OPERATIONAL REPRESENTATIONS OF THE GENERALIZED LAURICELLA FUNCTION OF SEVERAL VARIABLES

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The operational representations of the generalized Lauricella function of several variables defined by Srivastava and Daoust (1969) and, the generalized H-function of two variables defined by Munot and Kalla (1971) are given in the present note.

1. INTRODUCTION

The following notations will be employed throughout this note.

$(a)_n = a, a+1, \ldots, a+n-1.$

$\{B^{(n)}\}$ stands for $B^{(1)}, B^{(2)}, \ldots, B^{(n)}$.

$[(a); \{\theta^{(n)}\}]$ stands for A-parameters

$[a_1; \{\theta_1^{(n)}\}], [a_2; \{\theta_2^{(n)}\}], \ldots, [a_A; \{\theta_A^{(n)}\}]$.

$\{[(b^{(n)}); \phi^{(n)}]\}$ stands for $B \times n$ parameters

$[b_1^{(1)}; \phi_1^{(1)}], \quad [b_1^{(2)}; \quad \phi_1^{(2)}], \ldots, [b_1^{(n)}; \quad \phi_1^{(n)}],$

$[b_2^{(1)}; \quad \phi_2^{(1)}], \ldots [b_2^{(2)}; \quad \phi_2^{(2)}], \ldots, [b_2^{(n)}; \quad \phi_2^{(n)}],$

$\vdots$

$[b_B^{(1)}; \quad \phi_B^{(1)}], \quad [b_B^{(2)}; \quad \phi_B^{(2)}], \ldots, [b_B^{(n)}; \quad \phi_B^{(n)}].$

$$\Omega_j = z_j^2 \frac{d}{dz_j}, \qquad \omega_j = y_j^2 \frac{d}{dy_j},$$

$$\lambda_j^{(i)} = (x_j^{(i)})^2 \frac{d}{dx_j^{(i)}}, \qquad \mu_j^{(i)} = (w_j^{(i)})^2 \frac{d}{dw_j^{(i)}}. \qquad \ldots(1.1)$$

The object of the present note is to obtain the operational representations for the generalized Lauricella function of several variables defined by Srivastava and Daoust [1969, pp. 454 (4.1)]

$$F\,{}^{A:\,\{B^{(n)}\}}_{C:\,\{D^{(n)}\}}\left(\begin{matrix}[(a):\ \{\theta^{(n)}\}]:\ \{[(b^{(n)}):\ \phi^{(n)}]\} \\ [(c):\ \{\psi^{(n)}\}]:\ \{[(d^{(n)}):\ \delta^{(n)}]\}\end{matrix};\ z_1,\ldots,z_n\right)$$

$$= \sum_{m_1,\ldots,m_n=D}^{\infty} \frac{\prod_{j=1}^{A}(a_j)_{\sum_{i=1}^{n} m_i\theta_j^{(i)}} \prod_{i=1}^{n}\prod_{j=1}^{B^{(i)}}(b_j^{(i)})_{m_i\phi_j^{(i)}}}{\prod_{j=1}^{C}(c_j)_{\sum_{i=1}^{n} m_i\psi_j^{(i)}} \prod_{i=1}^{n}\prod_{j=1}^{D^{(i)}}(d_j^{(i)})_{m_i\delta_j^{(i)}}}$$

$$\times \frac{z_1^{m_1}\, z_2^{m_2} \ldots z_n^{m_n}}{m_1!\, m_2! \ldots m_n!} \qquad \ldots(1.2)$$

where the symbol Π stands for the product, and the empty product is interpreted as unity, for convergent θ's, ϕ's, ψ's and δ's are positive such that $\Delta_i \geqslant 0$; the equality holds when $|z_i| < \rho_i$, $i = 1, \ldots, n$, with ρ_i defined by eqn. (5.3) of Srivastava and Daoust (1972, pp. 159).

The following operational representations for the generalized Lauricella function of several variables developed in Section 1 are

$$X_1 \left\{ \frac{\prod_{j=1}^{A} z_j^{a_j} \prod_{i=s+1}^{n-1} \prod_{j=1}^{B^{(i)}} (x_j^{(i)})^{b_j^{(i)}}}{\prod_{j=1}^{C} y_j^{c_j-1} \prod_{i=1}^{n-1} \prod_{j=1}^{D^{(i)}} (w_j^{(i)})^{d_j^{(i)}-1}}\ {}^{A\ B^{(n)}}\psi_{C:\,D^{(n)}} \right.$$

$$\left. \times \left(\begin{matrix}[(a):\ \theta^{(n)}],\ [(b^{(n)}):\ \phi^{(n)}]; \\ [(c):\ \psi^{(n)}],\ [(d^{(n)}):\ \delta^{(n)}];\end{matrix}\ \prod_{j=1}^{A} z_j^{\theta_j^{(n)}}\ \prod_{j=1}^{C} y_j^{-\psi_j^{(n)}}\right)\right\}$$

$$= \prod_{i=1}^{s}\prod_{j=1}^{B^{(i)}} \Gamma(b_j^{(i)}) \prod_{j=1}^{A} (\Gamma(a_j)\, z_j^{a_j}) \prod_{i=1}^{n-1}\prod_{j=1}^{B^{(i)}} (x_j^{(i)})^{b_j^{(i)}} \prod_{j=1}^{B^{(n)}} \Gamma(b_j^{(n)})$$

$$\times \left[\prod_{j=1}^{D^{(n)}} \Gamma(d_j^{(n)}) \prod_{j=1}^{C} (\Gamma(c_j)\, y_j^{c_j-1}) \prod_{i=1}^{n-1}\prod_{j=1}^{D^{(i)}} (w_j^{(i)})^{d_j^{(i)}-1}\right]^{-1} F\,{}^{A:\{B^{(n)}\}}_{C:\{D^{(n)}\}}\begin{pmatrix} Z_1 \\ \cdot \\ \cdot \\ \cdot \\ Z_n \end{pmatrix} \qquad \ldots(1.3)$$

where θ's, ϕ's, ψ's and δ's are positive integers, $\Delta_i \geqslant 0$, for $i = 1, \ldots, n$, and $|Z_i| < \rho_i$, for $i = 1, \ldots, n$;

$$Z_i = \begin{cases} \prod_{j=1}^{A} z_j^{\theta_j^{(i)}} \prod_{j=1}^{B^{(i)}} (x_j^{(i)})^{\phi_j^{(i)}} \prod_{j=1}^{C} y_j^{-\psi_j^{(i)}} \prod_{j=1}^{D^{(i)}} (w_j^{(i)})^{-\delta_j^{(i)}}, \\ \qquad\qquad i = 1 + s, \ldots, n-1; \\ \prod_{j=1}^{A} z_j^{\theta_j^{(i)}} \prod_{j=1}^{C} y_j^{-\psi_j^{(i)}} \prod_{j=1}^{D^{(i)}} (w_j^{(i)})^{-\delta_j^{(i)}}, \quad i = 1, \ldots, s; \\ \prod_{j=1}^{A} z_j^{\theta_j^{(i)}} \prod_{j=1}^{C} y_j^{-\psi_j^{(i)}}, \quad i = n. \end{cases} \qquad \ldots(1.4)$$

$$X_1 = \prod_{i=1}^{s} B^{(i)} \psi_0 \left(\begin{matrix} [(b^{(i)}): \phi^{(i)}]; \\ \cdots\cdots \,; \end{matrix} \prod_{j=1}^{A} \Omega_j^{\theta_j^{(i)}} \prod_{j=1}^{C} (-\omega_j)^{-\psi_j^{(i)}} \prod_{j=1}^{D^{(i)}} (-\lambda_j)^{-\delta_j^{(i)}} \right)$$
$$\times \prod_{i=1+s}^{n-1} {}_0\psi_0 \left(\begin{matrix} \cdots; \\ \cdots; \end{matrix} \prod_{j=1}^{A} \Omega_j^{\theta_j^{(i)}} \prod_{j=1}^{C} (-\omega_j)^{-\psi_j^{(i)}} \prod_{j=1}^{B^{(i)}} (\mu_j)^{\phi_j^{(i)}} \right.$$
$$\left. \times \prod_{j=1}^{D^{(i)}} (-\lambda_j)^{-\delta_j^{(i)}} \right) \qquad \ldots(1.5)$$

and ${}_p\psi_q$ is Wright's function (1935) of the generalized hypergeometric function defined by

$${}_p\psi_q \left(\begin{matrix} [(a), \alpha]; \\ [(b), \beta]; \end{matrix} \; x \right) = \sum_{m=0}^{\infty} \frac{\prod_{j=1}^{p} \Gamma(a_j + \alpha_j m)\, x^m}{\prod_{j=1}^{q} \Gamma(b_j + \beta_j m)\, m!} \qquad \ldots(1.6)$$

$$X_2 \left\{ \frac{\prod_{j=1}^{A} z_j^{a_j} \prod_{i=1+s}^{n-1} \prod_{j=1}^{B^{(i)}} (x_j^{(i)})^{b_j^{(i)}}}{\prod_{j=1}^{C} y_j^{c_j-1} \prod_{i=1}^{s} \prod_{j=1}^{D^{(i)}} (x_j^{(i)})^{d_j^{(i)}-1}} \; {}_{A+B^{(n)}}\psi_{C+D^{(n)}} \right.$$
$$\left. \times \left(\begin{matrix} [(b^{(n)}): \phi^{(n)}],\ [(a): \theta^{(n)}]; \\ [(d^{(n)}): \delta^{(n)}],\ [(c): \psi^{(n)}]; \end{matrix} \prod_{j=1}^{A} z_j^{\theta_j^{(n)}} \prod_{j=1}^{C} y_j^{-\psi_j^{(n)}} \right) \right\}$$
$$= \frac{\prod_{j=1}^{A} z_j^{a_j} \prod_{i=1}^{n-1} \prod_{j=1}^{B^{(i)}} (x_j^{(i)})^{b_j^{(i)}} \prod_{i=1}^{s} \prod_{j=1}^{B^{(i)}} \Gamma(b_j^{(i)}) \prod_{j=1}^{A} \Gamma(a_j) \prod_{j=1}^{B^{(n)}} \Gamma(b_j^{(n)})}{\prod_{j=1}^{C} y_j^{c_j-1} \prod_{i=1}^{s} \prod_{j=1}^{D^{(i)}} (x_j^{(i)})^{d_j^{(i)}-1} \prod_{i=s+1}^{n} \prod_{j=1}^{D^{(i)}} \Gamma(d_j^{(i)}) \prod_{j=1}^{C} \Gamma(c_j)}$$
$$\times F_{C:\ \{D^{(n)}\}}^{A:\ \{B^{(n)}\}} \begin{pmatrix} Z_1 \\ \vdots \\ Z_n \end{pmatrix}, \qquad \ldots(1.7)$$

where θ's, ϕ's, ψ's and δ's are +ve integers, $\Delta_i \geqslant 0$, $|Z_i| < \rho_i$ for $i = 1, \ldots, n$;

$$Z_i \equiv \begin{cases} \prod_{j=1}^{A} z_j^{\theta_j^{(i)}} \prod_{j=1}^{C} y_j^{-\psi_j^{(i)}} \prod_{j=1}^{D^{(i)}} (x_j^{(i)})^{-\delta_j^{(i)}}\ ; & i = 1, \ldots, s, \\ \prod_{j=1}^{B^{(i)}} (x_j^{(i)})^{\phi_j^{(i)}} \prod_{j=1}^{C} (y_j)^{-\psi_j^{(i)}} \prod_{j=1}^{A} z_j^{\theta_j^{(i)}}\ ; & i = 1+s, \ldots, n-1\ ; \\ \prod_{j=1}^{A} z_j^{\theta_j^{(i)}} \prod_{j=1}^{C} y_j^{-\psi_j^{(i)}}\ ; & i = n, \end{cases} \qquad \ldots(1.8)$$

and the operator X_2

$$X_2 = \prod_{i=1}^{s} {}_{B^{(i)}}\psi_0 \left(\begin{matrix} [(b^{(i)}): \phi^{(i)}]; \\ \cdots\cdots \,; \end{matrix} \prod_{j=1}^{A} \Omega_j^{\theta_j^{(i)}} \prod_{j=1}^{C} (-\omega_j)^{-\psi_j^{(i)}} \prod_{j=1}^{D^{(i)}} (-\lambda^{(i)})^{-\delta_j^{(i)}} \right)$$
$$\times \prod_{i=1+s}^{n-1} {}_0\psi_{D^{(i)}} \left(\begin{matrix} \cdots\cdots \,; \\ [(d^{(i)}): \delta^{(i)}]; \end{matrix} \prod_{j=1}^{A} \Omega_j^{\theta_j^{(i)}} \prod_{j=1}^{C} (-\omega_j)^{-\psi_j^{(i)}} \prod_{j=1}^{B^{(i)}} (\lambda_j^{(i)})^{\phi_j^{(i)}} \right) \qquad \ldots(1.9)$$

$$X_3 \left\{ \prod_{j=1}^{A} z_j^{e_j} \prod_{j=1}^{C} y_j^{1-e_j} \; {}_{A+B^{(n)}}\psi_{C+D^{(n)}} \right.$$

$$\left. \times \left(\begin{matrix} [(a)\colon \theta^{(n)}],\ [(b^{(n)});\ \phi^{(n)}]; \\ [(c)\colon \psi^{(n)}],\ [(d^{(n)})\colon \delta^{(n)}]; \end{matrix} \; \frac{\prod_{j=1}^{A} z_j^{\theta_j^{(n)}}}{\prod_{j=1}^{C} y_j^{+\psi_j^{(n)}}} \right) \right\}$$

$$= \frac{\prod_{i=1}^{n} \prod_{j=1}^{B^{(i)}} \Gamma(b_j^{(i)}) \prod_{j=1}^{A} (\Gamma(a_j)\, z_j^{e_j})}{\prod_{i=1}^{n} \prod_{j=1}^{D^{(i)}} \Gamma(d_j^{(i)}) \prod_{j=1}^{C} (\Gamma(c_j)\, y_j^{e_j-1})} \; F^{A\colon \{B^{(n)}\}}_{C\colon \{D^{(n)}\}} \begin{pmatrix} Z_1 \\ \cdot \\ \cdot \\ \cdot \\ Z_n \end{pmatrix} \qquad \ldots(1.10)$$

where θ's, ϕ's, ψ's and δ's are +ve integers; the right-hand side is convergent, if $\Delta_i \geqslant 0$, $|Z_i| < \rho_i$, for $i = 1, \ldots, n$;

$$Z_i = \prod_{j=1}^{A} z_j^{\theta_j^{(i)}} \prod_{j=1}^{C} y_j^{-\psi_j^{(i)}}, \quad i = 1, \ldots, n; \qquad \ldots(1.11)$$

and

$$X_3 = \prod_{i=1}^{n-1} {}_{B^{(i)}}\psi_{D^{(i)}} \left(\begin{matrix} [(b^{(i)})\colon \phi^{(i)}]; \\ [(d^{(i)})\colon \delta^{(i)}]; \end{matrix} \; \prod_{j=1}^{A} \Omega_j^{\theta_j^{(i)}} \prod_{j=1}^{C} \omega_j^{-\psi_j^{(i)}} \right) \qquad \ldots(1.12)$$

PROOF: To prove (1.3), we start with the L.H.S. of (1.3). On expressing ${}_p\psi_q$-functions, also occurring in X_1, in terms of power series by means of (1.6) and after some adjustment, differentiating by the simple properties of differential operators

$$\left(z^2 \frac{d}{dz}\right)^n z^a = (a)_n z^{a+n}, \; \left(x^2 \frac{d}{dx}\right)^{-n} (x^{1-a}) = \frac{(-1)^n x^{1-a-n}}{(a)_n}, \qquad \ldots(1.13)$$

after some adjustment, it is found that

$$\frac{\prod_{i=1}^{s} \prod_{j=1}^{B} (x_j^{(i)})^{+b_j^{(i)}} \prod_{j=1}^{A} z_j^{a_j}}{\prod_{j=1}^{C} y_j^{e_j-1} \prod_{i=1}^{n} \prod_{j=1}^{D^{(i)}} (w_j^{(i)})^{d_j^{(i)}-1}} \sum_{m_1, m_2, \ldots, m_n=0}^{\infty}$$

$$\times \frac{\prod_{j=1}^{A} \Gamma(a_j + \sum_{i=1}^{n} \theta_j^{(i)} m_i) \prod_{i=s+1}^{n-1} \prod_{j=1}^{B^{(i)}} \Gamma(b_j^{(i)}) \prod_{i=1}^{n} \prod_{j=1}^{B^{(i)}} \Gamma(b_j^{(i)} + \phi_j^{(i)} m_i)}{\prod_{i=1}^{n} \prod_{j=1}^{D^{(i)}} \Gamma(d_j^{(i)} + \delta_j^{(i)} m_i)\; m_1! \cdots m_n!\; Z_1^{-m_1} \cdots Z_n^{-m_n}}$$

$$\ldots(1.14)$$

The R.H.S. of (1.3) now immediately follows from (1.14), by using (1.2)

2. Particular Cases

If we set α for $\phi^{(1)}$, γ for $\phi^{(2)}$, β for $\delta^{(1)}$, δ for $\delta^{(2)}$, θ for $\theta^{(1)}$ and $\theta^{(2)}$, ϕ for $\psi^{(1)}$ and $\psi^{(2)}$; $n = 2$, $s = 1$ in (1·3) and make use of the following property of

$$F_{C:\{D^{(n)}\}}^{A:\{B^{(n)}\}}\begin{pmatrix} Z_1 \\ \cdot \\ \cdot \\ \cdot \\ Z_n \end{pmatrix}$$

$$\underset{z_3,\ldots,z_n \to 0}{Lt} F_{F:B,D}^{E:A,C}\begin{pmatrix}[(e):\theta,\theta],:\ [(a):\alpha],\ [(c):\gamma]; \\ [(f):\phi,\phi]:\ [(b):\beta],\ [(d):\delta];\end{pmatrix} z_1,\ldots,z_n\Bigg)$$

$$= \frac{\prod_{j=1}^{B}\Gamma(b_j)\prod_{j=1}^{D}\Gamma(d_j)\prod_{j=1}^{F}\Gamma(f_j)}{\prod_{j=1}^{E}\Gamma(e_j)\prod_{j=1}^{A}\Gamma(a_j)\prod_{j=1}^{C}\Gamma(c_j)}$$

$$\times H_{E,[A:C],F,[B+1:D+1]}^{E,A,C,1,1}\left[\begin{matrix}-z_1\\-z_2\end{matrix}\middle|\begin{matrix}(e,\theta)\\(a,\alpha);\ (c,\gamma)\\(f,\phi)\\(0,1),(b,\beta);\ (0,1),(d,\delta)\end{matrix}\right], \qquad \ldots(2.1)$$

where (e,θ) denotes a sequence of E-parameters $(e_1,\theta_1),\ldots,(e_E,\theta_E)$; and, $H\begin{bmatrix}x\\y\end{bmatrix}$ is the generalized H-function of two variables (Munot and Kalla 1971, pp. 69), we obtain

$${}_A\psi_0\begin{pmatrix}[(a):\alpha]; \\ \ldots\ ;\end{pmatrix}\prod_{j=1}^{E}\Omega_j^{\theta_j}\prod_{j=1}^{F}(-\omega_j)^{-\phi_j}\prod_{j=1}^{B}(-\lambda_j)^{-\beta_j}\Bigg)$$

$$\times\left\{\frac{\prod_{j=1}^{E}z_j^{e_j}}{\prod_{j=1}^{F}y_j^{f_j-1}\prod_{j=1}^{B}(w_j)^{b_j-1}}\right.$$

$$\left.\times\ {}_{E+C}\psi_{F+D}\begin{pmatrix}[(e):\theta],\ [(c):\gamma], \\ [(f):\phi],\ [(d):\delta],\end{pmatrix}\prod_{j=1}^{A}z_j^{\theta_j}\prod_{j=1}^{C}y_j^{-\phi_j}\Bigg)\right\}$$

$$= \frac{\prod_{j=1}^{A}x_j^{a_j}\prod_{j=1}^{E}z_j^{e_j}\prod_{j=1}^{B}\Gamma(b_j)}{\prod_{j=1}^{F}y_j^{f_j-1}\prod_{j=1}^{B}(w_j)^{b_j-1}}$$

$$\times H_{E,[A:C],F,[1+B:1+D]}^{E,A,C,1,1}\left[\begin{matrix}-Z_1\\-Z_2\end{matrix}\middle|\begin{matrix}(e,\theta)\\(a,\alpha);\ (c,\gamma)\\(f,\phi)\\(0,1),(b,\beta);\ (0,1),(d,\delta)\end{matrix}\right] \qquad \ldots(2.2)$$

where

$$Z_1 = \prod_{j=1}^{E} z_j^{\theta_j} \prod_{j=1}^{F} y_j^{-\phi_j} \prod_{j=1}^{B} w_j^{-\beta_j},$$

and

$$Z_2 = \prod_{j=1}^{E} z_j^{\theta_j} \prod_{j=1}^{F} y_j^{-\phi_j}.$$

On taking $s = 0$, $n = 2$ in (1.3), we obtain

$$\exp\left[+ \prod_{j=1}^{A} \Omega_j^{\theta_j^{(1)}} \prod_{j=1}^{B^{(1)}} (\mu_j^{(1)})^{\phi_j^{(1)}} \prod_{j=1}^{C} (-\omega_j)^{-\psi_j^{(1)}} \prod_{j=1}^{D^{(1)}} (-\lambda_j^{(1)})^{-\delta_j^{(1)}}\right]$$

$$\times \left\{ \frac{\prod_{j=1}^{A} z_j^{a_j} \prod_{j=1}^{B^{(1)}} (x_j^{(1)})^{b_j^{(1)}}}{\prod_{j=1}^{C} y_j^{c_j-1} \prod_{j=1}^{D^{(1)}} (w_j^{(1)})^{d_j^{(1)}-1}} \, {}_{A+B^{(2)}}\psi_{C+D^{(2)}} \right.$$

$$\left. \times \left[\begin{matrix} [(a)\colon \theta^{(1)}],\ [(b^{(2)})\colon \phi^{(2)}]; \\ [(c)\colon \psi^{(1)}],\ [(d^{(2)})\colon \delta^{(2)}]; \end{matrix} \prod_{j=1}^{A} z_j^{\theta_j^{(1)}} \prod_{j=1}^{C} y_j^{-\psi_j^{(1)}}\right] \right\}$$

$$= \frac{\prod_{j=1}^{A} z_j^{a_j} \prod_{j=1}^{B^{(1)}} (x_j^{(1)})^{b_j^{(1)}}}{\prod_{j=1}^{C} y_j^{c_j-1} \prod_{j=1}^{D^{(1)}} (w_j^{(1)})^{d_j^{(1)}-1}} H^{A,\, B^{(1)},\, B^{(2)},\, 1,\, 1}_{A,\, [B^{(1)}\colon B^{(2)}],\, C,\, [1+D^{(1)}\colon 1+D^{(2)}]}$$

$$\times \left[\begin{matrix} -Z_1 \\ -Z_2 \end{matrix} \middle| \begin{matrix} (a, \theta^{(1)}) \\ (b^{(1)}, \phi^{(1)});\ (b^{(2)}, \phi^{(2)}) \\ (c, \psi^{(1)}) \\ (0, 1), (d^{(1)}, \delta^{(1)});\ (0, 1), (d^{(2)}, \delta^{(2)}) \end{matrix}\right] \qquad \ldots(2.3)$$

where $\Delta_i \geqslant 0$ and the equality holds when $|z_i| < \rho_i$, $i = 1, 2$; with ρ_i defined by eqn. (5.3) of Srivastava and Daoust (1972) in which

$$Z_1 = \prod_{j=1}^{A} z_j^{\theta_j^{(1)}} \prod_{j=1}^{B^{(1)}} (x_j^{(1)})^{\phi_j^{(1)}} \prod_{j=1}^{C} (y_j)^{-\psi_j^{(1)}} \prod_{j=1}^{D^{(1)}} (w_j)^{-\delta_j^{(1)}},$$

and

$$Z_2 = \prod_{j=1}^{A} z_j^{\theta_j^{(1)}} \prod_{j=1}^{C} (y_j)^{-\phi_j^{(1)}}.$$

Eqns. (2.2) and (2.3) represent an operational representation for the generalized H-function of two variables defined earlier by Munot and Kalla (1971) and Saxena (1971).

It is interesting to observe that the results for the operational representations of the Lauricella functions of n-variables, Kampé de Fériet function of two variables and confluent hypergeometric functions in several arguments can be obtained as special cases of our results. The results, given by Khan (1969, 1973) and Chandel (1969) recently, also follow as particular cases.

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