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INFLUENCE OF TRANSVERSE SHEAR AND ROTATORY INERTIA ON AXISYMMETRIC VIBRATIONS OF POLAR ORTHOTROPIC ANNULAR PLATES OF PARABOLICALLY VARYING THICKNESS

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The present paper deals with the free axisymmetric vibrations of polar orthotropic annular plates of non-uniform thickness under Mindlin's shear theory of plates. The coupled differential equations governing the transverse motion have been solved by the method of Chebyshev polynomials. Frequencies, mode shapes and moments have been computed for the first three modes of vibration for different plate parameters. A comparison of numerical results with those obtained by classical theory has been presented and the influence of the shear deformation and rotatory inertia has been clarified.

INTRODUCTION

Structural components of varying thickness are highly favoured these days due to economy, safety and durability considerations. Plates of variable thickness are often encountered in engineering applications and their use in machine design, nuclear reactor technology, naval structures and acoustical components is quite common. In the recent past, considerable attention has been devoted to the study of static and dynamic behaviour of isotropic, non-uniform plates of different geometries, references (Lal 1979, Banerjee 1979, Laura *et al.* 1979, Rao and Prasad 1975, Gupta and Lal 1980) to name a few. As regards the work on anisotropic, circular/annular plates, only one investigation by Soni and Amba Rao (1975) on vibrations of orthotropic circular plates of variable thickness has been found by the present authors. However, the static and dynamic behaviour of orthotropic circular/annular plates of uniform thickness have been considered by a number of workers. Out of these a few important ones are reported in references (Pardoen 1974, Greenberg and Stavsky 1978, Ginesu *et al.* 1979, Vijayakumar and Joga Rao 1971, Kirmser *et al.* 1972).

The object of the present paper is to investigate the effects of the shear deformation and the rotatory inertia on the axially symmetric free vibrations of polar orthotropic annular plates of parabolically varying thickness. The inclusion of rotatory inertia and transverse shear alongwith orthotropy and variable thickness further complicates the governing differential equations, with the result that coupled equations are obtained. These have been solved by Chebyshev collocation technique. The frequency determinants have been obtained for three cases of inner edge clamped and the outer edge either clamped or simply supported or free. Frequencies, mode shapes and moments have been computed for the first three modes of vibration for different plate parameters. The numerical calculations have been made with the values for fibre reinforced plastic (boron epoxy), taking it as an example of a polar orthotropic material. A comparison of results with that obtained from classical plate theory leads to interesting conclusions.

2. Equation of Motion

Taking into account both the shear deformation and rotatory inertia effects, the differential equations, for the free axisymmetric motion of homogeneous and cylindrically (polar) orthotropic circular or annular plate of thickness $h = h(r)$, referred to the polar coordinates (r, θ) are (Mindlin 1951, Soni and Amba Rao 1975, Deresiwicz and Mindlin 1955)

$$\frac{\partial M_r}{\partial r} + \frac{M_r - M_\theta}{r} - Q_r = \frac{\rho h^3}{12} \frac{\partial^2 \psi_r}{\partial t^2} \qquad \ldots(1)$$

$$\frac{1}{r} Q_r + \frac{\partial Q_r}{\partial r} = \rho h \frac{\partial^2 w}{\partial t^2} \qquad \ldots(2)$$

where t is the time, w the transverse deflection, ρ the mass density per unit volume, and ψ_r the angular rotation of the normal to the neutral surface in radial direction. The moment and shear resultants are

$$M_r = D_r \left(\frac{\partial \psi_r}{\partial r} + \frac{\nu_\theta}{r} \psi_r \right), \; M_\theta = D_\theta \left(\frac{\psi_r}{r} + \nu_r \frac{\partial \psi_r}{\partial r} \right)$$

$$Q_r = k_s G_{r\theta} h \left(\psi_r + \frac{\partial w}{\partial r} \right)$$

where $D_r = E_r h^3/12(1 - \nu_r \nu_\theta)$, $D_\theta = E_\theta h^3/12(1 - \nu_r \nu_\theta)$ are the flexural rigidities; E_r, E_θ, ν_r, ν_θ and $G_{r\theta}$ are the elastic constants in proper directions with $\nu_r E_\theta = E_r \nu_\theta$; and $k_s (= \pi^2/12)$ is an averaging shear coefficient.

For harmonic vibrations

$$w(r, t) = \bar{w}(r)\, e^{i\omega t} \text{ and } \psi_r(r, t) = \psi(r)\, e^{i\omega t} \qquad \ldots(3)$$

where ω is the radian frequency. Equations (1) and (2) now reduce to

$$\phi - 12 k_s G_{r\theta} h \frac{(1 - \nu_r \nu_\theta)}{E_r} \left(\psi + \frac{d\bar{w}}{dr} \right) = 0 \qquad \ldots(4)$$

$$\frac{d}{dr} [r(\phi)] + 12 \rho r h \frac{(1 - \nu_r \nu_\theta)}{E_r} \bar{w} = 0 \qquad \ldots(5)$$

where

$$\phi = h^3\left(\frac{d^2\psi}{dr^2} + \frac{1}{r}\frac{d\psi}{dr} - \frac{\psi}{r^2}\right) + 3h^2\frac{dh}{dr}\left(\frac{d\psi}{dr} + \frac{\nu_\theta}{r}\psi\right)$$
$$+ \rho h^3\frac{(1 - \nu_r\nu_\theta)\,\omega^2}{E_r}\psi.$$

Consider an annular plate of inner and outer peripheral radii b and a, respectively. Out of many research efforts made on the free vibrations of non-uniform plates, there are several (Gupta and Lal 1980, Olson and Hazell 1979, Jain and Soni 1973, Tomar and Gupta 1976) in which the variation of thickness is parabolic, and these appear to be of practical importance. Therefore, the discussion here is confined to the case in which the thickness variation in radial direction is given by

$$\bar{h} = h_0(1 - \alpha x^2) \qquad \ldots(6)$$

where $h_0 = \bar{h}\mid_{x=0}$, α is the taper parameter and

$$x = r/a,\quad W = \bar{w}/a,\quad \bar{h} = h/a \qquad \ldots(7)$$

are the dimensionless variables.

Substitution of the relations (6) and (7) into equations (4) and (5) and the elimination of the displacement function W from the resulting equations leads to a fourth order linear homogeneous differential equation with variable coefficients which can be written as

$$\sum_{i=0}^{4} A_i\frac{d^i\psi}{dx^i} = 0 \qquad \ldots(8)$$

where

$$A_4 = (1 - \alpha x^2)^2,\ A_3 = (2/x)(1 - \alpha x^2)(1 - 9\alpha x^2)$$

$$A_2 = (1/x^2)\Big[A_4\Big(\Omega^2x^2\Big\langle 1 + \frac{1}{K_0}\Big\rangle - 2 - p\Big)$$
$$-2\alpha x^2(1 - \alpha x^2)\langle 3\nu_\theta + 19\rangle + 48\alpha^2x^2\Big]$$

$$A_1 = (1/x^3)\Big[A_4\Big(\Omega^2x^2\Big\langle 1 + \frac{1}{K_0}\Big\rangle + 3p\Big) - 2\alpha x^2(1 - \alpha x^2)\Big\langle 3\nu_\theta - 5p$$
$$+ \Omega^2x^2\Big(5 + \frac{3}{K_0}\Big)\Big\rangle + 12\alpha^2x^4(3\nu_\theta + 7)\Big]$$

$$A_0 = x^{-4}\Big[A_4\Big(\Omega^2x^2\Big\langle(\Omega^2x^2 - p)\frac{1}{K_0} - 1\Big\rangle - 3p\Big) - 2\alpha x^2(1 - \alpha x^2)\Big(5p - 3\nu_\theta$$
$$+ \Omega^2x^2\Big\langle 5 + \frac{3\nu_\theta}{K_0}\Big\rangle\Big) + 12\alpha^2x^4(3\nu_\theta - p + \Omega^2x^2) - (\Omega^2/I)\,x^4\Big]$$

$$\Omega^2 = \rho a^2 \omega^2 (1 - \nu_r \nu_\theta)/E_r,\ K_0 = k_s G_{r\theta}\ (1 - \nu_r \nu_\theta)/E_r,$$

$$p = E_\theta / E_r \quad \text{and} \quad I = h_0^2/12.$$

In terms of ψ, we can write W as

$$W(x) = -\frac{I}{\Omega^2}\left[\sum_{i=0}^{3} B_i \frac{d^i\psi}{dx^i}\right] \qquad \ldots(9)$$

where

$$B_3 = A_4,\ B_2 = (2/x)(1 - \alpha x^2)(1 - 7\alpha x^2)$$

$$B_1 = (1/x^2)[B_3(\Omega^2 x^2 - p) - 6\alpha x^2(1 - \alpha x^2)(3 + \nu_\theta) + 24\alpha^2 x^4]$$

$$B_0 = (1/x^3)[B_3(\Omega^2 x^2 + p) + 6\alpha x^2(1 - \alpha x^2)(p - \Omega^2 x^2 - \nu_\theta) + 24\nu_\theta \alpha^2 x^4].$$

Let $\epsilon = b/a$. Then the solution of eqn. (8) together with the boundary conditions at the inner and outer edges of the plate constitutes a well defined two-point boundary value problem in the range (ϵ, 1). This has been solved by Chebyshev collocation technique.

3. Method of Solution

Let us take a linear transformation

$$2x = (1 - \epsilon) y + (1 + \epsilon) \qquad \ldots(10)$$

to transform the range $\epsilon \leqslant x \leqslant 1$ into the applicability range $-1 \leqslant y \leqslant 1$ of the present technique. In terms of y, eqns. (8) and (9) become

$$\sum_{i=0}^{4} V_i \frac{d^i\psi}{dy^i} = 0 \qquad \ldots(11)$$

and

$$W(y) = -(I/\Omega^2) \sum_{i=0}^{3} W_i \frac{d^i\psi}{dy^i} \qquad \ldots(12)$$

where $V_i = (2/(1 - \epsilon))^i A_i$, $W_i = (2/(1 - \epsilon))^i B_i$, $i = 0, 1, 2, 3, 4$.

Proceeding as in Gupta and Lal (1980), let us assume

$$\frac{d^4\psi}{dy^4} = \sum_{k=0}^{m-5} c_{k+5} T_k \qquad \ldots(13)$$

where $T_k (k = 0, 1, 2, ..., m - 5)$ are the Chebyshev polynomials.

Successive integrations of eqn. (13) lead to

$$\psi = c_1 + c_2 T_1 + c_3 T_1^1 + c_4 T_1^2 + \sum_{k=0}^{m-5} c_{k+5} T_k^4 \qquad \text{...(14)}$$

where $c_j (j = 1, 2, ..., m)$ are unknown constants and T_k^i represents the ith integral of T_k.

Substitution of ψ and its derivatives in eqn. (11) gives an equation in terms of the T's and the unknown constants c's. The satisfaction of this resultant equation at $(m - 4)$ collocation points given by

$$y_k = \cos\left(\frac{2k + 1}{m - 4} \frac{\pi}{2}\right), \quad k = 0, 1, ..., m - 5 \qquad \text{...(15)}$$

provides a set of $(m - 4)$ equations in terms of unknowns c_j $(j = 1, 2, ..., m)$, which can be written in the matrix form as

$$[M] [C] = [0] \qquad \text{...(16)}$$

where M and C are the matrices of order $(m - 4) \times m$ and $m \times 1$, respectively.

4. Boundary Conditions

The following three combinations of boundary conditions have been taken into consideration:

(i) $C - C$: clamped at both the inner and outer edges;
(ii) $C - S$: clamped at the inner and simply supported at the outer edge;
(iii) $C - F$: clamped at the inner and free at the outer edge.

The relations which should be satisfied at a clamped, at a simply supported, and at a free edge, are

$$W = \psi = 0; \; W = \frac{2}{1 - \epsilon} \frac{d\psi}{dy} + \frac{\nu_\theta}{x} \psi = 0;$$

and

$$\frac{2}{1 - \epsilon} \frac{d\psi}{dy} + \frac{\nu_\theta}{x} \psi = \psi + \frac{2}{1 - \epsilon} \frac{dW}{dy} = 0, \text{ respectively.}$$

5. Characteristic Equations

Applying the $C - C$ boundary condition at $y = -1$ and $y = 1$, a set of four homogeneous equations is obtained. These equations together with field equations (16) give a complete set of m equations in m unknowns, which can be denoted by

$$[M/Bcc] [C] = [0] \qquad \text{...(17)}$$

where B_{CC} is a matrix of order $4 \times m$.

For a non-trivial solution of eqn. (17), the frequency determinant must vanish and hence we get

$$| M/B_{CC} | = 0. \quad \text{...(18)}$$

Similarly, for $C - S$ and $C - F$ boundary conditions frequency determinants can be written as

$$| M/B_{CS} | = 0 \quad \text{...(19)}$$

$$| M/B_{CF} | = 0 \quad \text{...(20)}$$

6. Numerical Results and Discussion

Equations (18) - (20) are transcendental in the frequency parameter Ω and can be solved numerically for specific plate parameters. In the work reported here, the frequency parameters for the first three modes of vibration were computed for $\alpha = 0.5(-0.2) -0.5$, $\epsilon = 0.3, 0.5$ and $h_0 = 0.01, 0.05, 0.1, 0.2$ for $C-C$, $C-S$ and $C-F$ boundary conditions. For computations, the number of collocation points m was taken as 12 since further increase in m, does not improve the values even in the third place of decimal. Calculations were carried out with double precision arithmetic (16 significant digits) on the IBM 360/44 Computer. The numerical values of the elastic constants used for the plate material are taken from (Soni and Amba Rao 1975)

$$E_r = 8.0 \times 10^6 \text{ lb m}^{-2}, E_\theta = 2.7 \times 10^6 \text{ lb m}^{-2},$$

$$\nu_r = 0.25, \qquad G_{r\theta} = 1.25 \times 10^6 \text{ lb m}^{-2}.$$

During computations, it was found that the difference between the frequency parameters for $C - C$ and $C - S$ plates obtained by shear theory (ST) and classical plate theory (CPT) is appreciable even in the second mode for $\epsilon = 0.3$, $h_0 = 0.05$, $-0.5 \leqslant \alpha \leqslant 0.5$ and hence higher values of $h_0 > 0.1$ have not been considered. In case of $C - F$ plate, however, this difference for $h_0 < 0.1$, $\alpha \geqslant -0.3$, $\epsilon = 0.3$, is small, and hence a higher value of $h_0 = 0.2$ has been considered. Thus the plate parameters for $C - C$ and $C - S$ plates have been chosen as $\epsilon = 0.3, 0.5$, $h_0 = 0.01, 0.05, 0.1$ and for $C - F$ plate $\epsilon = 0.5$, $h_0 = 0.05, 0.1, 0.2$ to study the effects of rotatory inertia and transverse shear on the natural frequencies. The corresponding values are given in Tables I–V. A study of these tables and Fig. 1 bring out the following conclusions:

(i) The effects of shear and rotatory inertia are more pronounced in case of $C - C$ plate as compared to $C - S$ and $C - F$ plates for the same plate parmeters.

(ii) The values of frequency parameter predicted by CPT become less and less accurate as the taper parameter α decreases.

(iii) The difference $(\Omega_C - \Omega_S)$ increases with the increase in radii ratio ϵ where Ω_C and Ω_S denotes the values of frequency parameter obtained by CPT and ST, respectively.

(iv) The difference $(\Omega_C - \Omega_S)$ increases with the increase in h_0.

(v) The difference $(\Omega_C - \Omega_S)$ increases with the increase in mode number.

(vi) The plots for Ω_C for $C - C$, $C - S$ and $C - F$ plates show almost a linear variation with taper constant α. However, it is not so for Ω_S.

The Figs. 2(a, b, c) exhibit the plots for the frequency parameters Ω_S and Ω_C versus h_0 for $\alpha = -0.5$, $\epsilon = 0.3, 0.5$. The reason for choosing this value of α lies in

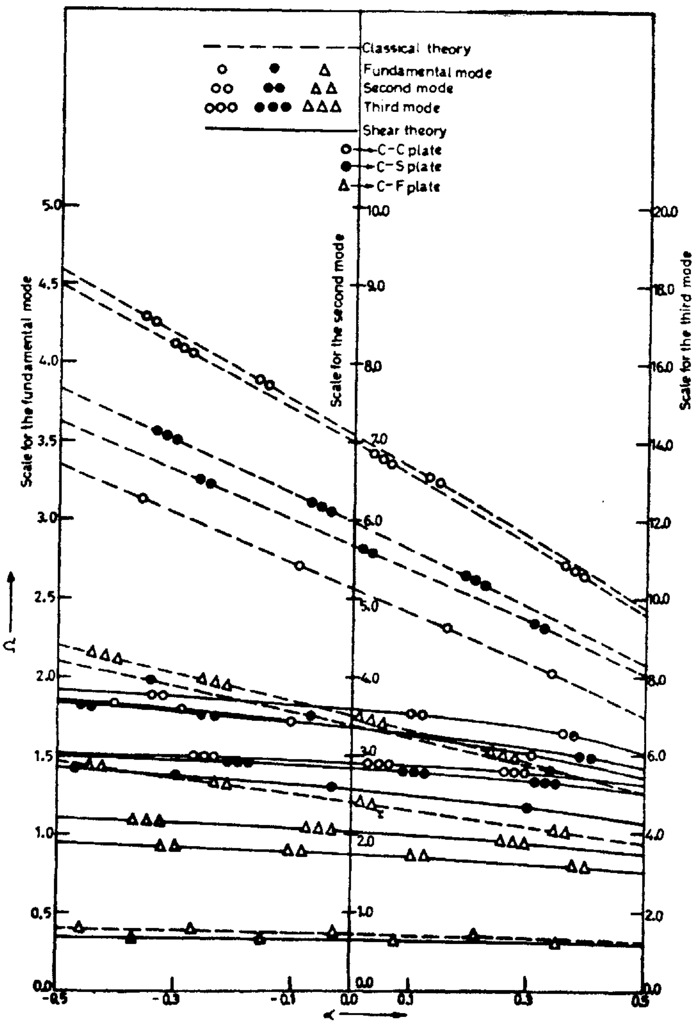

FIG. 1. Variation of natural frequencies with taper parameter for $h_0=0.1$, $\epsilon=0.5$ for the first three modes of vibration.

the fact that the error is maximum for it and thus it helps in deciding the applicability range of CPT.

The CPT predicts satisfactory results for $C - C$ boundary condition up to second mode provided $\alpha \geqslant 0.5$, $\epsilon = 0.3$ and $h_0 \leqslant 0.05$, and errors up to 10 per cent are neglected. For $\epsilon = 0.5, \alpha \geqslant 0.5, h_0 \leqslant 0.05$, the CPT results are reliable only in the first mode neglecting error up to 7 per cent. Thus for $C - C$ plate the CPT fails to predict reasonably good results even in the first mode for $\alpha < 0$ and $h_0 \geqslant 0.05$. In case of $C - S$ plate, the CPT gives satisfactory results for both $\epsilon = 0.3, 0.5$, $h_0 = 0.05$ for $\alpha \geqslant 0.5$ up to second mode while for $-0.3 < \alpha < 0.5$ only first mode will be reasonable when an error of 10 per cent is permissible. For $C - F$ plate, the CPT predicts the results satisfactorily up to second mode for $\epsilon = 0.3, \alpha \geqslant -0.3$, even for $h_0 = 0.1$ while for $\epsilon = 0.5$ it is not so [clear from Fig. 2(c)]. These discussions show that ϵ and α both play an important role in the effects of transverse shear and rotatory inertia. The CPT fails for modes higher than second in general for $h_0 \geqslant 0.05$.

In Figs. 3 and 4, normalised displacements and moments have been exhibited for $\alpha = \pm 0.5$ and $\epsilon = 0.5$ for the first three modes of vibration for all the three boundary conditions. Only curves for shear theory are shown in the figures as the displacements and moments computed from the ST and CPT differ in respective magnitudes only in

TABLE I

Values of frequency parameter Ω, for $C - C$ plate

$b/a = 0.3$

Mode	Values of α	Ω_C		Ω_S		
		*	$h_0 = 0.1$	$h_0 = 0.01$	$h_0 = 0.05$	$h_0 = 0.1$
I	−0.5	56.5030	1.6311	0.1623	0.7265	1.1404
	−0.3	51.9134	1.4986	0.1492	0.6775	1.0881
	−0.1	47.2193	1.3631	0.1358	0.6252	1.0284
	0.1	42.3859	1.2235	0.1220	0.5692	0.9597
	0.3	37.3562	1.0783	0.1076	0.5085	0.8799
	0.5	32.0252	0.9245	0.0923	0.4417	0.7854
II	−0.5	154.6063	4.4630	0.4413	1.7828	2.4766
	−0.3	142.8686	4.1242	0.4086	1.6916	2.4081
	−0.1	130.8114	3.7762	0.3747	1.5899	2.3260
	0.1	118.3252	3.4157	0.3394	1.4756	2.2257
	0.3	105.2276	3.0376	0.3022	1.3459	2.1003
	0.5	91.1842	2.6322	0.2625	1.1973	1.9400
III	−0.5	301.8734	8.7143	0.8561	3.1172	4.0302
	−0.3	280.6782	8.1024	0.7939	2.9902	3.9503
	−0.1	258.3156	7.4569	0.7302	2.8454	3.8544
	0.1	234.4250	6.7672	0.6646	2.6790	3.7362
	0.3	208.6752	6.0239	0.5958	2.4842	3.5849
	0.5	180.8931	5.2219	0.5195	2.2456	3.3792

*For general value of h_0.

TABLE II

Values of frequency parameter Ω, for C − C plate
$b/a = 0.5$

Mode	Values of α	Ω_C *	Ω_C $h_0 = 0.1$	Ω_S $h_0 = 0.01$	Ω_S $h_0 = 0.05$	Ω_S $h_0 = 0.1$
I	−0.5	116.0032	3.3487	0.3312	1.3431	1.8626
	−0.3	105.2731	0.0389	0.3012	1.2578	1.8002
	−0.1	94.4052	2.7252	0.2705	1.1631	1.7236
	0.1	83.3458	2.4059	0.2392	1.0576	1.6285
	0.3	72.0042	2.0785	0.2069	0.9398	1.5089
	0.5	60.2092	1.7380	0.1732	0.8070	1.3558
II	−0.5	318.3008	9.1885	0.8974	3.0890	3.8392
	−0.3	289.6400	8.3611	0.8199	2.9517	3.7600
	−0.1	260.5781	7.5222	0.7405	2.7902	3.6619
	0.1	230.9572	6.6671	0.6585	2.5989	3.5366
	0.3	200.5069	5.7881	0.5733	2.3701	3.3703
	0.5	168.7093	4.8702	0.4838	2.0939	3.1397
III	−0.5	623.9898	18.0130	1.7262	5.1662	6.0963
	−0.3	569.6992	16.4458	1.5815	4.9912	6.0055
	−0.1	514.0009	14.8379	1.4327	4.7812	5.8918
	0.1	456.3372	13.1733	1.2794	4.5252	5.7455
	0.3	396.0083	11.4318	1.1204	4.2075	5.5495
	0.5	332.3302	9.5935	0.9511	3.8015	5.2702

*For general value of h_0.

TABLE III

Values of frequency parameter Ω, for C − S plate
$b/a = 0.3$

Mode	Values of α	Ω_C *	Ω_C $h_0 = 0.1$	Ω_C $h_0 = 0.01$	Ω_C $h_0 = 0.05$	Ω_C $h_0 = 0.1$
I	−0.5	33.7986	0.9757	0.0973	0.4571	0.7819
	−0.3	31.9424	0.9221	0.0920	0.4352	0.7552
	−0.1	30.0036	0.8661	0.0865	0.4118	0.7252
	0.1	27.9541	0.8070	0.0806	0.3864	0.6908
	0.3	25.7472	0.7432	0.0742	0.3584	0.6507
	0.5	23.2967	0.6725	0.0672	0.3265	0.6024
II	−0.5	120.8240	3.4878	0.3464	1.5025	2.2730
	−0.3	112.4118	3.2450	0.3226	1.4203	2.1910
	−0.1	103.7702	2.9955	0.2981	1.3318	2.0979
	0.1	94.8242	2.7373	0.2726	1.2360	1.9911
	0.3	85.4508	2.4667	0.2458	1.1312	1.8667
	0.5	75.4301	2.1774	0.2172	1.0143	1.7185
III	−0.5	256.6910	7.4100	0.7284	2.8680	3.9184
	−0.3	238.6603	6.8895	0.6781	2.7344	3.8198
	−0.1	219.9566	6.3496	0.6266	2.5872	3.7028
	0.1	200.4302	5.7859	0.5731	2.4234	3.5618
	0.3	179.9014	5.1933	0.5164	2.2376	3.3880
	0.5	158.1134	4.5643	0.4542	2.0198	3.1659

*For general value of h_0.

TABLE IV

Values of frequency parameter Ω, *for C – S plate*
$b/a = 0.5$

Mode	Values of α	Ω_C		Ω_S		
		*	$h_0 = 0.1$	$h_0 = 0.01$	$h_0 = 0.05$	$h_0 = 0.1$
I	−0.5	72.8678	2.1035	0.2091	0.9248	1.4289
	−0.3	67.3564	1.9444	0.1935	0.8702	1.3785
	−0.1	61.7399	1.7822	0.1775	0.8115	1.3198
	0.1	55.9771	1.6159	0.1611	0.7479	1.2505
	0.3	49.9982	1.4433	0.1439	0.6785	1.1677
	0.5	43.6709	1.2606	0.1258	0.6013	1.0663
II	−0.5	251.6171	7.2635	0.7156	2.7481	3.6885
	−0.3	230.0375	6.6406	0.6558	2.5998	3.5811
	−0.1	208.1654	6.0092	0.5947	2.4340	3.4519
	0.1	185.8878	5.3661	0.5321	2.2479	3.2943
	0.3	163.0143	4.7058	0.4675	2.0375	3.0990
	0.5	139.1888	4.0180	0.3998	1.7969	2.8511
III	−0.5	531.9840	15.3570	1.4870	4.9279	6.0247
	−0.3	485.8330	14.0248	1.3639	4.7234	5.9157
	−0.1	438.8280	12.6679	1.2380	4.4847	5.7773
	0.1	390.7059	11.2787	1.1084	4.2044	5.5978
	0.3	341.1157	9.8471	0.9736	3.8711	5.3591
	0.5	289.5779	8.3593	0.8301	3.4654	5.0299

*For general value of h_0.

TABLE V

Values of frequency parameter Ω, *for C – F plate*
$b/a = 0.5$

Mode	Values of α	Ω_C		Ω_S		
		*	$h_0 = 0.2$	$h_0 = 0.05$	$h_0 = 0.1$	$h_0 = 0.2$
I	−0.5	14.0934	0.8136	0.1930	0.3515	0.5831
	−0.3	13.4211	0.7748	0.1841	0.3408	0.5754
	−0.1	12.8021	0.7392	0.1763	0.3313	0.5688
	0.1	12.2571	0.7076	0.1699	0.3234	0.5641
	0.3	11.8210	0.6824	0.1640	0.3172	0.5624
	0.5	11.5597	0.6674	0.1254	0.2870	0.5522
II	−0.5	101.8529	5.8804	1.2615	1.9133	2.4332
	−0.3	94.6315	5.4634	1.1951	1.8562	2.3967
	−0.1	87.3322	5.0420	1.1246	1.7911	2.3561
	0.1	79.9320	4.6148	1.0492	1.7160	2.3098
	0.3	72.3997	4.1800	0.9663	1.6280	2.2554
	0.5	64.6959	3.7352	0.8569	1.5154	2.1859
III	−0.5	306.4074	17.6904	3.2878	4.4303	5.0943
	−0.3	281.0130	16.2242	3.1279	4.3104	5.0389
	−0.1	255.3090	14.7402	2.9491	4.1674	4.9679
	0.1	232.1957	13.4059	2.7437	3.9926	4.8737
	0.3	202.5096	11.6918	2.5016	3.7750	4.7457
	0.5	174.9482	10.1006	2.2115	3.4989	4.5690

*For general value of h_0.

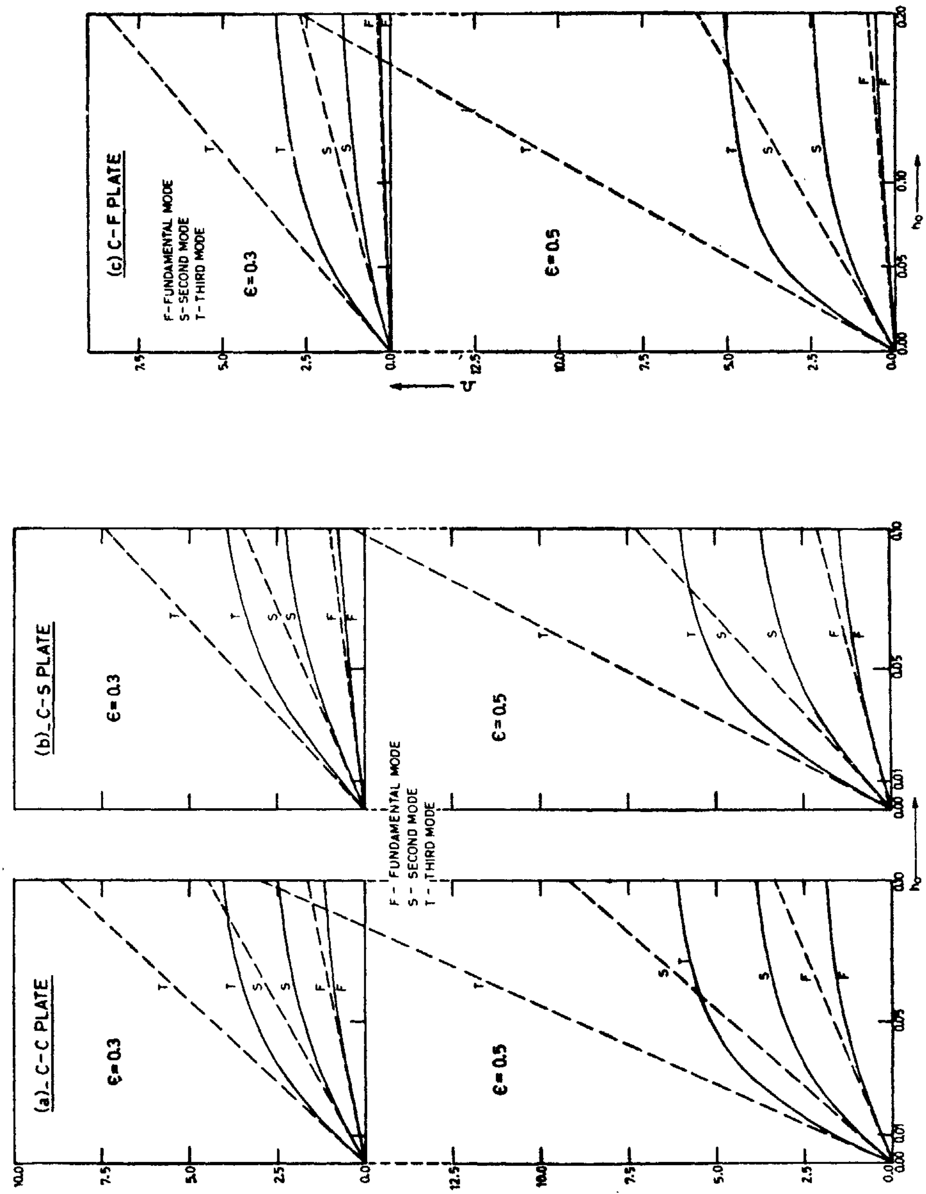

FIG. 2. Variation of natural frequencies with thickness. $\alpha = -0.5$; - - -, classical theory; ——, shear theory.

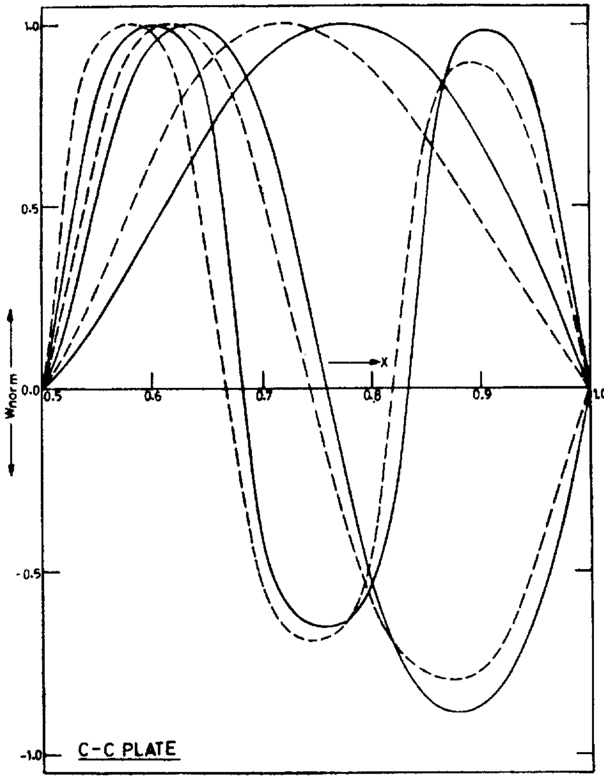

FIG. 3(a) Normalized displacements for the first three modes of vibration. $\epsilon = 0.5$; - - -, $\alpha = -0.5$; ——, $\alpha = 0.5$.

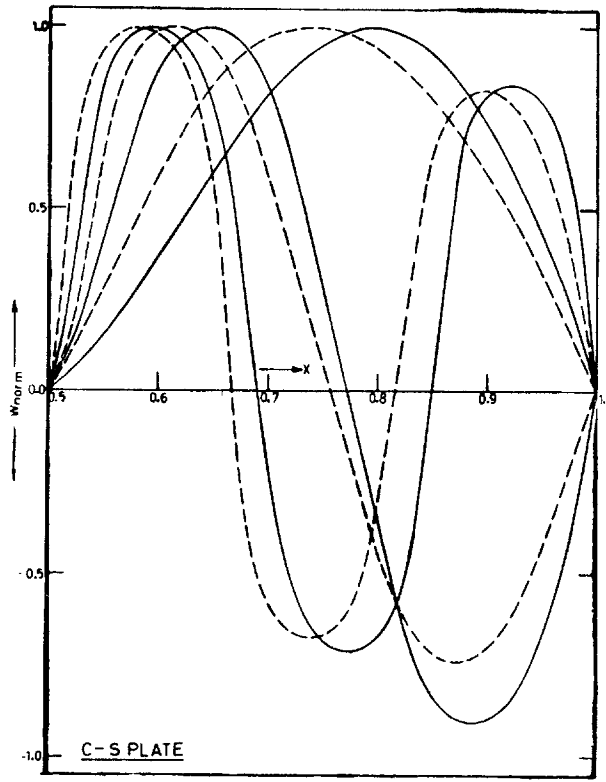

FIG. 3(b) Normalized displacements for the first three modes of vibration, $\epsilon = 0.5$; - - -, $\alpha = -0.5$; ——, $\alpha = 0.5$.

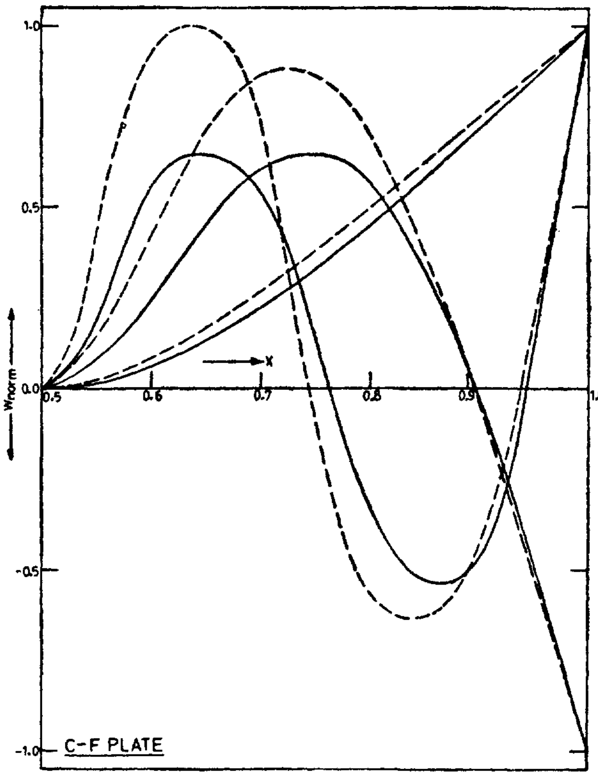

FIG. 3(c) Normalized displecements for the first three modes of vibration. $\epsilon = 0.5$; – – –, $\alpha = -0.5$; ——, $\alpha = 0.5$.

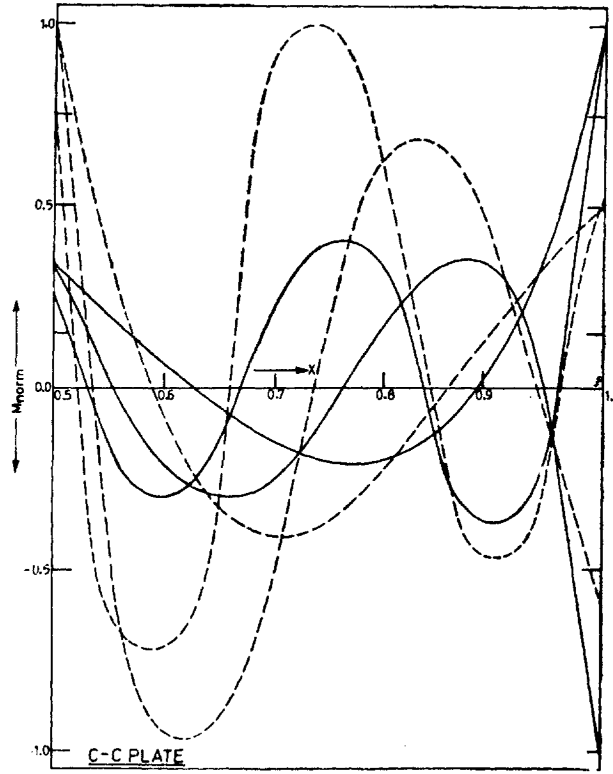

FIG. 4(a) Normalized displacements for the first three modes of vibration. $\epsilon = 0.5$; – – –, $\alpha = -0.5$; ——, $\alpha = 0.5$.

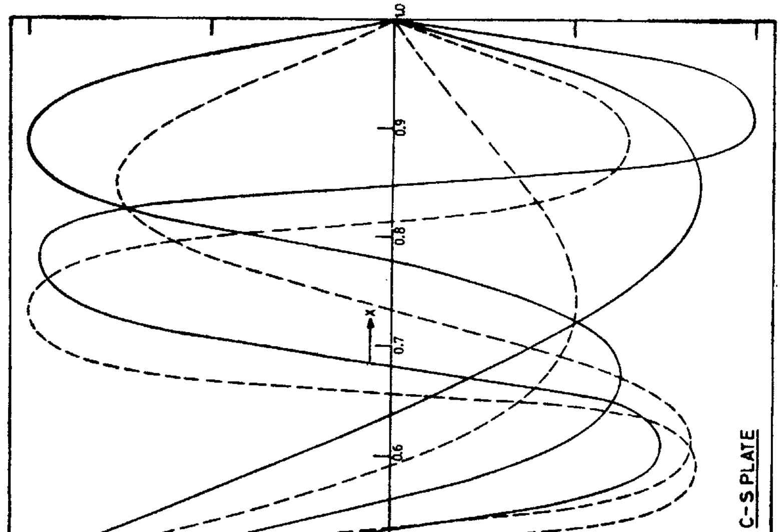

FIG. 4(b) Normalized displacements for the first three modes of vibration. $\epsilon = 0.5$; – – –, $\alpha = -0.5$; ——, $\alpha = 0.5$.

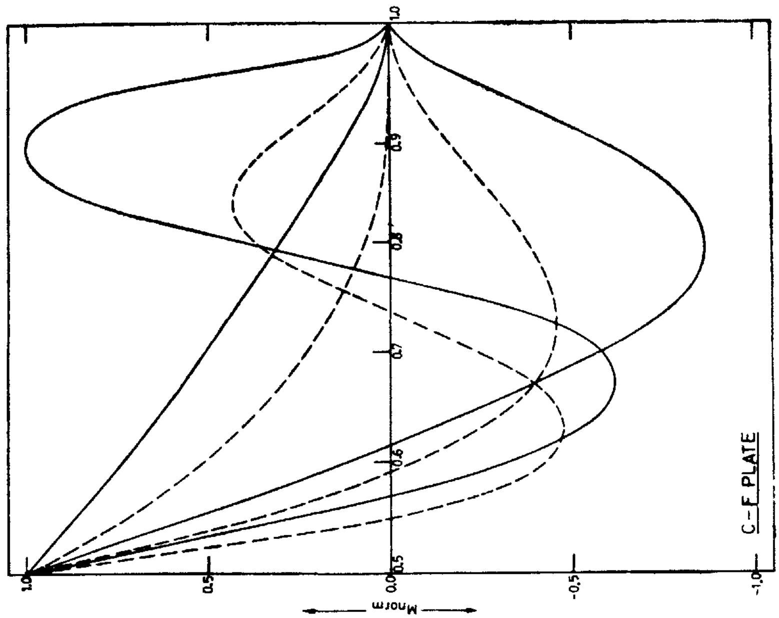

FIG. 4(c) Normalized displacements for the first three modes of vibration. $\epsilon = 0.5$; – – –, $\alpha = -0.5$; ——, $\alpha = 0.5$.

the second or higher decimal place. For the $C - C$ and $C - S$ boundary conditions it is seen that the transverse deflection for $\alpha = 0.5$ is less towards the inner edge and greater towards the outer edge than the corresponding deflection for $\alpha = -0.5$. However, in the case of the $C - F$ boundary condition, the transverse deflection for $\alpha = -0.5$ is greater than the corresponding deflection for $\alpha = 0.5$ towards both the edges except in the third mode at the outer edge. The radii of the nodal circles decrease as the outer edge becomes thicker and thicker for all the three boundary conditions. In case of $C - C$ boundary condition, the moments for $\alpha = -0.5$ are greater than the corresponding moments for $\alpha = 0.5$ towards the inner edge while it is otherwise towards the outer edge. For $C - S$ and $C - F$ boundary conditions, the moments for $\alpha = -0.5$ are less than those for $\alpha = 0.5$ towards both the edges except for $C - S$ case in the 2nd and 3rd mode at the inner edge. As regards the lines along which moments vanish, these are shifted towards the outer edge as α increases.

The frequency parameters computed by neglecting the rotatory inertia term differ negligibly from the corresponding values obtained by shear theory, leading to the conclusion that the transverse shear deformation accounts for almost the entire discrepancy. It is seen that shear theory gives lower values of Ω for all the three boundary conditions and for all plate parameters. Thus the effects of transverse shear and rotatory inertia must be taken into account in studying the dynamic response of polar orthotropic annular plates of parabolically varying thickness for large radii ratio (> 0.3) and moderately thick plates i.e. $h_0 > 0.5$.

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