}v\, \gamma_m = 0$, $\gamma_m v^m \neq$ const. and having the resolved curvature tensor K^i_{hjk} of the form (2.12), admits naturally a non-affine motion (i.e. $\mathcal{L}v\, \Gamma^{*i}_{jk} \neq 0$).

Secondly, let us consider the space of the second kind having $\rho = \gamma_m v^m = 0$. In this case according to Theorem 2.2, we have $\mathcal{L}v\, K^i_{hjk} = 0$ necessarily. Then, let us study the possibility of $\mathcal{L}v\, \Gamma^{*i}_{jk} = 0$. In an affinely connected space the Bianchi identity (1.8) takes the form

$$\gamma_j K^i_{hkl} v^l = -\gamma_k K^i_{hjl} v^l \qquad \ldots(4.7)$$

from which, taking case of $\gamma_j \neq 0$, we can put

$$K^i_{hkl} v^l = M^i_h \gamma_k. \qquad \ldots(4.8)$$

Now, being $\gamma_j \neq 0$, there exists a suitable vector ξ^m such that $\gamma_m \xi^m = 1$. Transvecting (4.8) by ξ^k, we get

$$K^i_{hkl} \xi^k v^l = M^i_h. \qquad \ldots(4.9)$$

Then introducing a non-symmetric tensor t^{kl} considered in the last half of §2 by $t^{kl} = v^k \xi^l$, from (4.8), we get

$$K^i_{hkl}\, t^{kl} = M^i_h \qquad \ldots(4.10)$$

which by virtue of (2.8) reduces to

$$v^i_{\,|h} = M^i_h\,. \qquad \ldots(4.11)$$

Consequetly (4.8), takes the form

$$K^i_{hkl}\, v^l = -\,\gamma_k v^i_{\,|h}. \qquad \ldots(4.12)$$

Introducing (4.12) into (1.13), we obtain

$$\mathcal{L}v\, \Gamma^{*i}_{jk} = v^i_{\,|jk} - \gamma_k v^i_{\,|j} + \dot{\partial}_s \Gamma^{*i}_{jk}\; v^s_{\,|r}\, \dot{x}^r. \qquad \ldots(4.13)$$

Therefore, when $v^i_{\,|j}$ denotes a recurrent tensor with respect to the gradient recurrent vector γ_k, we have

$$\mathcal{L}v\, \Gamma^{*i}_{jk} = \dot{\partial}_s \Gamma^{*i}_{jk}\; v^s_{\,|r}\, \dot{x}^r. \qquad \ldots(4.14)$$

Thus, we have

Theorem 4.2 — An *SRFn*-space defined by a gradient recurrence vector and characterised by $\mathcal{L}v\, \gamma_m = 0$ and $\gamma_m v^m = 0$, admits a non-affine motion when the space has a recurrent tensor $v^i_{\,|j}$ with respect to γ_k.

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