SOME THEOREMS ON MULTIDIMENSIONAL INTEGRAL TRANSFORMS

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In this paper, we establish three theorems concerning certain multidimensional integral transforms which are contained, as special cases, in one of the two general classes of multidimensional H-function transformations introduced and studied by Srivastava and Panda (1978). The first two theorems express the relationships between images and originals of related functions in these three special transforms, while the third gives interconnections between images of related functions in the special multidimensional H-function transform and the multidimensional Bessel transform. All three theorems are believed to be new, and they extend and unify a large number of similar theorems for single and double integral transforms obtained from time to time by several authors.

1. Introduction and Definitions

Recently, Srivastava and Panda (1978) introduced and studied two general multiple integral transforms whose kernels involve the *H*-function of several variables. These transforms were studied in great details by Srivastava and Panda (1978) in a series of papers. [For their definitions, see Srivastava and Panda (1978, Part I, p. 119, eqn. (1.1) and p. 121, eqn. (1.15)]].

In this paper we choose three important and useful special cases of the multidimensional *H*-function transform of the second kind [Srivastava and Panda 1978, Part I, p. 121, eqn. (1.15)] and establish three new and interesting theorems involving them. These special multidimensional integral transforms are listed below:

(i) The Multidimensional H-function Transform

$$T_{H} \{ f(x_{1},...,x_{r}) ; p_{1},...,p_{r} \} = p_{1} \quad p_{r} \int_{0}^{\infty} ... \int_{0}^{\infty} f(x_{1},...,x_{r})$$

$$0,0:(m',n');...;(m^{(r)}..n^{(r)})$$

$$.H$$

$$p,q:[p',q'];...;[p^{(r)},q^{(r)}]$$

$$\left[[(a):\theta',...,\theta^{(r)}]:[(b'):\phi'];...;[(b^{(r)}):\phi^{(r)}]; p_{1}x_{1},...,p_{r}x_{r} \right] dx_{1}.dx_{r}$$

$$\left[(c):\epsilon',...,\epsilon^{(r)}]:[(d'):\delta'];...;[(d^{(r)}):\delta^{(r)}]; p_{1}x_{1},...,p_{r}x_{r} \right] dx_{1}.dx_{r}$$

which follows from the abovementioned multidimensional *H*-function transform of the second kind [see Srivastava and Panda 1978, Part I, p. 121, eqn. (1.15)] in the special case $m_1 = ... = m_r = 1$.

The kernel of the above transform is a special case of the multivariable H-function defined by Srivastava and Panda [1976a, p. 271, eqn. (4.1)]. Also, from the work of Srivastava and Panda (1976b), it follows that

$$H = \begin{cases} 0, 0: (m', n'); \dots; (m(r), n(r)) & \begin{pmatrix} z_1 \\ \vdots \\ z_r \end{pmatrix} \\ = \begin{cases} 0 & |z_1| & |\alpha_1| \\ 0 & |z_1| & |\alpha_1| \\ \end{pmatrix}, \dots, |z_r| & |\alpha_r| \\ \end{pmatrix}, \min \left\{ |z_1|, \dots, |z_r| \right\} \rightarrow 0 \\ -\beta_1, \dots, |z_r| & |\alpha_r| \\ \end{pmatrix}, \min \left\{ |z_1|, \dots, |z_r| \right\} \rightarrow \infty \\ \dots (1.2)$$

where

$$\alpha^{i'} = \min_{1 \leq j \leq m^{(i)}} \left\{ \operatorname{Re} \left(\frac{d_j^{(i)}}{\delta_j^{(i)}} \right) \right\} \dots (1.3)$$

$$\beta_i = \min_{1 \leq j \leq n^{(i)}} \left\{ \operatorname{Re} \left(\frac{1 - b_j^{(i)}}{\phi_j^{(i)}} \right) \right\}$$

$$\forall i \in (1, \dots, r).$$

(ii) The Multidimensional G-function Transform

$$T_{G} \{ f(x_{1},...,x_{r}); p_{1},...,p_{r} \} = p_{1} ... p_{r} \int_{0}^{\infty} ... \int_{0}^{\infty} f(x_{1},...,x_{r})$$

$$\frac{M'+1,0}{M',M'+1} \left[p_{1} x_{1} \middle| \frac{(g'_{j}+h'_{j})_{1,M'}}{(h'_{j})_{1,M'},\gamma_{1}} \right] ...$$

$$\frac{M^{(r)}+1,0}{G} \left[p_{r} x_{r} \middle| \frac{(g'_{j})_{1}+h'_{j}}{(h'_{j})_{1},M'^{(r)},\gamma_{r}} \right] dx_{1} ... dx_{r} ...(1.4)$$

This transform is an analogue of the G-function transform in one variable studied by Bhise (1959); indeed, it is contained in (1.1) and hence also in the multidimensional H-function transform of the second kind Srivastava and Panda [1978 Part, I, p.12 1, eqn. (1.15)].

(iii) The Multidimensional Bessel Function Transform $T_{K} \{ f(x_{1},...,x_{r}); p_{1},...,p_{r} \}$ $= \left(\frac{2}{\pi} \right)^{r/2} p_{1} ... p_{r} \int_{0}^{\infty} ... \int_{0}^{\infty} f(x_{1},...,x_{r}) (p_{1} x_{1})^{1/2} ... (p_{r} x_{r})^{1/2}$ $K_{V^{1}} (p_{1} x_{1}) ... K_{V_{r}} (p_{r} x_{r}) dx_{1} ... dx_{r}, \qquad ... (1.5)$

The above transform in r variables is analogous to the Bessel function transform of one variable due to Meijer (1940) and is a special case of the multidimensional H-function transform due to Srivastava and Panda [1978, Part I, p. 121, eqn. 1.15)].

In each of the definitions (1.1), (1.4) and (1.5), it is assumed that the multiple integral converges absolutely.

2. RESULTS REQUIRED

The following results will be required in establishing our main theorems:

Theorem A (Gupta 1980) - If

$$h_1(p_1,...,p_r) = T_1\{h_2(x_1,...,x_2) g(x_1,...,x_r); p_1,...,p_r\}$$

$$= \int_0^\infty ... \int_0^\infty k_1(p_1 x_1,...,p_r x_r) h_2(x_1,...,x_r) \cdot g(x_1,...,x_r) dx_1 ... dx_r ... (2.1)$$

and
$$h_2(p_1^{\sigma_1}, ..., p_r^{\sigma_r}) = T_2\{f(x_1, ..., x_r); p_1, ..., p_r\}$$

$$= \int_0^\infty ... \int_0^\infty k_2(p_1x_1, ..., p_rx_r) f(x_1, ..., x_r) dx_1...dx ... (2.2)$$

then
$$h_1(\mathbf{p}_1, ..., \mathbf{p}_r) = (\sigma_1 ... \sigma_r) \int_0^\infty ... \int_0^\infty f(x_1, ..., x_r) \phi(x_1, ..., x_r;$$

$$p_1, ..., p_r) dx_1...dx_r ...(2.3)$$

where
$$\phi(p_1,...,p_r,\alpha_1,...,\alpha_r) = T_2 \left\{ x_1^{\sigma_1^{-1}} ... x_r^{\sigma_1^{-1}} g(x_r^{\sigma_1},...,x_r^{\sigma_r}) \right.$$

$$\left. k_1\left(\alpha_1 x_1^{\sigma_1},...,\alpha_r x_r^{\sigma_r}\right); p_1,...,p_r \right\} ... (2.4)$$

 $\sigma_1, \ldots, \sigma_r$ are non-zero real numbers of the same sign, each of $\alpha_1, \ldots, \alpha_r$ is independent of p_1, \ldots, p_r , and all the multiple integrals involved in (2.1) to (2.4) are assumed to be absolutely convergent.

Theorem B (Gupta 1980)-If

$$h_1(p_1,...,p_r) = T_1 \left\{ x_1^{(c_1-\sigma_1)/\sigma_1} ... x_r^{(c_r-\sigma_r)/\sigma_r} f(x_1,...,x_r); p_1,...,p_r \right\} ... (2.5)$$

and
$$h_2(p_1,...,p_r) = T_2 \left\{ f\left(x_1^{\sigma_1},...,x_r^{\sigma_r}\right); p_1,...,p_r \right\}$$
 ...(2.6)

then
$$h_1(\mathbf{p}_1, ..., \mathbf{p}_r) = (\sigma_1, ..., \sigma_r) \int_0^\infty ... \int_0^\infty h_2(x_1, ..., x_r)$$

$$\phi(x_1, ..., x_r, \mathbf{p}_1, ..., \mathbf{p}_r) dx_1 ... dx_r$$

$$\phi(x_1, ..., x_r, \mathbf{p}_1, ..., \mathbf{p}_r) dx_1 ... dx_r ... (2.7)$$
where $p_1^{-c_1-1} ... p_r^{-c_r+1} k_1 (\alpha_1 p_1^{-\sigma_1}, ..., \alpha_r p_r^{-\sigma_r})$

where
$$p_1$$
 ..., p_r k_1 (α_1 p_1 ,..., α_r p_r)
$$= T_2 \{ \phi(x_1, ..., x_r, \alpha_1, ..., \alpha_r) ; p_1, ..., p_r \} \qquad ... (2.8)$$

 σ_i and α_i (i = 1, ..., r) are of the same type as stated with Theorem A, and the various multiple integrals involved in (2.5) to (2.8) are assumed to be absolutely convergent.

Also, the following results easily obtainable from a recent result given by Gupta and Bhatt (1981) will be required in the sequel:

(I)
$$T_{G} \left\{ \begin{array}{l} x_{1}^{\rho_{1}} \dots x_{r}^{\rho_{r}} \stackrel{\mathbf{0}}{H}, \quad 0; \quad (m', n'); \dots; \quad (m(r), n(r)) \\ H_{p, q; [p', q']; \dots; [p(r), q(r)]} \stackrel{\sigma_{1}}{\bigcup} \frac{\sigma_{1}}{\alpha_{r} x_{r}}; \quad p_{1}, \dots, p_{r} \end{array} \right\}$$

$$= p_{1}^{-\rho_{1}} \dots p_{r}^{-\rho_{r}} \stackrel{\mathbf{0}}{\bigcup}, \quad 0: \quad (m', n' + M' + 1); \dots, (m(r), n(r) + M(r) + 1) \\ p_{1} \dots p_{r} \stackrel{\mathbf{H}}{\bigcup} \frac{H_{p, q; [p' + M' + 1, q' + M'], \dots, [p(r) + M(r) + 1, q(r) + M(r)]}}{\left[\begin{pmatrix} \sigma_{1} \\ \alpha_{1} / p_{1} \\ \sigma_{r} \\ \sigma_{r} \\ \vdots \\ \sigma_{r} \end{pmatrix} \right] \left[(a); \theta', \dots, \theta^{(r)} \right] : P_{1}; \dots; P_{r} \\ \frac{\sigma_{1}}{\bigcup} \frac{\sigma_{1}}{\bigcup} \left[(c): \epsilon', \dots, \epsilon^{(r)} \right] : Q_{1}; \dots; Q_{r} \right] \dots (2.9)$$

$$\forall i \in \{1, \dots, r\}$$

where
$$P_i$$
 stands for $[(b^{(i)}):\phi^{(i)}]_{i,n(i)}, [-h_j^{(i)}-\rho_i:\sigma_i]_{1,M}^{(i)},$

$$[-\gamma_i-\rho_i:\sigma_i], [(b^{(i)}):\phi^{(i)}]_{n(i)+j,p(i)}$$

 Q_i stands for $[(d^{(i)}):\delta^{(i)}]_{1,m^{(i)}}$, $[-g_j^{(i)}-h_j^{(i)}-\rho_i:\sigma_i]_{1,M^{(j)}}[(d^{(i)}):\delta^{(i)}]_{m^{(i)}+1,q^{(i)}}$ and Re $(p_i)>0$, $\sigma_i>0$

$$\min_{1 \leqslant j \leqslant m^{(i)}} \left\{ \operatorname{Re} \left(\left[\frac{d_{j}^{(i)}}{\delta_{j}^{(i)}} \right] \right\} + \min_{1 \leqslant j \leqslant M^{(i)}} \left\{ \operatorname{Re} \left(h_{j}^{(i)}, \gamma_{i} \right) \right\} > - \operatorname{Re} \left(\rho_{i} + 1 \right) \right\}$$

(II)
$$T_{K}$$
 $\begin{cases} x_{1}^{\rho_{1}} & \dots & x_{r}^{\rho_{r}} & H_{rp,q}^{0,0}; (m', n'); \dots; (m^{(r)}, n^{(r)}) \\ \vdots & \vdots & \vdots \\ \alpha_{r} & x_{r}^{\sigma_{r}} & \vdots \end{cases}; p_{1}, \dots, p_{r} \end{cases}$ $= (4\pi)^{-r/2} p_{1}^{-\rho_{1}} & \dots & p_{r}^{-\rho_{r}} & 0,0 : (mr,nr+2); \dots; (m^{(r)}, n^{(r)}, n^{(r)}+2) \\ & p_{1}^{-\rho_{1}} & \dots & p_{r}^{-\rho_{r}} & H_{p,q}^{-\rho_{1}}; \dots; [p^{(r)+2},q^{(r)}] \end{cases}$ $= (\alpha_{1}/p_{1}^{-\sigma_{1}}) \begin{bmatrix} (a) : \theta^{r}, \dots, \theta^{(r)} \end{bmatrix} : I_{1}; \dots; I_{r} \\ \vdots & \vdots & \vdots \\ \alpha_{r}/p_{r}^{-\sigma_{r}} & [(c) : \epsilon^{r}, \dots, \epsilon^{(r)} [: J_{1}; \dots; J_{r}) \end{bmatrix} \dots (2.10)$ $\forall i \in (1,\dots,r),$

where I_i stands for $[(b^{(i)}):\phi^{(i)}]_{1,n}$ (i), $\left[\frac{1}{4}\pm\frac{v_i}{2}-\frac{\rho_i}{2};\frac{\sigma_s}{2}\right]$, $[(b^{(i)}):\phi^{(i)}]_{n(i)+1,p(i)}$

 J_i stands for $[(d^{(i)}):\delta^{(i)}]_{1,q}(i)$ and Re $(p_i)>0$; $\sigma_i>0$

$$\sigma_i \quad \min_{1 \leqslant j \leqslant m^{(i)}} \quad \left\{ \operatorname{Re} \left(\frac{d_j^{(i)}}{\delta_j^{(i)}} \right) \right\} + \min_{1 \leqslant j \leqslant m^{(i)}} \left\{ \operatorname{Re} \left(\rho_i \pm \nu_i + \frac{3}{2} \right) \right\} > 0.$$

(III)
$$T_{K} \left\{ \left(\frac{x_{1}}{2} \right) \dots \left(\frac{x_{r}}{2} \right) \right\} = \begin{pmatrix} 0, 0 : (m', n'); \dots; (m^{(r)}, n^{(r)}) \\ H_{p, q} : [p', q' + 2]; \dots; [p^{(r)}, q^{[r]} + 2] \end{pmatrix}$$

$$\begin{pmatrix} \alpha_{1} \left(\frac{x_{1}}{2} \right) & M : K_{1}; \dots; K_{r} \\ \vdots & & \vdots \\ \alpha_{r} \left(\frac{x_{r}}{2} \right)^{\sigma_{r}} & N : L_{1}; \dots; L_{r} \end{pmatrix}; p_{1} \dots, p_{r} \right\},$$

$$= \pi^{-r/2} p_{1}^{-c_{1}^{-1}} p_{r}^{-c_{r}^{-1}} H_{p, q}^{0, 0} : (m', n'); \dots;$$

$$= m \begin{pmatrix} (m^{(r)}, n^{(r)}) & \cdots \\ p_{r} : [p', q']; \dots; \\ -\sigma_{1} & \cdots \\ p_{r} : [p^{(r)}, q^{(r)}] & \cdots \end{pmatrix}$$

$$= m \begin{pmatrix} (m^{(r)}, n^{(r)}) & \cdots \\ \vdots & \cdots \\ \alpha_{r} p_{r}^{-r} & \cdots \end{pmatrix}$$

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$$= m \begin{pmatrix} (m^{(r)}, n^{(r)}) & \cdots \\ \vdots & \cdots \\ \alpha_{r} p_{r}^{-r} & \cdots \end{pmatrix}$$

$$= m \begin{pmatrix} (m^{(r)}, n^{(r)}) & \cdots \\ \vdots & \cdots \\ \alpha_{r} p_{r}^{-r} & \cdots \end{pmatrix}$$

$$= m \begin{pmatrix} (m^{(r)}, n^{(r)}) & \cdots \\ \vdots & \cdots$$

 $\forall i \in (1,...,r),$

where M and N stand for $[(a):\theta',...,\theta^{(r)}]$ and

[(c) : ϵ' ,..., $\epsilon^{(r)}$] respectively :

 K_i stands for $[(b^{(i)}):\phi^{(i)}]_{1,p(i)}$

$$L_i$$
 stands for $\left[(d_j^{(i)}) : \delta^{(i)} \right]_{1,q(i)}$, $\left[\frac{1}{4} \pm \frac{\mathsf{v}_i}{2} - \frac{c_i}{2} \right]$,

and Re $(p_i) > 0$, $\sigma_i > 0$

$$\sigma_i \qquad \lim_{1 \leq j \leq m^{(i)}} \left\{ \operatorname{Re}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right) \right\} + \min \left\{ \operatorname{Re}\left(c_i \pm v_i + \frac{3}{2}\right) \right\} > 0.$$

3. MAIN THEOREMS

If in Theorem A, we take the transforms T_1 and T_2 to be the multidimensional transforms T_H and T_G defined by (1.1) and (1.4), respectively, and put

$$g(x_1,...,x_r) = \frac{(\rho_r + 1 - \sigma_1)/\sigma_1}{x_1} ... x_r \frac{(\rho_r + 1 - \sigma_r)/\sigma_r}{x_r} ... (3.1)$$

therein, we obtain the following theorem with the help of (2.9) and after a little simplification.

Theorem 1-If

$$h_1(p_1,...,p_r) = T_H \left\{ h_2(x_1, ..., x_r) \ x_{1_1}^{(\rho_1+1-\sigma_1)/\sigma_1} ... \ x_r^{(\rho_r+1-\sigma_r)/\sigma_r}; p_1, ..., p_r \right\} ...(3.2)$$

and
$$h_2 \begin{pmatrix} p_1^{\sigma_1}, ..., p_1^{\sigma_r} \end{pmatrix} = T_G \left\{ f(x_1, ..., x_r); p_1, ..., p_r \right\}_r;$$
 ...(3.3)

then
$$h_1(p_1, ..., p_r) = \sigma_1 p_1 ... \sigma_r p_r \int_{a}^{\infty} ... \int_{a}^{\infty} f(x_1, ..., x_r) x_1^{-\rho_1} ... x_r^{-\rho_r}$$

$$\begin{array}{c|c}
0,0; (m',n'+M'+1) & ; \dots; (m(r),n^{(r)}+M(r)+1) \\
H \\
p,q[p'+M'+1,q'+M']; \dots; [p(r)+M^{(r)}+l,q^{(r)}+M^{(r)}] \\
\begin{pmatrix}
p_1/x_1^{\sigma_1} & [(a):\theta', , \theta^{(r)}]:P_1, , P_r \\
p_r/x_r^{\sigma_r} & [(c):\epsilon', , \epsilon^{(r)}]:Q_1, , Q_r
\end{pmatrix}$$

$$dx_1 \dots dx_r \\
\dots (3.4)$$

provided that Re $(p_i) > 0$. $\sigma_i > 0$,

and
$$\sigma_i \min_{1 \leqslant j \leqslant m^{(i)}} \left\{ \operatorname{Re} \left(\frac{d_j^{(i)}}{\delta_j^{(i)}} \right) \right\} + \min_{1 \leqslant j \leqslant M^{(i)}} \left\{ \operatorname{Re} \left(h_j^{(i)}, \gamma_i \right) \right\}$$

$$> \operatorname{Re} \left(\rho_i + 1 \right), \ \forall \ i \in (1, ..., r),$$

the various integrals involved in (3.2) to (3.4) being assumed to be absolutely convergent.

The symbols P_i and Q_i are the same as those given with the result (2.9).

Again, if in Theorem A, we take the transforms T_1 and T_2 to be the multidimensional transforms T_H and T_K defined by (1.1) and (1.5), respectively, and $g(x_1, ..., x_r)$ the same function as given by (3.1), we obtain the following theorem with the help of (2.10)

Theorem 2—If $h_{1}(p_{1},...,p_{r}) = T_{H}(h_{2}(x_{1},...,x_{r}) x_{1}(\rho_{1}^{+1-\sigma_{1}})/\sigma_{1} ... x_{r}(\rho_{r}^{+1-\sigma_{r}})/\sigma_{r} p_{1} ...,p_{r}) ...(3.5)$ and $h_{2}(p_{1}^{\sigma_{1}}, p_{r}^{\sigma_{r}}) = T_{E}\left\{f(x_{1},...,x_{r}) : p_{1},...,p_{r}\right\} ...(3.6)$ then $h_{1}(p_{1},...,p_{r}) = \sigma_{1}p_{1},...,\sigma_{r}p_{r}(4\pi) \int_{0}^{-r/2} \int_{0}^{\infty} ... \int_{0}^{\infty} f(x_{1},...,x_{r})$ $x_{1}^{-\rho_{1}} ... x_{r}^{-\rho_{r}} H$ p, q: [p' + 2, q']; ...; [p(r) + 2, q(r)] $\left(\begin{array}{c|c} p_{1}/x_{1}^{\sigma_{1}} & [(a): \theta',...,\theta^{(r)}]: I_{1},...,I_{r} \\ p_{r}/x_{r} & [(c): \epsilon'_{1},...,\epsilon'_{r}]: I_{1},...,I_{r} \end{array}\right) dx_{1} ... dx_{r}$...(3.7)

provided that Re $(p_i) > 0$, $\sigma_i > 0$

$$\frac{\sigma_i \min}{1 \leqslant j \leqslant m^{(i)}} \left\{ \operatorname{Re} \left(\frac{d_j^{(i)}}{\delta_j^{(i)}} \right) \right\} + \min \left\{ \operatorname{Re} \left(\rho_i \pm \nu_i + \frac{3}{2} \right) \right\} > 0$$

 $\forall i \in (1,...,r)$

and the integrals occurring in (3.5) to (3.7) are assumed to be absolutely convergent. {The symbols I_i and J_i are defined with the result (2.10).}

Next, if in Theorem B, we take the transforms T_1 and T_2 to be the multidimensional transforms T_H and T_K defined by (1.1) and (1.5), respectively, we get the following theorem with the help of (2.11) and after certain simplifications.

Theorem 3-If

$$h_{1}(p_{1},...,p_{r}) = T_{H} \left\{ x_{1}^{(c_{1}-\sigma_{1})/\sigma_{1}} ... x_{r}^{(c_{r}-\sigma_{r},)/\sigma_{r}} f(x_{1},...,x_{r}); p_{1},...,p_{r} \right\}$$
...(3.8)
and
$$h_{2}(p_{1},...,p_{r}) = T_{K} \left\{ f x_{1}^{-\sigma_{1}},...,x_{r}^{-\sigma_{r}} \right\}; p_{1},...,p_{r} \right\}$$
...(3.9)
then
$$h_{1}(p_{1},...,p_{r}) = \sigma_{1} p_{1} ... \sigma_{r} p_{r} \pi^{r/2} \int_{9}^{\infty} ... \int_{0}^{\infty} h_{2}(x_{1},...,x_{r})$$

$$\left(\frac{x_{1}}{2} \right)^{c_{1}+1} ... \left(\frac{x_{r}}{2} \right)^{c_{r}+1} H_{p,q:[p',q'+2]; ...;[p(r),q(r)+2]}^{0,0:(m',n'); ...;(m(r),n(r))}$$

$$\vdots$$

$$p_{r} \left(\frac{x_{1}}{2} \right)^{\sigma_{1}} \left[(a) : \theta^{r}, ..., \theta^{(r)}] : K_{1}; ...; K_{r} \right] dx_{1} ... dx_{r}$$

$$[(c) : \epsilon^{r}, ..., \epsilon^{(r)}] : L_{1}; ...; L_{r}$$
...(3.10)

provided that Re $(P_i) > 0$, $\sigma_i > 0$

$$\frac{\sigma_{i} \min}{1 \leqslant j \leqslant m^{(i)}} \left\{ \operatorname{Re} \left(\frac{d_{j}^{(i)}}{\delta_{j}^{(i)}} \right) \right\} + \min \left\{ \operatorname{Re} \left(c_{i} \pm v_{i} + \frac{3}{2} \right) \right\} > 0$$

$$\forall i \in (1, ..., r)$$

and the multiple integrals in (3.8) to (3.10) are assumed to be absolutely convergent. {The symbols K_i and L_i stand for the parameters given with the result (2.11)}.

4. SPECIAL CASES

Since one of the kernels of the transforms involved in all of our theorems is a multivariable *H*-function which is very general in nature, a large number of other theorems involving several other multiple integral transforms follow as simple special cases of our theorems on suitably specializing the parameters of the multivariable *H*-function.

Again, the one- and two-dimensional analogues of our theorems are of interest in themselves. The two-dimensional analogues of Theorems 1, 2 and 3 were obtained recently by Handa [1976, p. 156, eqn. (3.6.27)] and Gupta [1978, p. 66, eqn. (1.6.7); p. 79, (1.10.5)].

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