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OSCILLATORY FLOW PAST AN INFINITE VERTICAL POROUS PLATE WITH MASS TRANSFER

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A two-dimensional unsteady flow of a viscous incompressible fluid past an infinite vertical porous plate is studied on taking into account the following: (1) constant suction velocity, (2) the free-stream velocity oscillates about a non-zero constant mean, (3) constant plate temperature, (4) presence of free convection currents near the plate, (5) presence of foreign mass like H_2, He, H_2O etc.

Approximate solutions to coupled nonlinear equations governing the flow are derived. The mean velocity and temperature field are shown to be affected by the frequency of the free-stream. These results are shown graphically. The numerical values of the mean skin-friction, mean rate of heat transfer, the amplitude and phase of the skin-friction and the rate of heat transfer are tabulated. During the course of discussion, the effects of the parameters like ω (frequency), γ (suction parameter), *Gr* (Grashof number), *N* (buoyancy ratio), *Sc* (Schmidt number), *E* (Eckert number) are discussed.

NOTATION

$|B|$ = amplitude of the skin-friction

C' = species concentration

C'_w = species concentration at the plate

C'_∞ = species concentration in the free-stream

C = non-dimensional species concentration

c_p = specific heat of the fluid at constant pressure

D = chemical molecular diffusivity

E = Eckert number

g_x = acceleration due to gravity

Gr = Grashof number

k	=	thermal conductivity
M_r, M_i	=	fluctuating parts of the velocity profile
N	=	boundary parameter
P	=	Prandtl number
p	=	pressure
$\mid Q_1 \mid$	=	amplitude of the first harmonic of the rate of heat transfer
$\mid Q_2 \mid$	=	amplitude of the second harmonic of the rate of heat transfer
q'	=	rate of heat transfer
Sc	=	Schmidt number
t'	=	time
t	=	dimensionless time
T'	=	temperature of the fluid
T'_w	=	temperature of the plate
T'_∞	=	temperature of the fluid in the free-stream
T_r, T_i	=	fluctuating parts of the temperature profile
u', v'	=	velocity components in the x' y' directions
u	=	dimensionless velocity in the x-direction
V_0	=	suction velocity
u'	=	free-stream velocity
U_0	=	mean of $u'(t')$
U	=	dimensionless free-stream velocity
ϵU_0	=	amplitude of free-stream fluctuations
u_0	=	mean velocity
u_1	=	fluctuating part of the velocity
x', y'	=	coordinate system
y	=	dimensionless coordinate normal to wall
ω'	=	frequency of the free-stream oscillations
ω	=	dimensionless frequency
τ'	=	skin-friction
θ	=	dimensionless temperature
θ_0	=	mean temperature
$\epsilon\theta_1$	=	amplitude of the temperature fluctuations
α	=	phase angle of the skin-friction

α_1 = phase angle of the first harmonic of rate of heat transfer

α_2 = phase angle of the second harmonic of rate of heat transfer

β = coefficient of volume expansion

β^* = coefficient of expansion with concentration

ρ' = density of the fluid in the boundary layer

ρ'_∞ = density of the fluid in the free-stream

ν = kinematic viscosity

μ = viscosity

ρ_1 = density of foreign mass

μ_1 = viscosity of foreign mass.

1. Introduction

Lin (1957) and Lighthill (1954) studied the effects of finite-amplitude and small amplitude free-stream oscillations on boundary layer flow past a semi-infinite body respectively and independently. Lin solved the problem by approximate method whereas Lighthill solved it by integral method. Lighthill's predictions were confirmed by experiment performed by Hill and Stenning (1960). Stuart (1955) studied the effects of free-stream oscillations on the flow past an infinite horizontal porous plate with constant suction. The effects of free-stream oscillations and free convection currents on flow past an infinite vertical porous plate with constant suction were studied by Soundalgekar (1973a, b). But Stuart, Soundalgekar did not consider the effects of suction on such an oscillatory flow in an explicit way. As such effects do play an important role in the phenomenon of separation, Stuart's problem was again studied by Soundalgekar (1980) by considering suction parameter in an explicit way. In two papers by Soundalgekar (1973a, b), the mean velocity and temperature field were not found to be affected by the free-stream oscillations. So in order to study the effects of suction, free-stream oscillations and the free-convection currents, Soundalgekar *et al.* (1980) again studied this problem and by modifying the representation of the velocity and temperature field, they showed that the mean flow is affected by the frequency of the free-stream oscillations. In considering the free-convection current effects, it was assumed that the density differences were caused purely by temperature differences and gradients.

But in many transport processes which occur in nature, in addition to temperature, the density-difference is also caused by chemical composition differences and gradients or by material or phase constitution. This occurs in nature where the atmospheric flow is driven appreciably by both temperature and H_2O concentration differences. In water, the density is considerably affected by the temperature differences and also by the concentration of dissolved materials or by suspended particulate matter. Such a flow caused by the density-difference which in turn is caused by the concentration difference is known as the mass transfer flow. These flows occur in industry as well as in nature, e.g., in chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields and groves of fruit trees, damage of crops due to freezing and pollution of the environment. Mass transfer effects on steady free-convection flows were studied by Gebhart and Pera (1971) by similarity analysis and by Carnahan and Marner (1976), Soundalgekar and Ganesan (1980 b) by explicit and implicit finite difference methods respectively. Eckert (1958) and Sparrow *et al.* (1964) studied this phenomenon as a mass-transfer cooling process. During early studies of mass transfer cooling, the main-stream gas and the coolant gas were assumed to be one and the same, viz., air. Later experimental and theoretical investigations suggested that more effective protection of the surface could be achieved if the molecular weight of the coolant gas were substantially lower than that of the main-stream gas. The use of a 'foreign' gas as coolant creates a binary (i.e., two-component) boundary layer. In the normal condition of the concentration level in the binary boundary layer, two phenomena, known as thermal diffusion and diffusion thermo (also known as the Soret and the Dufour effects) have to be taken into account. The study of both these effects in the binary boundary layer flow has been done by Eckert and his co-workers and an account of these studies has been presented by Sparrow (1964).

However, when the concentration level in the binary boundary layer is very low, the Soret and the Dufour effects are negligible. This assumption is reasonable in most terrestrial processes in air and water where the concentration and temperature differences are small, though they are important in causing buoyancy forces. Based on this assumption, equations of motion for the combined free and forced convective flow of binary gas have been derived by Gebhart and Pera (1971). But Gebhart, Eckert and his co-workers have studied the steady flow of binary gas in boundary layer flow past a semi-infinite plate. The flow of binary gas past an impulsively started or oscillating infinite vertical plate in its own plane was studied by Soundalgekar (1979) and Soundalgekar and Akolkar (1980a) respectively. It is now proposed to study the effects of mass-transfer on the oscillatory flow past an infinite vertical porous plate when the free-stream is oscillating in magnitude but not in direction about a non-zero constant mean. In subsequent papers, mass-transfer effects on oscillatory flow past semi-infinite plates will be studied.

In section 2, the mathematical equations governing the unsteady upward motion of an incompressible viscous fluid past an infinite vertical porous plate with constant suction are derived and under the assumption of small amplitude oscillations, the solutions are derived for the mean velocity and temperature field, the transient velocity and temperature field, the amplitude and phase of the rate of heat transfer and skin-friction. This is followed by a discussion where the effects of different parameters on the flow phenomenon are discussed. In section 3, the conclusions are set out.

2 Mathematical Analysis

A two-dimensional unsteady flow of a viscous incompressible fluid, in the upward direction, past an infinite vertical porous plate with constant suction is assumed. Let u' and v' be the components of velocity along x' and y'-axis which are taken in the upward direction along the plate and normal to the plate respectively. The species concentration C' is assumed to be small and hence the Soret and Dufour effects are negligible. Then following Soundalgekar (1973 a) and Gebhart and Pera (1971), we can show that the flow is governed by the following equations:

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \frac{\partial U'}{\partial t'} + g_x\beta(T' - T'_\infty) + g_x\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u'}{\partial y'^2} \quad \ldots(1)$$

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{\nu}{c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 \quad \ldots(2)$$

$$\frac{\partial v'}{\partial y'} = 0 \quad \ldots(3)$$

$$\frac{\partial C'}{\partial t'} + v'\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2}\,. \quad \ldots(4)$$

For constant suction and binary mixture, we can show that

$$v' = -v_0\frac{1 - \mu_1/\mu}{1 + (\mu_1/\mu)(\rho/\rho_1)} = -V'_0 \quad \ldots(5)$$

where μ_1 and ρ_1 are respectively the viscosity and density of foreign gas, assumed constant. Also the negative sign in (5) indicates that the suction velocity is directed towards the plate.

The boundary conditions are

$$u = 0,\ T' = T'_w,\ C' = C'_w \text{ at } y' = 0$$

$$u = U'(t') = U_0(1 + \epsilon e^{i\omega' t'}),\ T' \to T'_\infty,\ C' \to C'_\infty \text{ as } y' \to \infty. \quad \ldots(6)$$

On introducing the following non-dimensional quantities

$$y = y'U_0/\nu, \qquad t = t'U_0^2/4\nu, \qquad \omega = 4\nu\omega'/U_0^2,$$

$$u = u'/Gr.\,U_0,\ U = U'/Gr\cdot U_0,\ \gamma = V_0/U_0,$$

$$\theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},\ C = \frac{C' - C'_\infty}{C'_w - C'_\infty},\ Sc = \nu/D,$$

$$P = \frac{\rho' c_p}{k},\ Gr = \frac{\nu g\beta\,|T'_w - T'_\infty|}{U_0^3},\ N = \frac{\beta^*(C'_w - C'_\infty)}{\beta\,|T'_w - T'_\infty|},$$

$$E = \frac{U_0^2}{c_p|T'_w - T'_\infty|} \quad \ldots(7)$$

in eqns. (1) – (6) and taking account of (5), we have

$$\frac{1}{4}\frac{\partial u}{\partial t} - \gamma\frac{\partial u}{\partial t} = \frac{1}{4}\frac{\partial U}{\partial t} + \theta + NC + \frac{\partial^2 u}{\partial y^2} \quad ...(8)$$

$$\frac{P}{4}\frac{\partial \theta}{\partial t} - \gamma P\frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} + PE\ Gr^2\left(\frac{\partial u}{\partial y}\right)^2 \quad ...(9)$$

$$\frac{Sc}{4}\frac{\partial C}{\partial t} - \gamma\ Sc\cdot\frac{\partial C}{\partial y} = \frac{\partial^2 C}{\partial y^2} \quad ...(10)$$

with the following boundary conditions:

$$\left.\begin{aligned} &u = 0,\quad \theta = 1,\quad C = 1 \quad \text{at } y = 0 \\ &u = U(t) = \frac{1}{Gr}(1 + \partial e^{i\omega t}),\ \theta = 0,\ C = 0 \quad \text{as } y \to \infty. \end{aligned}\right\} \quad ...(11)$$

To solve these coupled nonlinear eqns. (8) – (10), subject to the boundary conditions (11), we now assume that the oscillations are of small amplitude and then assume the solutions in the neighbourhood of the wall as

$$u = u_0 + \frac{\epsilon}{2}(u_1 e^{i\omega t} + \bar{u}_1 e^{-i\omega t}) \quad ...(12)$$

$$\theta = \theta_0 + \frac{\epsilon}{2}(\theta_1 e^{i\omega t} + \bar{\theta}_1 e^{-i\omega t}) + \frac{\epsilon^2}{2}(\theta_2 e^{2i\omega t} + \theta_2 e^{-2i\omega t}) \quad ...(13)$$

$$C = C_0 + \frac{\epsilon}{2}(C_1 e^{i\omega t} + \bar{C}_1 e^{-i\omega t}) \quad ...(14)$$

and for the free-stream

$$U = 1 + \frac{\epsilon}{2}(e^{i\omega t} + e^{-i\omega t}) \quad ...(15)$$

where ϵ is a small quantity $\ll 1$. Here (—) denotes a complex conjugate of the corresponding preceding quantity.

Introducing eqns. (12) – (15) in eqns. (8) – (11), equating the coefficients of harmonic and non-harmonic terms, neglecting the coefficients of ϵ^2 in the momentum and species concentration and those of ϵ^3 in the energy equation, we have

$$u_0'' + \gamma u_0' = -\theta_0 - NC_0 \quad ...(16)$$

$$u_1'' + \gamma u_1' - \frac{i\omega}{4}u_1 = -\frac{i\omega}{4} - \theta_1 - NC_1 \quad ...(17)$$

$$\theta_0'' + \gamma P\theta_0' = -PEGr^2\left[u_0'^2 + \frac{\epsilon^2}{2}u_1'\bar{u}_1'\right] \quad ...(18)$$

$$\theta_1'' + \gamma P\theta_1' - \frac{i\omega P}{4}\theta_1 = -2PE\ Gr^2 u_0' u_1' \quad ...(19)$$

$$\theta_2'' + \gamma P\theta_2' - \frac{i\omega P}{2}\theta_2 = -\tfrac{1}{2}\ PE\ Gr^2 u_1'^2 \quad ...(20)$$

$$C_0'' + \gamma Sc C_0' = 0 \quad \ldots(21)$$

$$C_1'' + \gamma Sc C_1' - \frac{i\omega}{4} Sc C_1 = 0 \quad \ldots(22)$$

and the boundary conditions are

$$\left.\begin{array}{lll} u_0 = 0, & \theta_0 = 1, & \left.\begin{array}{l} C_0 = 1 \\ C_1 = 0 \end{array}\right\} y = 0 \\ u_1 = 0, & \theta_1 = 0, & \\ u_0 = 1/Gr, & \theta_0 = 0, & \left.\begin{array}{l} C_0 = 0 \\ C_1 = 0 \end{array}\right\} y \to \infty. \\ u_1 = 2/Gr, & \theta_1 = 0, & \end{array}\right\} \quad \ldots(23)$$

In eqns. (16) - (22), the primes denote derivatives with respect to y. Physically, u_0, θ_0, C_0 represent the mean velocity, the mean temperature and the mean species concentration respectively and it is found to be affected by the frequency of the free-stream oscillations. This was absent in Soundalgekar (1973a).

Part I — Mean Flow

We now study the mean flow—which is governed by eqns. (16), (18) and (21). These are coupled nonlinear equations whose exact solutions are not possible. Hence we expand u_0, θ_0 and ϵ_0 in powers of E, the Eckert number, for the Eckert number for incompressible fluids is always very small. Hence we assume

$$\left.\begin{array}{l} u_0(y) = u_{01}(y) + E u_{02}(y) \\ \theta_0(y) = \theta_{01}(y) + E^{\theta}_{02}(y) \\ C_0(y) = C_{01}(y) + E C_{02}(y). \end{array}\right\} \quad \ldots(24)$$

Substituting eqns. (24) in eqns. (16), (18) and (21), equating the coefficients of different powers of E, neglecting those of E^2, we get

$$u_{01}'' + \gamma u_{01}' = -\theta_{01} - N C_{01} \quad \ldots(25)$$

$$u_{02}'' + \gamma u_{02}' = -\theta_{02} - N C_{02} \quad \ldots(26)$$

$$\theta_{01}'' + \gamma P \theta_{01}' = 0 \quad \ldots(27)$$

$$\theta_{02}'' + \gamma P \theta_{02}' = -P\, Gr^2 \left[u_{01}'^2 + \frac{\epsilon^2}{2} u_{11}' \bar{u}_{11}' \right] \quad \ldots(28)$$

$$C_{01}'' + \gamma Sc C_{01}' = 0 \quad \ldots(29)$$

$$C_{02}'' + \gamma Sc C_{02}' = 0 \quad \ldots(30)$$

and the boundary conditions are

$$\left.\begin{array}{llll} u_{01}(0) = 0, & u_{02}(0) = 0, & \theta_{01}(0) = 1, & \theta_{02}(0) = 0, \\ C_{01}(0) = 1, & C_{02}(0) = 0, & & \\ u_{01}(\infty) = 1/Gr, & u_{02}(\infty) = 0, & \theta_{01}(\infty) = 0, & \theta_{02}(\infty) = 0, \\ C_{01}(\infty) = 0, & C_{02}(\infty) = 0. & & \end{array}\right\} \quad \ldots(31)$$

Solving eqns. (25) - (30) subject to the boundary conditions (31), the solutions for the mean flow are given below:

I : $Sc \neq 1$

$$\begin{aligned} u_0(y) = & \frac{1}{Gr} \cdot (1 - e^{-\gamma y}) + B_1(e^{-\gamma P y} - e^{-\gamma y}) + B_2(e^{-A_1 y} - e^{-\gamma y}) \\ & + E\,[B_{16}(e^{-\gamma P y} - e^{-\gamma y}) + B_{17}(e^{-2\gamma P y} - e^{-\gamma y}) \\ & + B_{18}(e^{-2A_1 y} - e^{-\gamma y}) + B_{19}(e^{-2\gamma y} - e^{-\gamma y}) \\ & + B_{20}(e^{-(\gamma P + A_1) y} - e^{-\gamma y}) + B_{21}(e^{-(\gamma + A_1) y} - e^{-\gamma y}) \\ & + B_{22}(e^{-(\gamma P + \gamma) y} - e^{-\gamma y}) + B_{23}(e^{-(A_4 + \bar{A}_4) y} - e^{-\gamma y})] \qquad \ldots(32) \end{aligned}$$

where

$$A_1 = \gamma Sc, \quad A_2 = \tfrac{1}{2}\,[\gamma P + ((\gamma^2 P^2) + i\omega P)^{1/2}]$$

$$A_3 = \tfrac{1}{2}\,[\gamma Sc + (\gamma^2 Sc^2 + i\omega Sc)^{1/2}]$$

$$A_4 = \tfrac{1}{2}\,[\gamma + (\gamma^2 + i\omega)^{1/2}]$$

$$A_5 = \tfrac{1}{2}\,[\gamma P + (\gamma^2 P^2 + 2i\omega P)^{1/2}],\ A_6 = \bar{A}_4$$

$$B_1 = -\frac{1}{\gamma^2 P(P-1)},\ B_2 = -\frac{N}{A_1(A_1 - \gamma)},\ B_3 = -\left(\frac{1}{Gr} + B_1 + B_2\right)$$

$$B_4 = -\frac{PGr^2 B_1^2}{2},\ B_5 = -\frac{PGr^2 A_1 B_2^2}{2(2A_1 - \gamma P)}$$

$$B_6 = -\frac{PGr^2 B_3}{2(2-P)},\ B_7 = -\frac{2PGr^2(\gamma P B_1 B_2)}{(\gamma P + A_1)}$$

$$B_8 = -\frac{2PGr^2(\gamma B_2 B_3 A_1)}{(\gamma + A_1)(\gamma + A_1 - \gamma P)},\ B_9 = -\frac{2P^2 Gr^2 B_1 B_3}{(P+1)}$$

$$B_{10} = -\frac{2P\epsilon^2 A_4 \bar{A}_4}{(A_4 + \bar{A}_4)(A_4 + \bar{A}_4 - \gamma P)}$$

$$B_{11} = -(B_4 + B_5 + B_6 + B_7 + B_8 + B_9 + B_{10})$$

$$B_{12} = \frac{4\gamma P^2 Gr B_1 A_4}{(\gamma P + A_4)^2 - \gamma P(\gamma P + A_4) - \frac{1}{4} i\omega P}$$

$$B_{13} = \frac{4PGrB_2 A_1 A_4}{(A_1 + A_2)^2 - \gamma P(A_1 + A_2) - \frac{1}{4} i\omega P}$$

$$B_{14} = \frac{4\gamma PGrB_3 A_4}{(\gamma + A_4)^2 - \gamma P(\gamma + A_4) - \frac{1}{4} i\omega P}$$

$$B_{15} = -(B_{12} + B_{13} + B_{14}),\ B_{16} = \frac{-B_{11}}{\gamma^2 P(P-1)}$$

(*equation continued on p.* 401)

$$B_{17} = \frac{-B_4}{2\gamma^2 P(2P-1)},\ B_{18} = \frac{-B_5}{2A_1(2A_1-\gamma)}$$

$$B_{19} = \frac{-B_6}{2\gamma^2},\ B_{20} = \frac{-B_7}{(\gamma P + A_1)(\gamma P + A_1 - \gamma)},\ B_{21} = \frac{-B_8}{A_1(\gamma + A_1)}$$

$$B_{22} = \frac{-B_9}{\gamma P(\gamma P + \gamma)},\ B_{23} = \frac{-B_{10}}{(A_4 + \bar{A}_4)(A_4 + \bar{A}_4 - \gamma)},$$

$$B_{24} = -(B_{16} + B_{17} + B_{18} + B_{19} + B_{20} + B_{21} + B_{22} + B_{23})$$

$$\theta_0(y) = e^{-\gamma P y} + E\,[B_4(e^{-2\gamma P y} - e^{-\gamma P y}) + B_5(e^{-2A_1 y} - e^{-\gamma P y}) + B_6(e^{-2\gamma y} - e^{-\gamma P y}) + B_7(e^{-(\gamma P + A_1)y} - e^{-\gamma P y}) + B_8(e^{-(\gamma + A_1)y} - e^{-\gamma P y}) + B_9(e^{-(\gamma P + \gamma)y} - e^{-\gamma P y}) + B_{10}(e^{-(A_4+\bar{A}_4)\cdot y} - e^{-\gamma P y})]. \quad \ldots(33)$$

Similarly, we have calculated u_0, θ_0 for $Sc = 1$.

Numerical values of u_0 and θ_0 are calculated and are shown in Figs. 1, 2 for different values of the parameters. Here P is chosen as 0.71 for air. In selecting values of Sc, the Schmidt number, the diffusing chemical species of most common interest in air are considered. We have listed the values of the Schmidt number in Table I for different concentration species. In water the value of the Schmidt number

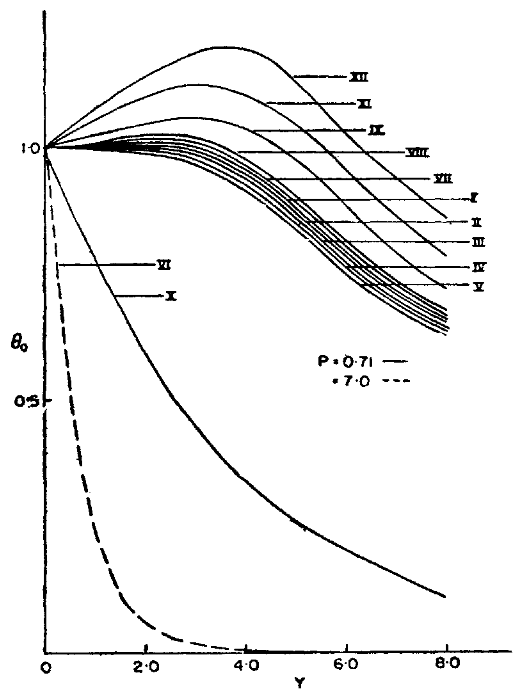

FIG. 1. Mean temperature profiles, $\epsilon = 0.2$

is 445 for ammonia, 453 for CO_2, 523 for sulphur dioxide, etc. Hence, we have considered one value of Schmidt number, viz., 500 approximately. The mean temperature profiles are shown on Fig. 1 for different values of *Sc*. We observe from Fig. 1 that an increase in *Sc* leads to a decrease in the mean temperature when *Gr*, γ, *E*, *N* or ω are constant. Also an increase in *N* or *Gr* or the frequency parameter ω leads to a rise in the mean temperature. Greater viscous dissipative heat also causes a rise in the mean temperature. The mean temperature decreases with increasing γ the suction parameter.

The mean velocity profiles are shown on Fig. 2 for all values of *Sc*. We observe from Fig. 2 that an increase in *Sc* leads to a decrease in the mean velocity but the mean velocity increases with increasing ω. The mean velocity also increases with increasing *N*, *E*, *Gr* or γ.

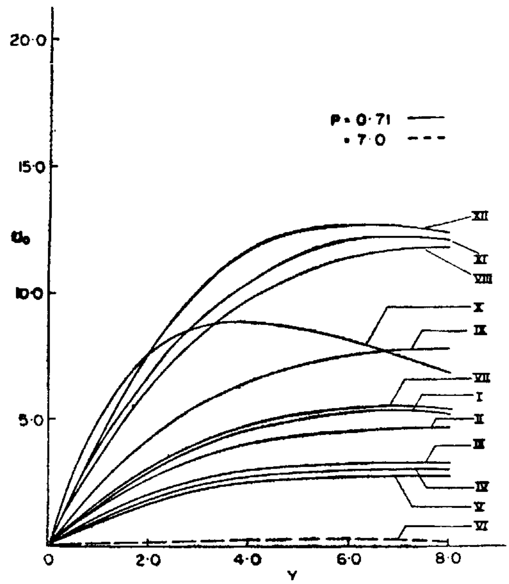

FIG. 2. Mean velocity profiles, $\epsilon = 0.2$.

TABLE I

Thermodynamic and transport properties at 25 °C and 1 atmosphere

Species	*Sc*	*P*
H_2	0.24	0.71
He	0.30	0.71
H_2O	0.60	0.71
NH_3	0.78	0.71
CO_2	1.00	0.71
	500.00	7.0

Table for Figures 1, 2, 3 and 4

Gr	γ	E	N	Sc	ω	
0.5	0.2	0.01	0.2	0.24	5.0	I
0.5	0.2	0.01	0.2	0.30	5.0	II
0.5	0.2	0.01	0.2	0.60	5.0	III
0.5	0.2	0.01	0.2	0.78	5.0	IV
0.5	0.2	0.01	0.2	1.00	5.0	V
0.5	0.2	0.01	0.2	500.00	5.0	VI
0.5	0.2	0.01	0 2	0.24	10.0	VII
0.5	0.2	0.01	0.5	0.24	5.0	VIII
0.5	0.2	0.02	0.2	0.24	5.0	IX
0.5	0.4	0.01	0.2	0.24	5.0	X
1.0	0.2	0.01	0.2	0.24	5.0	XI
1.0	0.2	0.01	0.2	1.0	5.0	XII

Knowing the mean velocity field, we can now study the effects of mass transfer and frequency on the mean skin-friction. It is given by

$$\tau_m = \tau'/\rho U_0^2 = -\left.\frac{du_0}{dy}\right|_{y=0}. \qquad \ldots(34)$$

From (32) and (34), we have

$$\tau_m = \frac{\gamma}{Gr} + B_1\gamma(1-P) + B_2(\gamma - A_1) + E\,[B_{16}\gamma(1-P) + B_{17}\gamma(1-2P)$$
$$+ B_{18}(\gamma - 2A_1) + B_{19}(-\gamma) + B_{20}(\gamma(1-P) - A_1)$$
$$+ B_{21}(-A_1) + B_{22}(-\gamma P) + B_{23}(\gamma - (A_4 + \bar{A}_4))]. \qquad \ldots(35)$$

The numerical values of τ_m are calculated for different values of the parameters and they are entered in Table II. We observe from this table that an increase in Sc leads to a decrease in the mean skin-friction. In case of water, the mean skin-friction is observed to be negative and hence we conclude that there may occur separation of flow near the plate. The mean skin-friction also increases with increasing N, E or Gr. But an increase in the suction parameter γ leads to a decrease in τ_m such that it is negative. This indicates that there may occur separation. At small values of ω, τ_m increases with increasing ω and at large values of ω, τ_m decreases with increasing ω.

We now study the mean rate of heat transfer. It is given by

$$q_m = \left.\frac{\partial\theta_0}{\partial y}\right|_{y=0}$$

where

$$q_m = -\frac{q'_m \nu}{U_0 k(T'_w - T'_\infty)}. \qquad \ldots(36)$$

From (33) and (36), we have

$$q_m = (-\gamma P) + E\,[B_4(-\gamma P) + B_5(\gamma P - 2A_1) + B_6\gamma(P-2) + B_7(-A_1) + B_8(\gamma(P-1) - A_1) + B_9(-\gamma) + B_{10}(\gamma P - (A_4 + \bar{A}_4))]. \quad \ldots(37)$$

The numerical values of q_m are entered in Table II. We observe that q_m also decreases with increasing Sc and ω. An increase in N, E or Gr leads to an increase in q_m but q_m decreases very fast with increasing the suction parameter γ.

TABLE II

Gr	γ	E	N	$\omega/Sc =$	$P = 0.71$ 0.24	0.30	0.60	0.78	1.0	$P = 7$ 500
					Values of q_m					
0.5	0.2	0.01	0.2	5.0	0.2434	0.1828	0.0767	0.0550	0.0306	−1.3826
0.5	0.2	0.01	0.2	10.0	0.2436					
0.5	0.2	0.01	0.2	75.0	0.0319					
0.5	0.2	0.01	0.5	5.0	0.8378					
0.5	0.2	0.02	0.2	5.0	0.6288					
0.5	0.4	0.01	0.2	5.0	−0.2241					
1.0	0.2	0.01	0.2	5.0	1.3430					
					Values of τ_m					
0.5	0.2	0.01	0.2	5.0	19.5331	17.3781	13.3340	12.4438	11.4004	1.1276
0.5	0.2	0.01	0.2	10.0	19.5308					
0.5	0.2	0.01	0.2	75.0	11.4280					
0.5	0.2	0.01	0.5	5.0	40.2176					
0.5	0.2	0.02	0.2	5.0	27.4527					
0.5	0.4	0.01	0.2	5.0	6.6747					
1.0	0.2	0.01	0.2	5.0	42.7089					

Part II — Unsteady Flow

We now solve eqns. (17), (19), (20) and (22) by substituting as follows:

$$\left.\begin{aligned} u_1 &= u_{11} + Eu_{12}, \quad \theta_1 = \theta_{11} + E\theta_{12} \\ \theta_2 &= \theta_{21} + E\theta_{22}, \quad C_1 = C_{11} + EC_{12} \end{aligned}\right\} \quad \ldots(38)$$

and taking into account the solutions of eqns. (25) – (30), we get the following:

$$u_1(y) = \frac{2}{Gr}(1 - e^{-A_4 y}) + E\,[B_{25}(e^{-A_2 y} - e^{-A_4 y}) + B_{26}(e^{-(\gamma P + A_4)y} - e^{-A_4 y}) + B_{27}(e^{-(A_1 + A_2)y} - e^{-A_4 y}) + B_{28}(e^{-(\gamma + A_4)y} - e^{-A_4 y})] \quad \ldots(39)$$

where

$$B_{25} = \frac{-B_{15}}{(A_2)^2 - \gamma A_2 - \frac{1}{4}i\omega}$$

$$B_{26} = \frac{-B_{12}}{(\gamma P + A_4)^2 - \gamma(\gamma P + A_4) - \frac{1}{4}i\omega}$$

$$B_{27} = \frac{-B_{13}}{(A_1 + A_2)^2 - \gamma(A_1 + A_2) - \frac{1}{4}i\omega}$$

$$B_{28} = \frac{-B_{14}}{(\gamma + A_4)^2 - \gamma(\gamma + A_4) - \frac{1}{4}i\omega}$$

$$B_{29} = -(B_{25} + B_{26} + B_{27} + B_{28}).$$

$$\theta_1(y) = E\,[B_{12}(e^{-(\gamma P + A_4)y} - e^{-A_2 y}) + B_{13}(e^{-(A_1 + A_2)y} - e^{-A_2 y}) + B_{14}(e^{-(\gamma + A_4)y} - e^{-A_2 y})] \quad \ldots(40)$$

$$\theta_2(y) = E \,.\, B_{30}(e^{-2A_4 y} - e^{-A_5 y})$$

where

$$B_{30} = -\frac{2P(A_4)^2}{4(A_4)^2 - 2\gamma P \cdot (A_4) - \frac{1}{2}i\omega P} \quad \ldots(41)$$

$$C_1(y) = 0. \quad \ldots(42)$$

We now express these solutions in terms of the fluctuating parts of the unsteady components of velocity and temperature as follows:

$$u(y, t) = u_0(y) + \epsilon(M_r \cos \omega t - M_i \sin \omega t) \quad \ldots(43)$$

$$\theta(y, t) = \theta_0(y) + \epsilon(\theta_{1r} \cos \omega t - \theta_{1i} \sin \omega t) + \epsilon^2(\theta_{2r} \cos 2\omega t - \theta_{2i} \sin 2\omega t) \quad \ldots(44)$$

where

$$M_r + iM_i = u_1, \quad \theta_{1r} + i\theta_{1i} = \theta_1, \quad \theta_{2r} + i\theta_{2i} = \theta_2.$$

The unsteady part of C is zero. From eqns. (43) and (44), we now derive the expressions for the transient velocity and transient temperature, for $\omega t = \pi/2$, as

$$\left.\begin{aligned} u(y, \pi/2\omega) &= u_0(y) - \epsilon M_i \\ \theta(y, \pi/2\omega) &= \theta_0(y) - \epsilon\theta_{1i} - \epsilon^2\theta_{2r}. \end{aligned}\right\} \quad \ldots(45)$$

The transient velocity profiles and temperature profiles calculated from (45) are shown in Figs. 3 and 4 respectively. The transient velocity and temperature field behave in the same manner as in case of the mean velocity and temperature field. In Fig. 5, we have shown the mean concentration profiles. We conclude from this figure that C_0 decreases with increasing Sc or γ.

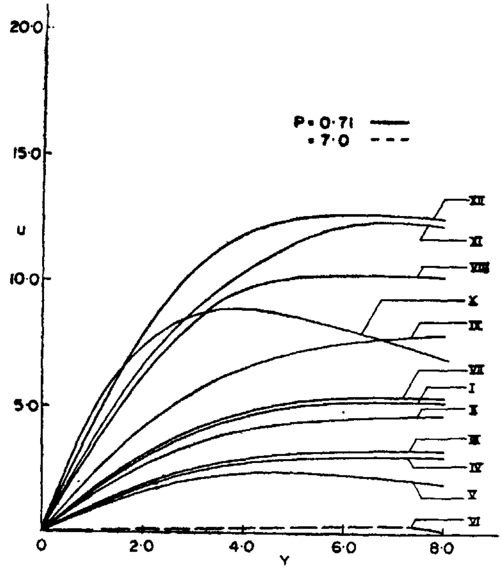

FIG. 3. Transient velocity profiles, $\omega t = \pi/2$, $\epsilon = 0.2$.

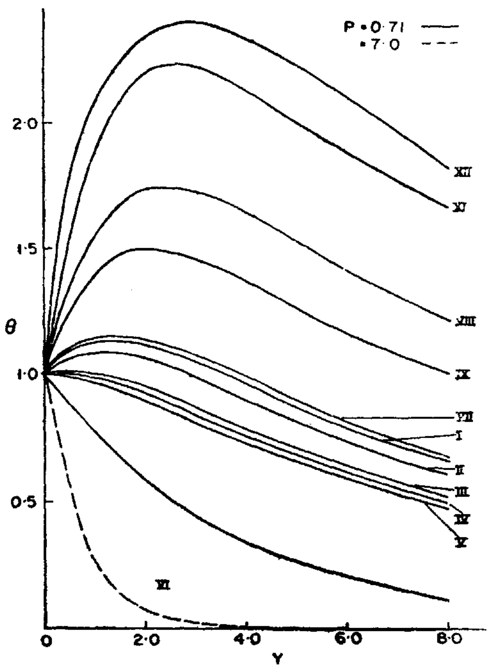

FIG. 4. Transient temperature profiles, $\omega t = \pi/2$, $\epsilon = 0.2$.

We now study the amplitude and phase of the skin-friction. It is given by

$$\tau = \tau_m + \epsilon e^{i\omega t} \left. \frac{du_1}{dy} \right|_{y=0} \qquad \ldots(46)$$

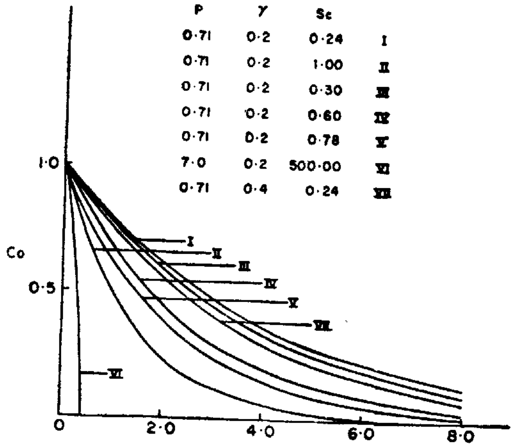

FIG. 5. Mean concentration profiles.

which can be written in terms of the amplitude and phase as

$$\tau = \tau_m + \epsilon \mid B \mid \cos(\omega t + \alpha), \qquad \ldots(47)$$

where $$B = B_r + iB_i = \left.\frac{du_1}{dy}\right|_{y=0}$$

and $$\tan \alpha = B_i/B_r.$$

TABLE III

Gr	γ	E	N	ω/Sc =	P = 0.71					P = 7
					0.24	0.30	0.60	0.78	1.0	500
					Values of | B |					
0.5	0.2	0.01	0.2	5.0	4.7555	4.7563	4.7581	4.7589	4.7520	4.7634
0.5	0.2	0.01	0.2	10.0	6.6089					
0.5	0.2	0.01	0.2	75.0	17.6049					
0.5	0.2	0.01	0.5	5.0	4.7506					
0.5	0.2	0.02	0.2	5.0	4.7466					
0.5	0.4	0.01	0.2	5.0	5.0755					
1.0	0.2	0.01	0.2	5.0	2.3648					
					Values of tan α					
0.5	0.2	0.01	0.2	5.0	0.8759	0.8763	0.8715	0.8775	0.8742	0.8799
0.5	0.2	0.01	0.2	10.0	0.9125					
0.5	0.2	0.01	0.2	75.0	0.9678					
0.5	0.2	0.01	0.5	5.0	0.8732					
0.5	0.2	0.02	0.2	5.0	0.8708					
0.5	0.4	0.01	0.2	5.0	0.7731					
1.0	0.2	0.01	0.2	5.0	0.8609					

The values of $|B|$ and $\tan\alpha$ calculated numerically are entered in Table III. We observe from this table that the amplitude $|B|$ increases with increasing Sc. An increase in ω, γ leads to an increase in the value of the amplitude whereas an increase in N, E or Gr leads to a decrease in the value of $|B|$. Again we observe from this table that the values of $\tan\alpha$ being positive for all values of Sc, ω, N, E, γ or Gr, we conclude that there is always a phase-lead.

It is now proposed to study the amplitude and phase of the rate of heat transfer. It is given by

$$q = q_m + \epsilon e^{i\omega t} \left.\frac{d\theta_1}{dy}\right|_{y=0} + \epsilon e^{2i\omega t} \left.\frac{d\theta_2}{dy}\right|_{y=0} \qquad \ldots(48)$$

which can be written in terms of the amplitude and phase as

$$q = q_m + \epsilon \, |Q_1| \cos(\omega t + \alpha_1) + \epsilon^2 \, |Q_2| \cos(\omega t + \alpha_2)$$

where

$$\left.\begin{aligned} Q_1 &= Q_{1r} + iQ_{1i} = \left.\frac{d\theta_1}{dy}\right|_{y=0} \\ Q_2 &= Q_{2r} + iQ_{2i} = \left.\frac{d\theta_2}{dy}\right|_{y=0} \\ \tan\alpha_1 &= Q_{1i}/Q_{1r}, \quad \tan\alpha_2 = Q_{2i}/Q_{2r}. \end{aligned}\right\} \qquad \ldots(49)$$

The numerical values of the amplitude of the first harmonic $|Q_1|$ and the amplitude of the second harmonic $|Q_2|$ are given in Table IV. We observe from this table that the amplitude of the first harmonic $|Q_1|$ decreases with increasing Sc but increases with increasing ω. An increase in N, E or Gr leads to an increase in the value of $|Q_1|$ but $|Q_1|$ decreases with increasing γ.

TABLE IV

					$P = 0.71$					$P = 7$
Gr	γ	E	N	$\omega/Sc =$	0.24	0.30	0.60	0.78	1.0	500
					Values of $\vert Q_1 \vert$					
0.5	0.2	0.01	0.2	5.0	0.08397	0.0776	0.0641	0.0597	0.0446	0.04181
0.5	0.2	0.01	0.2	10.0	0.0853					
0.5	0.2	0.01	0.2	75.0	0.0873					
0.5	0.2	0.01	0.5	5.0	0.1290					
0.5	0.2	0.02	0.2	5.0	0.1679					
0.5	0.4	0.01	0.2	5.0	0.0443					
1.0	0.2	0.01	0.2	5.0	0.1649					

(*Table Continued on p.* 409)

Values of $|Q_2| \times 10^2$

$\gamma/\omega=$	5	10	75
0.2	0.550	0.755	1.9773
0.4	0.609		

The amplitude of the second harmonic $|Q_2|$ is not affected by Sc or Gr as θ_{22} is independent of Sc and Gr. It increases with increasing ω and γ.

In Table V, the numerical values of the phases of the first and second harmonic of the rate of heat transfer are entered. We conclude from this table that there is always a phase-lead when $Sc \gtrsim 1$ but at $Sc = 1$, there is a phase-lag.

TABLE V

					$P = 0.71$					$P = 7$
Gr	γ	E	N	$\omega/Sc =$	0.24	0.30	0.60	0.78	1.0	500
					Values of $\tan \alpha_1$					
0.5	0.2	0.01	0.2	5.0	0.0479	0.0508	0.0589	0.0631	−0.0256	0.0205
0.5	0.2	0.01	0.2	10.0	0.0349					
0.5	0.2	0.01	0.2	75.0	0.0133					
0.5	0.2	0.01	0.5	5.0	0.0432					
0.5	0.2	0.02	0.2	5.0	0.0479					
0.5	0.4	0.01	0.2	5.0	0.0801					
1.0	0.2	0.01	0.2	5.0	0.0486					
					Values of $\tan \alpha_2$					
0.5	0.2	0.01	0.2	5.0	0.8202					
0.5	0.2	0.01	0.2	10.0	0.8687					
0.5	0.2	0.01	0.2	75.0	0.9494					
0.5	0.4	0.01	0.2	5.0	0.6752					

3. CONCLUSIONS

(1) The mean velocity and the mean temperature decrease with increasing Sc. (2) An increase in ω leads to an increase in the mean velocity and the mean temperature. (3) An increase in γ leads to a decrease in the mean temperature and an increase in mean velocity. (4) An increase in N, E or Gr leads to an increase in the mean velocity and the mean temperature. (5) τ_m and q_m decrease with increasing Sc. But at large values of Sc, there may occur separation of water at the plate. (6) τ_m, q_m decrease with increasing γ. (7) An increase in N leads to an increase in q_m and a decrease in τ_m. (8) τ_m, q_m increase with increasing E or Gr. (9) $|B|$ increases with increasing

Sc. (10) $|B|$ increases with increasing ω and γ but decreases with increasing *N*, *E* or *Gr*. (11) There is always a phase lead in case of the skin-friction but for the rate of heat transfer, there is a phase-lead when $Sc \gtrsim 0$. (12) $|Q_1|$ decreases with increasing *Sc* or γ and increases with increasing *N*, *E* or *Gr*. (13) $|Q_2|$ is not affected by *Sc* or *Gr*. It increases with increasing ω and γ.

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