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ON MULTIDIMENSIONAL MELLIN CONVOLUTIONS AND *H*-FUNCTION TRANSFORMATIONS

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In the present paper we first establish a multiple integral formula involving the product of two multivariable H-functions which were introduced and studied in a series of papers by Srivastava and Panda (1976, 1976a, 1978, 1979). This integral formula is then used to establish, in a very elegant form, an interesting multivariable generalization of the Srivastava-Buschman theorem [see Srivastava and Buschman (1976), p. 335], which expresses the multiple Mellin convolution of the multidimensional H-function transforms of two functions as the multidimensional H-function transforms of multiple Mellin convolutions of the functions. The present study extends and unifies a number of results on single, double and multiple integrals and Mellin convolutions involving special functions of one and two variables. Thus the results recently obtained by Srivastava and Buschman (1976), Singhal and Bhati (1977), Gupta and Goyal (1979), etc., follow as particular cases of our main findings.

1. INTRODUCTION, DEFINITION AND NOTATIONS

(a) *The Multivariable H-function*

The multivariable H-function occurring in this paper is a special case of the general H-function of several complex variables, introduced and studied in a series of papers by Srivastava and Panda (1976, 1976a, 1978, 1979). The parameters of this function will be displayed in the following contracted notation, which is due essentially to Srivastava and Panda [1976a, p. 130, eqn. (1.3)] :

$$H_1[x_1, \ldots, x_r] \equiv H^{0,0:m_1,n_1;\ldots;m_r,n_r}_{p,q:p_1,q_1;\ldots;p_r,q_r}\left[\begin{matrix}x_1\\ \vdots\\ x_r\end{matrix}\left|\begin{matrix}(a_j;\alpha'_j,\ldots,\alpha^{(r)}_j)_{1,p}:\\ (b_j;\beta'_j,\ldots,\beta^{(r)}_j)_{1,q}:\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(c'_j,\epsilon'_j)_{1,p_1};\ldots;(c^{(r)}_j,\epsilon^{(r)}_j)_{1,p_r}\\ (d'_j,\delta'_j)_{1,q_1};\ldots;(d^{(r)}_j,\delta^{(r)}_j)_{1,q_r}\end{matrix}\right]$$

$$= 1/(2\pi\omega)^r \int_{L_1}\cdots\int_{L_r}\phi(\xi_1,\ldots,\xi_r)\prod_{i=1}^{r}\{\theta_i(\xi_i)\,(x_i)^{\xi_i}\,d\xi_i\} \qquad \ldots(1.1)$$

where $\omega = \sqrt{-1}$

$$\phi(\xi_1, \ldots, \xi_r) = [\prod_{j=1}^{q} \Gamma(1 - b_j + \sum_{i=1}^{r} \beta_j^{(i)} \xi_i) \prod_{j=1}^{p} \Gamma(a_j - \sum_{i=1}^{r} \alpha_j^{(i)} \xi_i)]^{-1} \quad \ldots(1.2)$$

$$\theta_i(\xi_i) = \prod_{j=1}^{m_i} \Gamma(d_j^{(i)} - \delta_j^{(i)} \xi_i) \prod_{j=1}^{n_i} \Gamma(1 - c_j^{(i)} + \epsilon_j^{(i)} \xi_i)$$

$$\times [\prod_{j=m_i+1}^{q_i} \Gamma(1 - d_j^{(i)} + \delta_j^{(i)} \xi_i) \prod_{j=n_i+1}^{p_i} \Gamma(c_j^{(i)} - \epsilon_j^{(i)} \xi_i)]^{-1}, \quad \ldots(1.3)$$

$$\forall\, i \in \{1, \ldots, r\}.$$

Throughout the present paper, the symbol $(a_j; \alpha_j', \ldots, \alpha_j^{(r)})_{1,p}$ abbreviates the p-member array $(a_1; \alpha_1', \ldots, \alpha_1^{(r)}), \ldots, (a_p; \alpha_p', \ldots, \alpha_p^{(r)})$, and $(c_j, \epsilon_j)_{1,p}$ the p-member array $(c_1, \epsilon_1), \ldots, (c_p, \epsilon_p)$, $p \geqslant 0$, the array being empty if $p = 0$. Further, p, q, m_i, n_i, p_i and q_i are non-negative integers constrained by the inequalities:

$$p, q \geqslant 0, \quad 0 \leqslant n_i \leqslant p_i, \quad 1 \leqslant m_i \leqslant q_i, \quad \forall\, i \in \{1, \ldots, r\},$$

all the $a_j, b_j, c_j^{(i)}, d_j^{(i)}$ are assumed to be complex numbers and Greek letters are assumed to be positive numbers for standardization purposes; the definition of the multivariable H-function given by (1.1) will, however, be meaningful even if some of these quantities are zero.

The multiple integral (1.1) converges absolutely if

$$U_i > 0 \quad \text{and} \quad |\arg x_i| < \tfrac{1}{2} U_i \pi \quad \ldots(1.4)$$

where

$$U_i = -\sum_{j=1}^{p} \alpha_j^{(i)} - \sum_{j=1}^{q} \beta_j^{(i)} + \sum_{j=1}^{n_i} \epsilon_j^{(i)} - \sum_{j=n_i+1}^{p_i} \epsilon_j^{(i)}$$

$$+ \sum_{j=1}^{m_i} \delta_j^{(i)} - \sum_{j=m_i+1}^{q_i} \delta_j^{(i)}, \quad \forall\, i \in \{1, \ldots, r\}. \quad \ldots(1.5)$$

Throughout the present paper we shall assume that the conditions corresponding appropriately to the aforementioned ones are satisfied by all multivariable H-functions occurring in this paper.

Also, we shall require the known asymptotic expansions [Srivastava and Panda 1976a, p. 131, eqn. (1.9)] :

$$H_1[x_1, \ldots, x_r] = \begin{cases} O(|x_1|^{\mu_1} \ldots |x_r|^{\mu_r}), \ \max\{|x_1|, \ldots, |x_r|\} \to 0 \\ O(|x_1|^{\nu_1} \ldots |x_r|^{\nu_r}), \ \min\{|x_1|, \ldots, |x_r|\} \to \infty \end{cases} \quad \ldots(1.6)$$

where

$$\mu_i = \min_{1 \leq j \leq m_i} [\text{Re}\,\{d_j^{(i)}/\delta_j^{(i)}\}],\ \nu_i = \max_{1 \leq j \leq n_i} [\text{Re}\,\{(c_j^{(i)} - 1)/\epsilon_j^{(i)}\}], \quad \ldots(1.7)$$

$$\forall\ i \in \{1, \ldots, r\}.$$

(b) *Multidimensional Mellin Convolutions and H-function Transformations*

The multiple Mellin convolution of two functions g and f will be defined and represented as follows:

$$(g * \ldots * f)\,(u_1, \ldots, u_r)$$

$$= \int_0^\infty \ldots \int_0^\infty \{x_1^{-1} \ldots x_r^{-1}\}\, g(u_1/x_1, \ldots, u_r/x_r)\, f(x_1, \ldots, x_r)\, dx_1 \ldots dx_r \quad \ldots(1.8)$$

provided that the integral in (1.8) converges absolutely.

Again, on account of the presence of the factor $\prod_{i=1}^{r} (x_i)^{-1}$ in (1.8), the multidimensional H-function transform will be defined and represented in the following form, which is an obvious special case of one of the two general multidimensional H-function transformations defined by Srivastava and Panda [1978, p. 121, eqn. (1.15)]:

$$H_1\{f(x_1, \ldots, x_r);\ s_1, \ldots, s_r\}$$

$$\equiv H^{0,0:(a_j;\alpha_j',\ldots,\alpha_j^{(r)});\ m_1,n_1:(c_j',\ \epsilon_j');\ldots;m_r,n_r:(c_j^{(r)},\epsilon_j^{(r)})}_{p,q:(b_j;\ \beta_j',\ldots,\beta_j^{(r)});\ p_1,q_1:(d_j',\delta_j');\ldots;\ p_r,q_r:(d_j^{(r)},\delta_j^{(r)})}\ \{f(x_1, \ldots, x_r);\ s_1, \ldots, s_r\}$$

$$= \int_0^\infty \ldots \int_0^\infty \{x_1^{-1} \ldots x_r^{-1}\}\, H_1\,[s_1x_1, \ldots, s_rx_r]\, f(x_1, \ldots, x_r)\, dx_1 \ldots dx_r, \quad \ldots(1.9)$$

provided that the multiple integral occurring on the right-hand side of (1.9) is absolutely convergent.

2. The Integral Formula

The following integral formula will be established in this paper:

$$\int_0^\infty \ldots \int_0^\infty \{(x_1)^{\rho_1-1} \ldots (x_r)^{\rho_r-1}\}\, H_1'[z_1x_1^{-\sigma_1}, \ldots, z_rx_r^{-\sigma_r}]$$

$$\times\ H_1[y_1x_1, \ldots, y_rx_r]\, dx_1 \ldots dx_r$$

$$= \{(y_1)^{-\rho_1} \ldots (y_r)^{-\rho_r}\}\ H^{0,0:m_1+m_1',n_1+n_1';\ldots;m_r+m_r',n_r+n_r'}_{p+p',q+q':p_1+p_1',q_1+q_1';\ldots;p_r+p_r',q_r+q_r'} \left\{ \begin{matrix} Z_1 \\ \vdots \\ Z_r \end{matrix} \middle| \begin{matrix} U \\ \\ V \end{matrix} \right\} \quad \ldots(2.1)$$

where

$$Z_i = z_i y_i^{\sigma_i}\ (i = 1, \ldots, r)$$

and

$$H_1'[x_1, \ldots, x_r] \equiv H^{0,0:m_1',n_1';\ldots;m_r',n_r'}_{p',q':p_1',q_1';\ldots;p_r',q_r'} \left[\begin{matrix} x_1 \\ \vdots \\ x_r \end{matrix} \middle| \begin{matrix} (a_j'; A_j', \ldots, A_j^{(r)})_{1,p'}: \\ (b_j'; B_j', \ldots, B_j^{(r)})_{1,q'}: \end{matrix} \right.$$

$$\left. \begin{matrix} (e_j', E_j')_{1,q_1'}; \ldots; (e_j^{(r)}, E_j^{(r)})_{1,p_r'} \\ (f_j', F_j')_{1,q_1'}; \ldots; (f_j^{(r)}, F_j^{(r)})_{1,q_r'} \end{matrix} \right] \qquad \ldots(2.2)$$

U stands for the parameter sequence:

$$(a_j'; A_j', \ldots, A_j^{(r)})_{1,p'}, (a_j + \sum_{i=1}^{r} \alpha_j^{(i)} \rho_i; \alpha_j' \sigma_1, \ldots, \alpha_j^{(r)} \sigma_r)_{1,p} : (e_j', E_j')_{1,n_1'},$$

$$(c_j' + \epsilon_j' \rho_1, \epsilon_j' \sigma_1)_{1,p_1}, (e_j', E_j')_{n_1'+1,p_1'}; \ldots; (e_j^{(r)}, E_j^{(r)})_{1,n_r'},$$

$$(c_j^{(r)} + \epsilon_j^{(r)} \rho_r, \epsilon_j^{(r)} \sigma_r)_{1,p_r'}, (e_j^{(r)}, E_j^{(r)})_{n_r'+1,p_r'}.$$

V stands for the parameter sequence:

$$(b_j'; B_j', \ldots, B_j^{(r)})_{1,q'}, (b_j + \sum_{i=1}^{r} \beta_j^{(i)} \rho_i; \beta_j' \sigma_1, \ldots, \beta_j^{(r)} \sigma_r)_{1,q} : (f_j', F_j')_{1,m_1'},$$

$$(d_j' + \delta_j' \rho_1, \delta_j' \sigma_1)_{1,q_1}, (f_j', F_j')_{m_1'+1,q_1'}; \ldots; (f_j^{(r)}, F_j^{(r)})_{1,m_r'},$$

$$(d_j^{(r)} + \delta_j^{(r)} \rho_r, \delta_j^{(r)} \sigma_r)_{1,q_r}, (f_j^{(r)}, F_j^{(r)})_{m_r'+1,q_r'}.$$

The integral (2.1) converges under the following (sufficient) conditions:

$$\sigma_i > 0, -\mu_i + \sigma_i \nu_i' < \text{Re}(\rho_i) < -\nu_i + \sigma_i \mu_i', \quad \forall\, i \in \{1, \ldots, r\},$$

where μ_i, ν_i are given by (1.7) and

$$\mu_i' = \min_{1 \leqslant j \leqslant m_i'} [\text{Re}(f_j^{(i)}/F_j^{(i)})], \; \nu_i' = \max_{1 \leqslant j \leqslant n_i'} [\text{Re}\{(e_j^{(i)} - 1)/E_j^{(i)}\}].$$

Derivation of (2.1) — We suppose in the right-hand side of (1.8),

$$g(x_1, \ldots, x_r) = H_1'[x_1^{\sigma_1}, \ldots, x_r^{\sigma_r}] \qquad \ldots(2.3)$$

and

$$f(x_1, \ldots, x_r) = \{x_1^{\rho_1} \ldots x_r^{\rho_r}\} H_1[y_1 x_1, \ldots, y_r x_r] \qquad \ldots(2.4)$$

and evaluate the multidimensional Mellin transforms of g and f with the help of a recent result by Panda [1977, p. 160, eqn. (2.1)]. We then substitute these Mellin transforms in the left-hand side of (1.8) and interpret the result thus obtained with the help of the inversion formula for multidimensional Mellin transform [see, for instance, Srivastava and Panda (1978), p. 125, eqn. (3.4)] and (1.1), we easily get our integral formula (2.1).

We remark in passing that the integral (2.1) is very general in nature and a number of multiple integrals involving a large spectrum of special functions (or product of several such functions) can be obtained by specializing the parameters of the multivariable H-functions involved therein.

For example, if we set in (2.1) $p = q = 0$, the multivariable H-function will reduce immediately to the product of $2r$ different H-functions of single variable. Again, if we put $r = 2$ in (2.1), we shall get the integral established by Singhal and Bhati (1977, p. 73).

We shall require the following special case of (2.1) ($\rho_i = 0$, $\sigma_i = 1$, $i = 1, \ldots, r$) to establish a useful result on Mellin convolutions contained in the next section:

$$\int_0^\infty \cdots \int_0^\infty \{x_1^{-1} \ldots x_r^{-1}\} H_1'[z_1 x_1^{-1}, \ldots, z_r x_r^{-1}] H_1[y_1 x_1, \ldots, y_r x_r] \, dx_1 \ldots dx_r$$

$$= H^{0,0:m_1+m_1',n_1+n_1';\ldots;m_r+m_r',n_r+n_r'}_{p+p',q+q':p_1+p_1',q_1+q_1';\ldots;p_r+p_r',q_r+q_r'} \left[\begin{array}{c|l} z_1 y_1 & (a_j'; A_j', \ldots, A_j^{(r)})_{1,p'}, \\ \vdots & \\ z_r y_r & (b_j'; B_j', \ldots, B_j^{(r)})_{1,q'}, \end{array} \right.$$

$$(a_j; \alpha_j', \ldots, \alpha_j^{(r)})_{1,p} : (e_j', E_j')_{1,n_1'}, (c_j', \epsilon_j')_{1,p_1}, (e_j', E_j')_{n_1'+1,p_1'}; \ldots;$$

$$(b_j; \beta_j', \ldots, \beta_j^{(r)})_{1,q} : (f_j', F_j')_{1,m_1'}, (d_j', \delta_j')_{1,q_1}, (f_j', F_j')_{m_1'+1,q_1'}; \ldots;$$

$$\left. \begin{array}{l} (e_j^{(r)}, E_j^{(r)})_{1,n_r}, (c_j^{(r)}, \epsilon_j^{(r)})_{1,p_r}, (e_j^{(r)}, E_j^{(r)})_{n_r'+1,p_r'} \\ (f_j^{(r)}, F_j^{(r)})_{1,m_r'}, (d_j^{(r)}, \delta_j^{(r)})_{1,q_r}, (f_j^{(r)}, F_j^{(r)})_{m_r'+1,q_r'} \end{array} \right]$$

$$\equiv H_1^{*}[y_1 z_1, \ldots, y_r z_r], \text{ (say)} \qquad \ldots(2.5)$$

where $H_1'[\ldots]$ is defined by (2.2) and the conditions of validity easily obtainable from those given with (2.1) are satisfied.

3. Multiple Mellin Convolutions

In this section we shall obtain the following interesting result which expresses multiple Mellin convolution of the multidimensional H-function transform given by (1.9) of two functions as the multidimensional H-function transform of the multiple Mellin convolution.

Main Result

$$(H_1\{g(x_1, \ldots, x_r); s_1, \ldots, s_r\} * \ldots * H_1'\{f(x_1, \ldots, x_r); s_1, \ldots, s_r\})(u_1, \ldots, u_r)$$

$$= H_1''\{(g * \ldots * f)(t_1, \ldots, t_r); u_1, \ldots, u_r\}, \qquad \ldots(3.1)$$

where

(i) $H_1\{\ldots\}$ stands for the multidimensional H-function transform of $g(x_1, \ldots, x_r)$ defined by (1.9).

(ii) Similarly, $H_1'\{\ldots\}$ and $H_1''\{\ldots\}$ stand for the multidimensional H-function transforms analogous to (1.9) wherein the kernels are the multivariable H-function defined in (2.2) and (2.5), respectively.

The result (3.1) is valid under the following sets of conditions:

(i) The multidimensional Mellin transform of $H_1\{\ldots\}$, $H_1'\{\ldots\}$ and $(g * \ldots * f)$ $(u_1, \ldots, u_r)$ exist.

(ii) The multidimensional Mellin transform of the expressions on both the sides of (3.1) exist.

The proof of the result (3.1) is quite straight-forward and similar to those of the results established earlier by Srivastava and Buschman (1976, pp. 334–37) and Gupta and Goyal (1979). We, therefore, omit the details.

4. Special Cases

I. If we put $p' = p$, $q' = q$, $m_i' = m_i$, $n_i' = n_i$, $p_i' = p_i$, $q_i' = q_i$, $A_j^{(i)} = \alpha_j^{(i)}$, $B_j^{(i)} = \beta_j^{(i)}$, $E_j^{(i)} = \epsilon_j^{(i)}$, $F_j^{(i)} = \delta_j^{(i)}$; $e_j^{(i)} = c_j^{(i)} + \frac{1}{2}$, $f_j^{(i)} = d_j^{(i)} + \frac{1}{2}$ $(i = 1, \ldots, r)$, $a_j' = a_j + \frac{1}{2}$, $b_j' = b_j + \frac{1}{2}$ in (3.1) and apply the well-known duplication formula for the Gamma functions in the contour integral format of the multivariable H-function involved on the right-hand side of (3.1), we easily get the following result after a little simplification:

$$\Bigg(H_1\{g(x_1, \ldots, x_r); s_1, \ldots, s_r\} * \ldots *$$

$$\times\ H^{0,0:(a_j+\frac{1}{2};\alpha_j', \ldots, \alpha_j^{(r)});m_1,n_1:(c_j'+\frac{1}{2},\epsilon_j');\ldots;m_r,n_r:(c_j^{(r)}+\frac{1}{2},\epsilon_j^{(r)})}_{p,q:(b_j+\frac{1}{2};\beta_j', \ldots, \beta_j^{(r)});p_1,q_1:(d_j'+\frac{1}{2},\delta_j');\ldots;p_r,q_r:(d_j^{(r)}+\frac{1}{2},\delta_j^{(r)})}$$

$$\times\ \{f(x_1, \ldots, x_r); s_1, \ldots, s_r\}\Bigg)(u_1, \ldots, u_r)$$

$$= \pi^{\delta/2} 2^{\phi} H^{0,0:(2a_j;\alpha_j', \ldots, \alpha_j^{(r)});m_1,n_1:(2c_j',\epsilon_j');\ldots;m_r,n_r:(2c_j^{(r)},\epsilon_j^{(r)})}_{p,q:(2b_j;\beta_j', \ldots, \beta_j^{(r)});p_1,q_1:(2d_j',\delta_j');\ldots;p_r,q_r:(2d_j^{(r)},\delta_j^{(r)})}$$

$$\{(g * \ldots * f)(t_1^2, \ldots, t_r^2); 2^{-\eta_1}u_1^{1/2}, \ldots, 2^{-\eta_r}u_r^{1/2}\}, \qquad \ldots(4.1)$$

where

$$\delta = 2 \sum_{i=1}^{r} (m_i + n_i) - \sum_{i=1}^{r} (p_i + q_i) - p - q,$$

$$\phi = \sum_{i=1}^{r} (m_i + n_i) - \sum_{i=1}^{r} p_i - p + 2 \{\sum_{j=1}^{p} a_j - \sum_{j=1}^{q} b_j$$

$$+ \sum_{i=1}^{r} (\sum_{j=1}^{p_i} c_j^{(i)} - \sum_{j=1}^{q_i} d_j^{(i)})\},$$

$$\eta_i = \sum_{j=1}^{p} \alpha_j^{(i)} - \sum_{j=1}^{q} \beta_j^{(i)} + \sum_{j=1}^{p_i} \epsilon_j^{(i)} - \sum_{j=1}^{q_i} \delta_j^{(i)}, \ \forall\ i \in \{1, \ldots, r\}.$$

On using the following relation:

$$H^{0,0:-;1,0:-;\ldots;1,0:-}_{0,0:-;0,1:(a_1,1);\ldots;0,1:(a_r,1)} \{f(x_1, \ldots, x_r); s_1, \ldots, s_r\}$$

$$= \prod_{i=1}^{r} \{(s_i)^{a_i}\} L\left[\prod_{i=1}^{r} \{(x_i)^{a_i-1}\} f(x_1, \ldots, x_r); s_1, \ldots, s_r\right], \qquad \ldots(4.2)$$

in (3.1), we get

$$\left(\prod_{i=1}^{r} \{(s_i)^{a_i}\} L\left[\prod_{i=1}^{r} \{(x_i)^{a_i-1}\} g(x_1, \ldots, x_r); s_1, \ldots, s_r\right] * \ldots *\right.$$

$$\left.\prod_{i=1}^{r} (s_i)^{a_i} L\left[\prod_{i=1}^{r} \{(x_i)^{b_i-1}\} f(x_1, \ldots, x_r); s_1, \ldots, s_r\right]\right) (u_1, \ldots, u_r)$$

$$= 2^{3r/2} \prod_{i=1}^{r} \{(u_i)^{(a_i/2)+(b_i/2)-(1/4)}\} K\left[\prod_{i=1}^{r} \{(t_i)^{a_i+b_i-(3/2)}\}\right.$$

$$\left.\times (g * \ldots * f) (t_1^2, \ldots, t_r^2); a_1 - b_1, \ldots, a_r - b_r; 2u_1^{1/2}, \ldots, 2u_r^{1/2}\right] \qquad \ldots(4.3)$$

where

$$K[f(x_1, \ldots, x_r); \eta_1, \ldots, \eta_r; s_1, \ldots, s_r]$$

$$= \int_0^\infty \ldots \int_0^\infty f(x_1, \ldots, x_r) \prod_{i=1}^{r} \{s_i x_i^{1/2} K\eta_i (s_i x_i) \, dx_i\}$$

is the multidimensional analogue of Meijer's Bessel transform.

Again, if we let $b_i = a_i + \frac{1}{2}$, $(i = 1, \ldots, r)$ in (4.3), and invoking (1.8) therein, we get after a little simplification:

$$\left(\prod_{i=1}^{r} \{(s_i)^{a_i}\} L\left[\prod_{i=1}^{r} \{(x_i)^{a_i-1}\} g(x_1, \ldots, x_r); s_1, \ldots, s_r\right] * \ldots *\right.$$

$$\left.\prod_{i=1}^{r} \{s_i)^{a_i+(1/2)}\} L\left[\prod_{i=1}^{-r} \{(x_i)^{a_i-(1/2)}\} f(x_1, \ldots, x_r); s_1, \ldots, s_r\right]\right) (u_1, \ldots, u_r)$$

$$= 2^r \pi \prod_{i=1}^{r} \{(u_i)^{a_i}\} L\left[\prod_{i=1}^{r} \{(t_i)^{2a_i-1}\} (g * \ldots * f)(t_1^2, \ldots, t_r^2);\right.$$

$$\left. 2u_1^{1/2}, \ldots, 2u_r^{1/2}\right]. \qquad \ldots(4.4)$$

The relation (4.4) easily reduces to the following elegant form, with the help of (1.8), after a little simplification:

$$\left(L\left[\prod_{i=1}^{r} \{(x_i)^{a_i-1}\} g(x_1, \ldots, x_r); s_1, \ldots, s_r\right] * \ldots * \prod_{i=1}^{r} \{(s_i)^{1/2}\}\right.$$

$$\left. \times L\left[\prod_{i=1}^{r} \{(x_i)^{a_i-(1/2)}\} f(x_1, \ldots, x_r); s_1, \ldots, s_r\right]\right) (u_1, \ldots, u_r)$$

$$= 2^r \pi L\left[\prod_{i=1}^{r} \{(t_i)^{2a_i-1}\} (g * \ldots * f)(t_1^2, \ldots, t_r^2); 2u_1^{1/2}, \ldots, 2u_r^{1/2}\right]. \qquad \ldots(4.5)$$

Finally, setting $a_i = \frac{1}{2}$ in (4.5), we get the following result:

$$L[(g * \ldots * f)(t_1^2, \ldots, t_r^2); u_1, \ldots, u_r]$$

$$= 2^{-r} \pi^{-1} \left(L\left[\prod_{i=1}^{r} \{(x_i)^{-1/2}\} g(x_1, \ldots, x_r); s_1, \ldots, s_r\right] * \ldots * \prod_{i=1}^{r} \{(s_i)^{1/2}\}\right.$$

$$\left. \times L[f(x_1, \ldots, x_r); s_1, \ldots, s_r]\right) (u_1^2/4, \ldots, u_r^2/4). \qquad \ldots(4.6)$$

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