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EFFECTS OF EXTERNAL CIRCUIT ON HEAT TRANSFER IN MHD COUETTE FLOW

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An exact solution of energy equation in fully-developed MHD Couette flow has been derived. Temperature profiles are shown in open- and short-circuit cases. It has been observed that in short circuit case, temperature and Nusselt number (Nu) increase with increasing M, whereas in open-circuit case, with increasing M, the temperature decreases. Also in open-circuit case, Nu increases with increasing M when M is small, but at large values of M, Nu decreases with increasing M.

1. INTRODUCTION

The problem of MHD Couette flow of an electrically conducting, incompressible, viscous fluid between two infinite parallel plates was studied by Agarwal (1962) and heat transfer aspect of this problem was studied by Soundalgekar (1966). The generalized MHD Couette flow and its heat transfer aspect was also studied by Soundalgekar (1967). However, in all these problems, the effect of external circuit has not been considered. Hughes and Young (1966), in their text book, studied MHD Couette flow on taking into account the external circuit consisting of terminal voltage. The effects of such an external circuit on heat transfer aspect of such Couette flow have not been studied in the literature. Hence the motivation to study it. In section 2, mathematical solutions are derived and in section 3, conclusions have been set out.

2. MATHEMATICAL ANALYSIS

Consider the problem of an incompressible, viscous, electrically conducting fluid between two infinite parallel plates separated by a distance of y_0. If x-axis is taken along the lower stationary plate and y-axis taken normal to it and assuming the upper plate moving uniformly with velocity U_0, Hughes and Young have derived the following expressions for velocity and current density in non-dimensional form :

$$U^* = [(G^* - MV^*_T)/M^2] (\cosh (My^*) - 1) + \frac{1 - [(G^* - MV^*_T)/M^2] (\cosh M - 1)}{\sin h M} \sinh (My^*) \quad ...(1)$$

$$J_Z^* = - V_T^* + M\left[\frac{1-[(G^*-MV^*_T)/M^2](\cosh M-1)}{\sinh M}\right](\sinh My^*). \quad ...(2)$$

Here the non-dimensional quantities are defined as follows :

$$U^* = u/U_0,\ G^* = \left\{\frac{y_0^2}{\mu U_0}\ \frac{\partial p}{\partial x}\right\},\ y^* = y/y_0,$$

$$E_z^* = \frac{y_0}{U_0}\sqrt{\sigma/\mu}\ E_z,\ V_T^* = -E_z^*,\ V_T^* = (y_0/2z_0\ U_0)\sqrt{\sigma/\mu}\ V_T$$

$$J_z^* = (y_0/U_0\ \sqrt{\sigma/\mu})\ J_z,\ M = y_0 B_0\ \sqrt{\sigma/\mu}. \quad ...(3)$$

Here σ is the electrical conductivity of gas, V_T the terminal voltage, μ the viscosity, E_z the electric field, B_0 the magnetic induction, $\partial p/\partial x$ the applied pressure gradient and J_z is the current density.

Now the energy equation for the fully developed flow is given by

$$k\frac{d^2T}{dy^2} + \mu\left(\frac{du}{dy}\right)^2 + \frac{J_z^2}{\sigma} = 0 \quad ...(4)$$

where k is the thermal conductivity, T the temperature of the gas, J_z the current density and σ the electrical conductivity of the gas.

The boundary conditions of the problem are

$$\left.\begin{array}{l} T = T_1 \quad \text{at } y = 0 \\ T = T_2 \quad \text{at } y = y_0 \end{array}\right\}. \quad ...(5)$$

On introducing the following non-dimensional quantities

$$\theta = \frac{T-T_1}{T_2-T_1},\ P = \frac{\mu c_p}{k},\ E = \frac{U_0^2}{c_p(T_2-T_1)}$$

and those in (3) in eqns. (4) and (5), we get

$$\frac{d^2\theta}{dy^2} + Pr\ E\left[\left(\frac{\partial u^*}{\partial y}\right)^2 + M^2 J_Z^{*2}\right] = 0 \quad ...(6)$$

and boundary conditions $\theta(1) = 1,\ \theta(0) = 0.$ ...(7)

The solution of eqn. (6) subject to boundary condition (7) is given as follows :

$$\theta(y) = y^* + Pr\ E\left[\frac{A_1}{4M^2}\left\{(\cosh 2M-1)y^* + (1-\cosh 2My^*)\right\}\right.$$
$$+ \frac{A_2}{4M^2}\left\{y^* \sinh 2M - \sinh 2My^*\right\} + \frac{A_3}{M^2}\left\{(\cosh M-1)y^*\right.$$
$$\left.\left. + (1-\cosh My^*)\right\} + \frac{A_4}{M^2}\left\{y^*. \sinh M - \sinh My^*\right\} + \frac{A_5}{2}(y^*-y^{*2})\right] \quad ...(8)$$

where
$$A_1 = \tfrac{1}{2}[A^2M^4 + B^2M^4 + B^2M^2 + A^2M^2],$$
$$A_2 = (ABM^4 + ABM^2),\ A_3 = (2AM^3C - 2A^2M^2),$$
$$A_4 = (2M^2CB - 2ABM^4),$$
$$A_5 = \tfrac{1}{2}[M^2C^2 - 4AM^3C + 3A^2M^4 - 2M^4B^2 + B^2M^2 - A^2M^2],$$
$$A = \frac{G^*-MV^*_T}{M^2},\ C = -V_T^* \quad ...(8a)$$

$$B = \frac{1-[(G^*-MV_T^*)/M^2](\cosh M-1)}{\sinh M}.$$

Numerical calculations for θ (y^*) have been calculated in both short and open circuit cases and they are shown in Figs. 1-4. Figure 1 shows temperature profiles in short circuit case and $G^* = 0$. We observe from Fig. 1 that an increase in M or $Pr\ E$

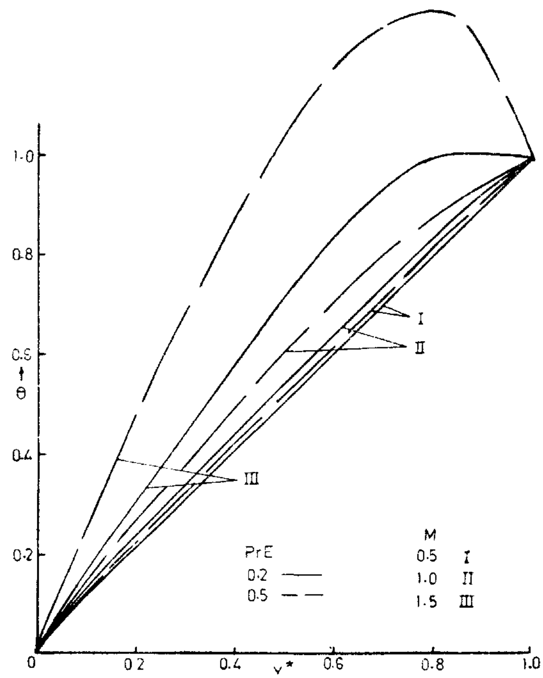

FIG. 1. Temperature profiles (Short-circuit) $G^* = 0$.

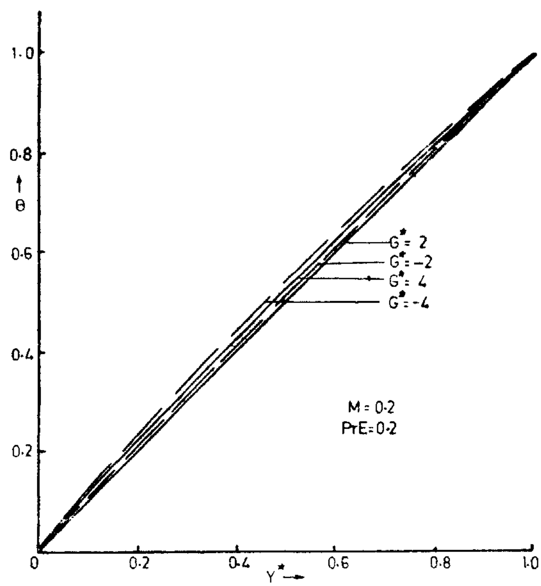

FIG. 2. Temperature Profiles (Short Circuit).

leads to a rise in temperature. In Fig. 2, temperature profiles are shown for different values of G^*. We observe from this figure that an increase in G^* leads to an increase in temperature. Figure 3 represents temperature profiles for open circuit and $G^*=0$. It is interesting to note here that an increase in M or PrE leads to a fall in temperature. In Fig. 4, the temperature profiles are shown for open circuit case and different values of G^*. We observe from this figure that an increase in G^* (>0) leads to an

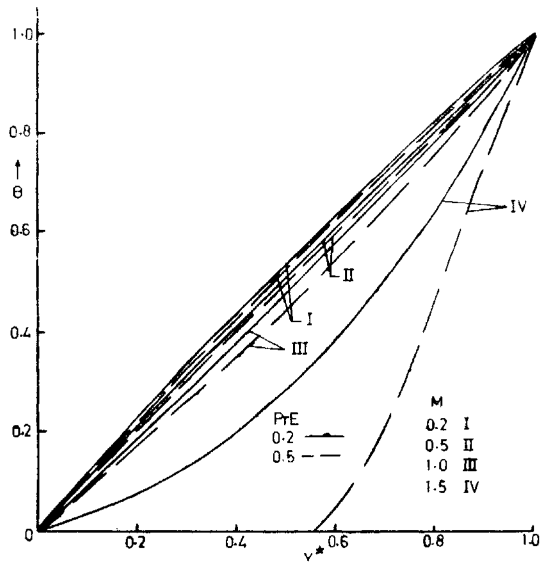

FIG. 3, Temperature Profiles (Open Circuit) $G^*=0$.

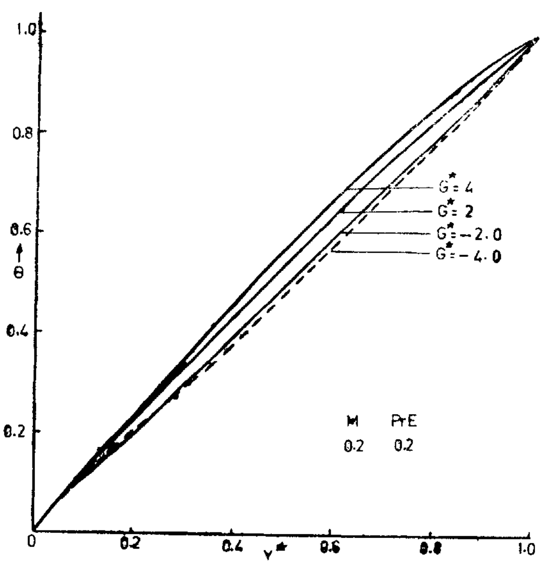

FIG. 4. Temperature Profiles (Open circuit).

increase in temperature, whereas a decrease in G^* (<0) leads to a fall in temperature when M and PrE are constant.

Knowing the temperature field, we now calculate the rate of heat transfer at the lower plate. It is expressed in terms of Nusselt number which is given in non-dimensional form as

$$Nu = \left(\frac{\partial\theta}{\partial y^*}\right)_{y^*} = 0. \qquad \ldots(9)$$

From (8) and (9), we have

$$Nu = 1 + P\,E\left[\frac{A_1}{4M^2}(\cosh 2M-1)+\frac{A_2}{4M^2}(\sin\text{h}\ 2M-2M)\right.$$
$$\left.+\frac{A_3}{M^2}(\cosh\ M-1)+\frac{A_4}{M^2}(\sinh M\quad M)+\frac{A_5}{2}\right]$$

where A_1, A_2, A_3, A_4 and A_5 are as defined in (8a).

Numerical values of Nu have been calculated and are entered in Table I. We observe from this table that in short-circuit case, Nu increases with increasing M or PrE, but in open-circuit case, Nu first increases with M when M is very small and with further increase in M, there is a decrease in the value of Nu. In open-circuit case, values of Nu are greater than those in short-circuit case when M is small but when M is large, the values of Nu in open-circuit case are less than those in short-circuit case.

TABLE I

PrE	M	Nu Short circuit	Nu Open circuit
0.2	0.2	1.0040	1.0514
	0.4	1.0165	1.0530
	0.5	1.0263	1.0514
	1.0	1.1381	1.93674
	1.5	1.5961	0.26980
0.5	0.2	1.0101	1.1286
	0.4	1.0413	1.1325
	0.5	1.0659	1.1284
	1.0	1.3453	0.84185
	1.5	2.4903	−0.82550

3. CONCLUSIONS

(1) An increase in M or PrE leads to a rise in temperature in short-circuit case and a fall in temperature in open-circuit case.

(2) The rate of heat transfer Nu increases with increasing M or PrE in short-circuit case, but in open-circuit case, Nu increases with M for small values of M, but

a further increase in M leads to a decrease in the value of Nu. The values of Nu are less in open-circuit case than in short-circuit case for small values of M, but for large values of M, they are greater than those in short-circuit case.

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