STEADY FLOW BETWEEN A ROTATING AND A POROUS STATIONARY DISC IN THE PRESENCE OF TRANSVERSE MAGNETIC FIELD

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In this paper an attempt has been made to study the steady axisymmetric flow of a viscous incompressible electrically conducting fluid between two coaxial non-conducting infinite discs, one rotating and the other stationary, with uniform suction at the stationary disc, in the presence of transverse magnetic field. Numerical solutions of Navier-Stokes equations are obtained for arbitrary suction Reynolds number by series expansion method. The velocity profiles have been shown graphically.

INTRODUCTION

Several authors have discussed the steady flow of an incompressible viscous fluid between two infinite rotating discs theoretically as well as experimentally. It was first discussed by Batchelor (1951) generalizing the solutions of Von Karman (1921) and Bodewadt (1940) for the flow over a single infinite rotating disc. Stewartson (1953) obtained approximate solutions for large and small values of Reynolds numbers. The numerical solutions for this problem have been obtained by Lance and Rogers (1962), Pearson (1965) and Mellor *et. al.* (1968).

Stuart (1954) examined the flow due to a rotating single disc of infinite radius with uniform suction at the disc and obtained numerical solutions for small and asymptotic solutions for large values of the suction parameter. Rogers and Lance (1960) found that the steady flow is possible only when there is suction at the disc and its effect is to prevent the boundary layer from leaving the disc and attaching itself to the disturbing agency at infinity.

Narayana and Rudraiah (1972) studied axisymmetric steady flow of a viscous incompressible fluid between two coaxial circular discs, one rotating and the other stationary, with uniform suction at the stationary disc. They obtained the solutions separately for small and large suction Reynolds numbers.

In the present paper we discuss axisymmetric steady flow of a viscous incompressible electrically conducting fluid between two non-conducting coaxial circular discs, one rotating constantly and the other stationary, in the presence of the transverse magnetic field. There is constant suction at the stationary disc. The problem is solved numerically for arbitrary suction Reynolds number. Two functions defining axial and azimuthal velocities are respectively expanded in terms of power series in ξ, the dimensionless coordinate normal to the discs. Three of the coefficients are determined using the boundary conditions at the stationary disc and the conditions at the other disc give three infinite series for another three coefficients which have been solved using iteration procedure. The remaining coefficients are expressed in terms of those using two recursion relations. The velocity profiles have been shown graphically.

Equations of Motion

Let us suppose that u, v and w are the velocities in r, θ and z directions respectively. Let us suppose that axes of the discs coincide with z-axis. And origin of the cylindrical coordinates is taken at stationary disc. Under transverse magnetic field of constant intensity B, equations of motion, leaving induced magnetic fields, for steady flow are

$$u \frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left[\nabla^2 u - \frac{u}{r^2} - \frac{2}{r^2}\frac{\partial v}{\partial \theta}\right] - \frac{\sigma B^2 u}{\rho} \qquad \text{...(1)}$$

$$u \frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \theta} + w\frac{\partial v}{\partial z} + \frac{uv}{r} = -\frac{1}{\rho}\frac{1}{r}\frac{\partial p}{\partial \theta} + \nu\left[\nabla^2 v + \frac{2}{r^2}\frac{\partial u}{\partial \theta} - \frac{v}{r^2}\right] - \frac{\sigma B^2 v}{\rho} \qquad \text{...(2)}$$

$$u \frac{\partial w}{\partial r} + \frac{v}{r}\frac{\partial w}{\partial \theta} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\nabla^2 w \qquad \text{...(3)}$$

where

$$\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2},$$

ν, ρ and σ are respectively the coefficient of kinematic viscosity, density and the conductivity of the fluid considered.

Analysis

Let us define, following set of velocity distribution,

$$z = l\xi$$

$$u = \frac{Ur}{2l} f'(\xi)$$

$$v = \Omega r g(\xi)$$

$$w = -Uf(\xi) \qquad \text{...(4)}$$

which satisfies the equation of continuity. Here l, Ω and U are respectively the gap width, angular velocity of the rotating disc and the suction velocity at the stationary disc; and a dash denotes differentiation with respect to ξ. The equations of motion, under these transformations, reduce to:

$$\frac{f'''}{R} + ff'' - \frac{f'^2}{2} + \frac{2R_1^2}{R^2} g^2 - \frac{l^2\sigma B^2}{\mu} \cdot \frac{f'}{R} = \frac{2l^2}{\rho U^2 r} \frac{\partial p}{\partial r} \quad \ldots(5)$$

$$\frac{g''}{R} + fg' - gf' - \frac{l^2\sigma B^2}{\mu} \cdot \frac{g}{R} = 0 \quad \ldots(6)$$

and

$$\frac{f''}{R} + ff' = -\frac{l}{\rho U^2} \frac{\partial p}{\partial z} \quad \ldots(7)$$

where $R = \frac{Ul}{\nu}$ (suction Reynolds number) and $R_1 = \frac{\Omega l^2}{\nu}$ (rotational Reynolds number).

Boundary conditions for functions f and g are

$$\left.\begin{aligned} f'(0) = 0, \quad g(0) = 0, \quad f(0) = 1 \\ f'(1) = 0, \quad g(1) = 1, \quad f(1) = 0. \end{aligned}\right\} \quad \ldots(8)$$

Let

$$\frac{2R_1^2}{R^2} = \lambda \text{ and } \frac{l^2\sigma B^2}{\mu} = m^2$$

where λ and m are real constants.

Then eqns. (5) and (6) become,

$$\frac{f'''}{R} + ff'' - \frac{f'^2}{2} + \lambda g^2 - m^2 \frac{f'}{R} = \frac{2l^2}{\rho U^2 r} \frac{\partial p}{\partial r} \quad \ldots(9)$$

$$\frac{g''}{R} + fg' - gf' - m^2 \frac{g}{R} = 0. \quad \ldots(10)$$

SOLUTION

From eqn. (7), the pressure gradient $\frac{\partial p}{\partial z}$ is independent of r, hence $\frac{\partial p}{\partial r}$ in eqn. (9) is independent of ξ. Therefore the right-hand side of (9) should be a constant. Differentiating equation (9) once with respect to ξ, we get

$$\frac{f^{IV}}{R} + ff''' + 2\lambda gg' - m^2 \frac{f''}{R} = 0. \quad \ldots(11)$$

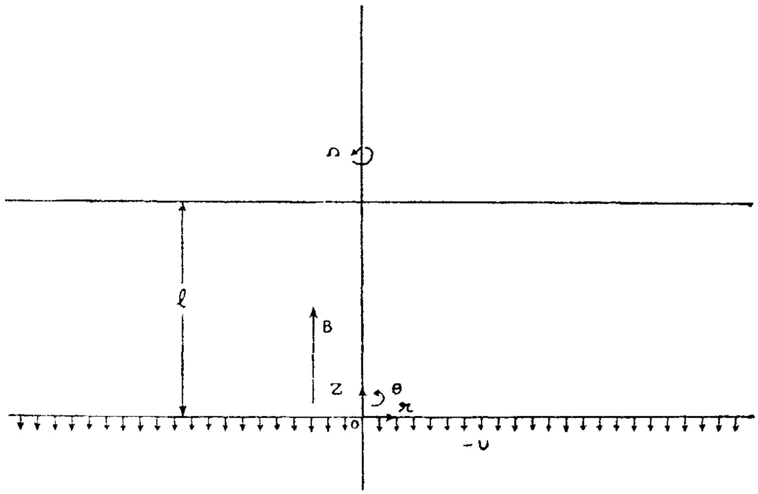

FIG. 1. Schematic diagram and coordinates.

Let us assume following power series for f and g :

$$f = \frac{1}{R}\,[B_0 + B_1\xi + B_2\xi^2 + \dots + B_n\xi^n + \dots] \qquad \dots(12)$$

$$g = \frac{1}{R}\,[D_0 + D_1\xi + D_2\xi^2 + \dots + D_n\xi^n + \dots] \qquad \dots(13)$$

where B's and D's are independent of ξ and may be functions of R_1 and R. Substituting eqns. (12) and (13) into eqns. (11) and (10) and collecting coefficients of each ξ^n and equating them to zero, we see that the general equations in the two sets satisfy the following two recursion relations.

$$B_{n+4} = -\sum_{k=0}^{n} \frac{(n-K+1)}{(n+4)(n+3)(n+2)(n+1)} \times \{(n-K+3)(n-K+2)\,B_{n-K+3}B_K + 2\lambda D_{n-K+1}D_K\} + \frac{m^2 B_{n+2}}{(n+4)(n+3)} \qquad \dots(14)$$

$$D_{n+2} = \sum_{k=0}^{n} \frac{(n-K+1)}{(n+2)(n+1)}\,\{B_{n-K+1}D_K - D_{n-K+1}B_K\} + \frac{m^2 D_n}{(n+2)(n+1)}. \qquad \dots(15)$$

It can be seen from relations (14) and (15) that coefficients $B_4, B_5, \dots, B_n, \dots$ and $D_2, D_3, \dots, D_n, \dots$ can be expressed in terms of B_0, B_1, B_2, B_3 and D_0 and D_1, which can be determined using the six boundary conditions.

The conditions at $\xi = 0$ give,

$$B_0 = R, B_1 = 0, D_0 = 0.$$

The remaining conditions give the following three infinite series.

$$B_0 + B_2 + B_3 + \ldots + B_n + \ldots = 0 \qquad \ldots(16)$$

$$2B_2 + 3B_3 + 4B_4 + \ldots + nB_n + \ldots = 0 \qquad \ldots(17)$$

$$D_1 + D_2 + D_3 + \ldots + D_n + \ldots = R. \qquad \ldots(18)$$

Thus, in view of eqns. (14) and (15) all B's and D's in eqns. (12) and (13) and hence the solutions for $f(\xi)$ and $g(\xi)$ are known in principle, if B_2, B_3 and D_1 are known which are to be obtained from eqns. (16) - (18).

The three infinite series (16) - (18) are algebraic, non-linear simultaneous equations for B_2, B_3 and D_1 and are solved here by the method of iteration.

For the purpose of iteration, eqns. (16) - (18) are written as,

$$B_2 = -(B_0 + B_3 + B_4 + \ldots + B_n + \ldots) \qquad \ldots(19)$$

$$B_3 = -\tfrac{1}{3}(2B_2 + 4B_4 + 5B_5 + \ldots + nB_n + \ldots) \qquad \ldots(20)$$

$$D_1 = R - (D_2 + D_3 + D_4 + \ldots + D_n + \ldots) \qquad \ldots(21)$$

To start with only terms up to $n = 6$ are considered and B_3 and D_1 are expressed in terms of B_2. For given values of R_1 and R and an assumed value of B_2, B_3 and D_1 are determined. Equations (19) - (21) are iterated with these as initial values. Once convergence for a particular n is achieved to a desired accuracy, n is increased till the values of B_2, B_3 and D_1 do not substantially change for a change in n. Generally n is given the values 6, 10, 15, 20, 30, 50, ... It is found that for low values of R and R_1, $n = 20$ would be sufficient. For $R > 1.25$ and $R_1 > 5$ the iteration even for $n = 10$ is found to be unstable. Once B_2, B_3 and D_1 are obtained, the remaining B's and D's are obtained using (14) and (15) and f, f' and g are obtained using (12) and (13).

To extend the solution for values of $R > 1.25$ and $R_1 > 5$ the following procedure using the Runge Kutta method is employed.

Equations (11) and (10) are

$$f^{IV} + Rff''' + 4\frac{R_1^2}{R}gg' - m^2 f'' = 0 \qquad \ldots(22)$$

$$g'' + R(fg' - gf') - m^2 g = 0 \qquad \ldots(23)$$

which are to be solved under the boundary conditions (8), therefore differential eqns. (22) and (23) are reduced to six first order differential equations. For given values of R_1 and R the conditions $f''(0) = \alpha_1, f'''(0) = \alpha_2$ and $g'(0) = \alpha_3$ required at the stationary disc, are roughly estimated from those known for previous values of R_1 and R (to start with these are estimated from the above series expansion method) and a generalized Newton Raphson iterative procedure is employed to get accurate values.

CONCLUSIONS

The effect of R on velocity distributions for fixed R_1 is seen in Figs. 2 and 3.

In Fig. 2 it is seen that g rises steeply. Figure 3 shows that f and f' are not affected much, but corresponding velocities w and u should be affected considerably being non-dimensionalized with respect to the suction velocity U. Thus when the suction velocity is increased, there is an increase in all the velocity components. Also, it is clear from Figs. 2 and 3 that velocity components are unidirectional, hence no cells are formed.

In Figs. 4 and 5, R is kept fixed and effect of R_1 on the velocity distributions is seen. Figure 4 shows that there is a strong radial inflow in the lower half and an outflow in the upper half of the gap. For small R_1 the flow is radially inward. It is quite evident from Figure 5, that for large R_1, there is large axial flow near and towards the rotating disc. Also the radial and axial velocity components change directions and points of zero velocities occur showing cellular type of flow.

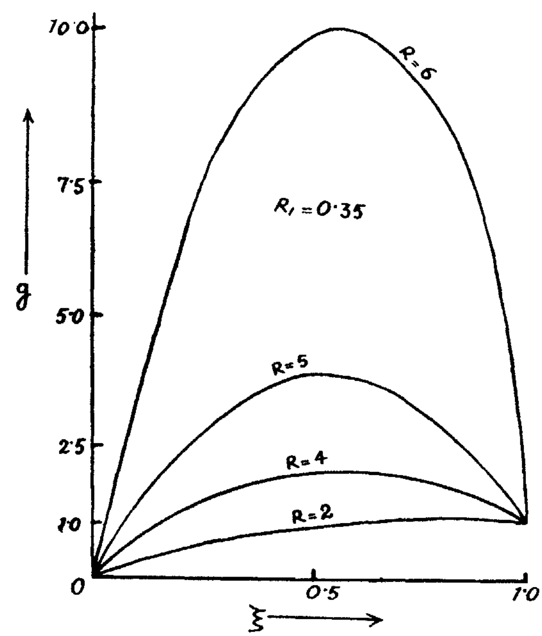

FIG. 2. Azimuthal velocity distribution for $R_1 = 0.35$ and $R = 2, 4, 5$ and 6.

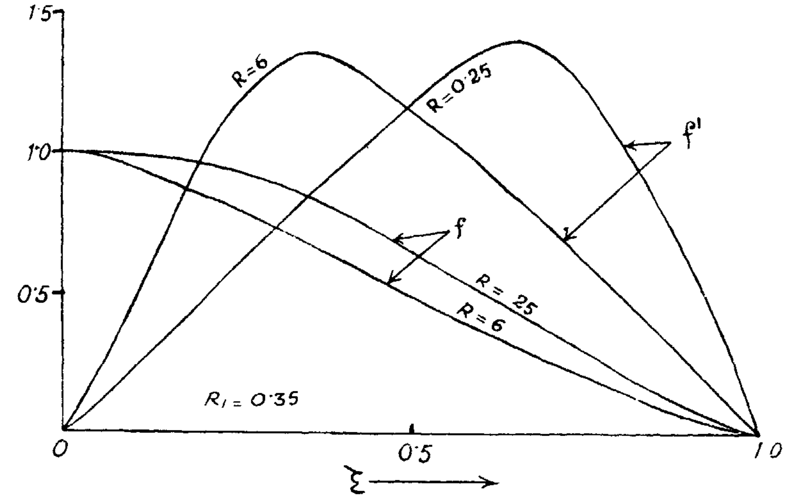

FIG. 3. Radial (f') and axial (f) velocity distributions for $R_1 = 0.35$, and $R = 0.25$ and 6.

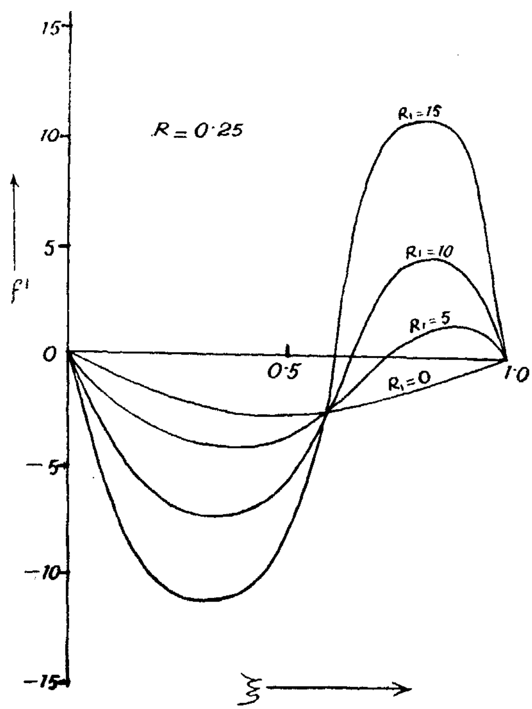

FIG. 4. Radial velocity distribution for $R = 0.25$ and $R_1 = 0, 5, 10, 15$.

The comparison of the velocity profiles with those obtained by Narayana and Rudraiah shows that there has been slight increase in the axial velocity and decrease in radial velocity.

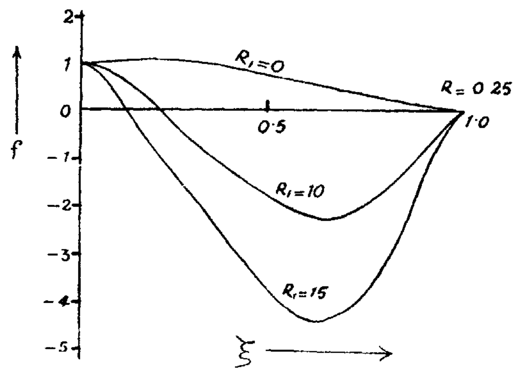

FIG. 5. Axial velocity distribution for $R = 0.25$ and $R_1 = 0$, 10 and 15.

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