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HEAT AND MASS TRANSFER IN POROUS BODIES OF SIMPLE GEOMETRY

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Integral balance method is applied to determine the transfer potentials for heat and mass in an unidimensional capillary-porous body of simple geometry under boundary conditions of the third kind, assuming temperature of the surrounding atmosphere to be a linear function of time. The whole analysis is presented in dimensionless form and influence of Predvoditelev criterion over the average transfer potentials and the generalized time rate of average transfer potentials, is shown graphically.

NOMENCLATURE

a_m = diffusion coefficient of moisture in the material

a_q = thermal diffusion coefficient

c_m = specific isothermal mass capacity of the material

c_q = specific heat capacity of the material

r = length coordinate

$2R$ = thickness of the plate or diameter of the cylinder or sphere

t = time variable

$U_1(r, t)$ = heat transfer potential

$U_2(r, t)$ = mass transfer potential

λ_m = mass transfer coefficient

λ_q = heat transfer coefficient

γ_0 = density of the porous skeleton

ϵ = phase conversion coefficient

ρ = specific heat of evaporation

δ_s = Soret coefficient

α = coefficient of heat exchange

β = coefficient of mass exchange

Γ = a shape factor (for an infinite plate $\Gamma = 0$ for a cylinder $\Gamma = 1$; for a sphere $\Gamma = 2$).

$D \equiv \frac{d}{dF_0}$ = derivative w.r.t. F_0.

Subscripts

a = surrounding medium

m = mass transfer characteristics

p = equilibrium value

q = heat-transfer characteristics

$;t$ = partial derivative w.r.t. t.

Superscripts

0 = initial conditions.

INTRODUCTION

Man has been keenly aware of the importance of a porous body since the first clay bowl was shaped by hand and set in the sun to dry. Under the proper conditions, a rockhard product was obtained and the art of pottery was advanced. The dehydration of foodstuffs has also been practised for centuries. Besides the proper management of our agricultural products, our energy resources now require an improved understanding of the drying process in biological materials as well. At low temperature drying, the transfer moisture in the material goes on only in the liquid phase and is due to the moisture gradient or the temperature gradient. This phenomenon is different from the individual transfer phenomenon of heat or matter and needs a simultaneous consideration of the transfer phenomena of both. Usually the shape of the porous matrix in which the transfer of heat and mass takes place is quite complicated and cannot be easily described by means of well defined boundaries; it is therefore proper to consider them in simple geometrical configuration like plate, cylinder and sphere for the analytical solution. In general, the differential equations governing the coupled phenomena of heat and mass transfer in porous media are complicated in nature. Rai and Pandey (1976) have applied a technique which is an extension of Goodman's (1964) technique to obtain the solutions of certain problems in coupled phenomena of heat and mass transfer in an infinite cylindrical porous body of finite radius. In this paper, the authors have applied a similar approach as in Rai and Pandey (1976) to obtain the transfer potentials for heat and mass in an unidimensional capillary-porous body of simple geometry like plate, cylinder and sphere under boundary conditions of the third kind, assuming the temperature of the surrounding atmosphere to be a linear function of time. The whole analysis is presented in dimensionless form and the influence of the predvoditelev number on average transfer potentials and generalized time rate of average transfer potentials have been shown graphically. The numerical results have been obtained on a TDC 12 computer.

Statement of the Problem

In Luikov's (1964) paper the analytical theory of heat and mass transfer in capillary-porous bodies is systematically developed, and is described by a set of partial differential equations of the parabolic type. This set in potential form may be represented as follows:

$$U_{1;t} = a_q \left(U_{1;rr} + \frac{\Gamma}{r} U_{1;r} \right) + \epsilon\rho\,(c_m/c_q)\, U_{2;t} \qquad \ldots(1)$$

$$U_{2;t} = a_m \left(\left(U_{2;rr} + \frac{\Gamma}{r} U_{2;r} \right) + \delta_s \left(U_{1;rr} + \frac{\Gamma}{r} U_{1;r} \right) \right),$$

$$0 < r < R,\ t > 0. \qquad \ldots(2)$$

Equations (1) and (2) are subjected to the following conditions:

Initial conditions

$$U_i(r, 0) = U_i^0,\ i = 1, 2,\ 0 \leqslant r \leqslant R. \qquad \ldots(3)$$

Heat balance on the free surface

$$-\lambda_q U_{1;r} + \alpha\,[U_{1a} - U_1] - (1 - \epsilon)\,\rho\beta(U_2 - U_{2p}) = 0,\ r = R, t > 0 \qquad \ldots(4)$$

where

$$U_{1a} = U_{10} - bt$$

b is an arbitrary constant.

Mass balance on the free surface

$$\lambda_m(U_{2;r} + \delta_s U_{1;r}) + \beta(U_2 - U_{2p}) = 0, r = R, t > 0. \qquad \ldots(5)$$

The conditions due to symmetry

$$U_{i;r}(0, t) = 0, i = 1, 2, t > 0. \qquad \ldots(6)$$

Now we shall define some dimensionless variables

$$x = r/R$$

$$F_0 = a_q(t/R^2)$$

$$\theta_1(x, F_0) = (U_1 - U_1^0)/(U_{1a} - U_1^0)$$

$$\theta_2(x, F_0) = (U_2 - U_2^0)/(U_{2p} - U_2^0)$$

and similarity criteria:

(i) the Luikov criterion of the field of transfer potential for mass with respect to the temperature field

$$Lu = a_m/a_q,$$

(ii) the Posnov criterion for bound matter

$$Pn = \delta_s(U^0_{1a} - U^0_1)/(U^0_2 - U_{2p}),$$

(iii) the Kossovich criterion for bound matter

$$K_o = \rho(c_m/c_q)\,(U^0_2 - U_{2p})/(U^0_{1a} - U^0_1),$$

(iv) the Biot criterion for heat transfer

$$B_{i\beta} = \alpha R/\lambda_q,$$

(v) the Biot criterion for mass transfer

$$B_{im} = \beta R/\lambda_m,$$

(vi) the Predvoditelev criterion

$$Pd = bR^2/a_q(U^0_{1a} - U^0_1).$$

Equations (1) – (6) can be written in the dimensionless form with the aid of the dimensionless variables and similarity criteria as defined above:

$$\theta_{1;F_0} = \theta_{1;xx} + \frac{\Gamma}{x}\,\theta_{1;x} - \epsilon K_o \theta_{2;F_0} \qquad \text{...(7)}$$

$$\theta_{2;F_0} = Lu\left(\theta_{2;xx} + \frac{\Gamma}{x}\,\theta_{2;x} - Pn\left(\theta_{1;xx} + \frac{\Gamma}{x}\,\theta_{1;x}\right)\right),$$

$$0 < x < 1,\ F_0 > 0 \qquad \text{...(8)}$$

$$\theta_i(x, 0) = 0, \qquad i = 1, 2, \qquad 0 \leqslant x \leqslant 1 \qquad \text{...(9)}$$

$$\theta_{1;x} - B_{iq}(1 - PdF_0 - \theta_1) + (1 - \epsilon)\,LuK_oB_{im}\,(1 - \theta_2) = 0 \qquad \text{...(10)}$$

$$\theta_{2;x} - Pn\theta_{1;x} - B_{im}(1 - \theta_2) = 0,\ \ x = 1,\ F_0 > 0 \qquad \text{...(11)}$$

$$\theta_{i;x}(0, F_0) = 0,\ \ F_0 > 0. \qquad \text{...(12)}$$

Solution

We define two quantities $\delta_1(F_0)$, the thickness of thermal boundary layer, and $\delta_2(F_0)$, the thickness of mass boundary layer. The thickness of both layers grow with generalized time. Its properties are such that for $x > \delta_i(F_0)$ $(i = 1, 2)$, the body, for all practical purposes, is at an equilibrium and there is no transfer of potentials beyond this point. Therefore, as long as the thicknesses of these layers are less than unity it behaves as an infinite medium, because the boundary conditions at $x = 0$ do not matter. At the generalized transition times when the thicknesses of two layers are separately equal to the unity—the boundary conditions prescribed on the surface of the body come into play. Therefore, for the solution, these problems are split up into two phases—one valid for the ranges $0 < F_0 < F_{01}$, $0 < F_0 < F_{02}$ (where F_{01} and F_{02} are the generalized transition times for heat and mass transfer potentials) and the other valid for the ranges $F_0 > F_{01}$, $F_0 > F_{02}$.

First Phase — We know that for any value of generalized time F_0 the nondimensional temperature and mass transfer potential have penetrated inside the body upto distances $\delta_1(F_0)$ and $\delta_2(F_0)$ respectively measured from $x = 1$. The concentration and temperature beyond the penetration depths remain at initial values, i.e. the conditions at $x = 1 - \delta_i (i = 1, 2)$ are

$$\theta_i(1 - \delta_i, F_0) = 0 \qquad ...(13)$$

$$\theta_{i;x}(1 - \delta_i, F_0) = 0. \qquad ...(14)$$

The criterion Pn in practice affects only the mass transfer potential fields, but K_o affects the field of thermal potential. With increase of K_o and Pn both the mean and local heat and mass transfer potentials decrease linearly; furthermore, the linearity holds for all values of generalized time. Therefore, for small value of generalized time both Pn and K_o have a negligible influence and in the first phase these criteria can be neglected whereas in the second phase these will be kept in the transfer equations and the boundary conditions. When Pn and K_o are zero, eqns. (7), (8), (10) and (11) can be written as

$$\theta_{1;F_0} = \theta_{1;xx} + \frac{\Gamma}{x}\theta_{1;x} \qquad ...(15)$$

$$\theta_{2;F_0} = Lu\left(\theta_{2;xx} + \frac{\Gamma}{x}\theta_{2;x}\right) \qquad ...(16)$$

$$\theta_{1;x} - B_{iq}(1 - PdF_0 - \theta_1) = 0, \qquad x = 1, F_0 > 0 \qquad ...(17)$$

$$\theta_{2;x} - B_{im}(1 - \theta_2) = 0. \qquad ...(18)$$

The internal heat and mass transfer in porous bodies are expressed by a set of partial differential equations of the parabolic type. Let us assume parabolic polynomial profiles for transfer potentials as

$$\theta_i(x, F_0) = \sum_{J=0}^{2} A_{ij}(1 - x)^j \qquad ...(19)$$

where $A_{ij} (i = 1, 2;\ j = 0, 1, 2)$ may be functions of generalized time as well as the similarity criteria.

Using eqns. (13), (14), (17), (18) and (19) the transfer potentials for heat and mass in this phase come out to be

$$\theta_1(x, F_0) = B_{iq}(1 - PdF_0)\,(\delta_1 - 1 + x)^2/\delta_1(2 + B_{iq}\delta_1) \qquad \ldots(20)$$

and

$$\theta_2(x, F_0) = B_{im}(\delta_2 - 1 + x)^2/\delta_2\,(2 + B_{im}\delta_2) \qquad \ldots(21)$$

where δ_1 and δ_2 are given by the differential equations

$$\frac{d\delta_1^2}{dF_0} = \frac{2(2 + B_{iq}\delta_1)\,[3(2 + \Gamma\delta_1)\,(1 - PdF_0) + Pd\,\delta_1^2]}{(4 + B_{iq}\delta_1)\,(1 - PdF_0)} \qquad \ldots(22)$$

and

$$\frac{d\delta_2^2}{dF_0} = \frac{12\,Lu(2 + B_{im}\delta_2)\,(1 - \Gamma)\,\delta_2 - 2}{B_{im}\delta_2^2 + (3 - 2B_{im})\,\delta_2 - 8} \qquad \ldots(23)$$

respectively.

Using Picard's method of successive approximation the value of δ_1 up to the first approximation is

$$\delta_1^2 \simeq 6F_0. \qquad \ldots(24)$$

Integrating the differential equation (23) under the initial condition $\delta_2(0) = 0$, we get following implicit relations:

$$\delta_2^2 + \frac{2}{B_{im}}\,\delta_2 - \frac{4}{B_{im}}\left(\frac{1}{B_{im}} + \frac{1}{1 + B_{im}}\right)\log\left(1 + \tfrac{1}{2}\,B_{im}\delta_2\right)$$

$$- \frac{4}{1 + B_{im}}\log\left(1 - \tfrac{1}{2}\,\delta_2\right) = 12\,LuF_0 \quad (\text{for } \Gamma = 0)$$

$$\delta_2^3 + 3\left(\frac{1}{2B_{im}} - 1\right)\delta_2^2 - \frac{6}{B_{im}}\left(2 + \frac{1}{B_{im}}\right)\delta_2$$

$$+ \frac{12}{B_{im}^2}\left(2 + \frac{1}{B_{im}}\right)\log\left(1 + \tfrac{1}{2}\,B_{im}\delta_2\right) = -36LuF_0 \quad (\text{for } \Gamma = 1)$$

and

$$\delta_2^2 + 2\left(\frac{1}{B_{im}} - 4\right)\delta_2 + \frac{4(1 + 2B_{im})}{B_{im}^2(B_{im} - 1)}\log\left(1 + \tfrac{1}{2}B_{im}\delta_2\right)$$

$$+ \frac{4(4B_{im} - 7)}{B_{im} - 1}\log\left(1 + \tfrac{1}{2}\,\delta_2\right) = -12LuF_0 \quad (\text{for } \Gamma = 2). \qquad \ldots(25)$$

Second phase — When the energy and mass penetration depth reaches at $x = 0$ the idea of penetration depth fails and therefore we have to take into account the boundary conditions at $x = 0$, and the profiles have to be redetermined to include the effect of this boundary.

Let us assume that thermal and mass layers reach the centre at generalized time F_{0_1} and F_{0_2} respectively. Thus, the initial distributions of the potentials for the second phase become

$$\theta_1(x, F_{0_1}) = B_{iq}(1 - PdF_{0_1})\, x^2/(2 + B_{iq}) \qquad \ldots(26)$$

and

$$\theta_2(x, F_{0_2}) = B_{im}x^2/(2 + B_{im}) \qquad \ldots(27)$$

where

$$F_{0_1} \simeq \tfrac{1}{6} \qquad \ldots(28)$$

and

$$\begin{aligned} F_{0_2} &= \frac{1}{6Lu}\left[\tfrac{1}{2} + \frac{1}{B_{im}} - \frac{2}{B_{im}}\left(\frac{1}{B_{im}} + \frac{1}{1 + B_{im}}\right)\log\left(1 + \tfrac{1}{2}B_{im}\right) \right. \\ &\quad \left. + \frac{2}{1 + B_{im}}\log 2\right], \quad (\text{for } \Gamma = 0) \\ &= \frac{1}{6Lu}\left[\tfrac{1}{3} + \frac{7}{4B_{im}} + \frac{1}{B^2_{im}} - \frac{2}{B^2_{im}}\left(2 + \frac{1}{B_{im}}\right)\right. \\ &\quad \left. \times \log\left(1 + \tfrac{1}{2}B_{im}\right)\right], \quad (\text{for } \Gamma = 1) \\ &= \frac{1}{6Lu}\left[\tfrac{7}{2} - \frac{1}{B_{im}} - \frac{2(1 + 2B_{im})}{B^2_{im}(B_{im} - 1)}\log\left(1 + \tfrac{1}{2}B_{im}\right)\right. \\ &\quad \left. - \frac{2(4B_{im} - 7)}{B_{im} - 1}\log\tfrac{3}{2}\right], \quad (\text{for } \Gamma = 2). \end{aligned} \qquad \ldots(29)$$

To determine the distribution of potentials in this phase we again assume polynomials of second degree in x. That is

$$\theta_i(x, F_0) = \sum_{j=0}^{2} B_{iJ}x^J \qquad \ldots(30)$$

where B_{iJ} $(i = 1, 2;\ J = 0, 1, 2)$ may be functions of generalized time F_0 as well as the similarity criteria.

On using eqns. (10) to (12) and (30), we get

$$B_{11} = B_{21} = 0 \qquad \ldots(31)$$

$$B_{20} = 1 + a_2 B_{12} - (1 - B_{10} - PdF_0)\, a_1 \qquad ...(32)$$

$$B_{22} = (1 - B_{10} - PdF_0)\, a_3 - B_{12} a_4 \qquad ...(33)$$

where

$$a_1 = (2 + B_{im})\, B_{iq}/2\,(1 - \epsilon)\, LuK_0 B_{im}$$

$$a_2 = \left(1 + \frac{2}{B_{iq}}\right) a_1 - Pn$$

$$a_3 = \frac{B_{im}}{2 + B_{im}}\, a_1, \quad a_4 = \left(1 + \frac{2}{B_{iq}}\right) a_3 - Pn.$$

Integrating eqns. (7) and (8) with respect to x from $x = 0$ to $x = 1$ and using eqns. (10) - (12) and (30) - (33), we get two simultaneous differential equations for the determination of B_{10} and B_{12} as follows :

$$\left[D - \frac{(1 + \Gamma)\,\epsilon}{1 - \epsilon}\, B_{iq}\right] B_{10} + \left[\tfrac{1}{3}D - \frac{(1 + \Gamma)\,(2 + \epsilon B_{iq})}{1 - \epsilon}\right] B_{12}$$

$$= -\frac{(1 + \Gamma)\,\epsilon\, B_{iq}}{1 - \epsilon}\,(1 - PdF_0) \qquad ...(34)$$

$$\left[\frac{B_{im} + 3}{3B_{im}}\, D + (1 + \Gamma)\, Lu\right] a_3 B_{10} + \left[\left\{\left(\tfrac{1}{3} + \frac{1}{B_{im}}\right)(a_4 + Pn) - Pn\right\} D\right.$$

$$\left. + (1 + \Gamma)\, Lu(a_4 + Pn)\right] B_{12}$$

$$= a_3 Lu\left[(1 + \Gamma)\,(1 - PdF_0) - \frac{(3 + B_{im})\, Pd}{3Lu\, B_{im}}\right]. \qquad ...(35)$$

Solving eqns. (34), (35) and using eqns. (26), (27), (32), (33), we obtain the values of B_{10}, B_{12}, B_{20}, B_{22}. Substituting the values of B_{10}, B_{12}, B_{20} and B_{22} in equation (30), we get temperature and mass transfer potentials in the nondimensional form as:

$$\theta_1(x, F_0) = 1 - Pd\left[F_0 - \frac{1}{2(\Gamma + 1)}\left(1 - x^2 + \frac{2}{B_{iq}}\right)\right] + (c_1 x^2 + c_3)$$

$$\exp(-m_1 F_0) + (c_2 x^2 + c_4)\exp(-m_2 F_0), \qquad ...(36)$$

$$\theta_2(x, F_0) = 1 + \frac{1}{2(\Gamma + 1)}\, PnPd\,(1 - x^2) + a_3\left(1 + \frac{2}{B_{im}} - x^2\right)$$

$$\times\, (c_3 \exp(-m_1 F_0) + c_4 \times \exp(-m_2 F_0))$$

$$+ \left\{(a_4 + Pn)\left(1 + \frac{2}{B_{im}} - x^2\right) - (1 - x^2)\, Pn\right\}$$

$$\times\, \{c_1 \exp(-m_1 F_0) + c_2 \exp(-m_2 F_0)\}. \qquad ...(37)$$

where

$$m_i = \tfrac{1}{2}\left[\frac{v_2}{v_1} + (-1)^i\left\{\left(\frac{v_2}{v_1}\right)^2 - \frac{v_3}{v_1}\right\}^{1/2}\right],\quad i = 1, 2$$

$$v_1 = \tfrac{1}{3}\left[2\,Pn - \frac{(3 + B_{im})(3 + B_{iq})}{3(1-\epsilon)\,LuK_oB_{im}}\right],$$

$$v_2 = -\frac{2(1+\Gamma)\,B_{iq}}{3(1-\epsilon)\,K_o}\left[1 + \epsilon K_oPn + \frac{1}{Lu} + \frac{3}{LuB_{im}} + \frac{3}{B_{iq}}\right],$$

$$v_3 = -\frac{2(1+\Gamma)^2\,B_{iq}}{(1-\epsilon)\,K_o}$$

$$c_k = \frac{\Delta_k}{\Delta},\ k = 1, 2, 3, 4$$

$$\Delta = \begin{vmatrix} \exp(-m_1F_{01}) & \exp(-m_2F_{01}) & 0 & 0 \\ 0 & 0 & \exp(-m_1F_{01}) & \exp(-m_2F_{01}) \\ a_2\exp(-m_1F_{02}) & a_2\exp(-m_2F_{02}) & a_1\exp(-m_1F_{02}) & a_1\exp(m_2F_{02}) \\ a_4\exp(-m_1F_{02}) & a_4\exp(-m_2F_{02}) & a_3\exp(-m_1F_{02}) & a_3\exp(-m_2F_{02}) \end{vmatrix}$$

and Δ_k is the determinant obtained by replacing all the elements of kth column of Δ by b_1, b_2, b_3 and b_4 respectively. Here

$$b_1 = \frac{Pd}{2(1+\Gamma)} + \frac{B_{iq}(1 - PdF_{01})}{B_{iq} + 2}$$

$$b_2 = -1 + PdF_{01} - \frac{Pd(2 + B_{iq})}{2(1+\Gamma)\,B_{iq}}$$

$$b_3 = \frac{Pd}{2(1+\Gamma)}\left(a_2 - \frac{B_{iq}+2}{B_{iq}}\,a_1\right) - 1$$

$$b_4 = \frac{Pd}{2(1+\Gamma)}\left(a_4 - \frac{B_{iq}+2}{B_{iq}}\,a_3\right) - \frac{B_{im}}{2 + B_{im}}.$$

Particular Cases

(i) *Infinite Plate* — Putting $\Gamma = 0$ in equations (36) and (37), the transfer potentials of heat and mass in case of an infinite plate may be written as:

$$\theta_1(x, F_0) = 1 - Pd\left[F_0 - \tfrac{1}{2}\left(1 - x^2 + \frac{2}{B_{iq}}\right)\right] + (C_{10}x^2 + C_{30})\exp(-m_{10}F_0) + (C_{20}x^2 + C_{40})\exp(-m_{20}F_0) \quad \text{...(38)}$$

and

$$\theta_2(x, F_0) = 1 + \tfrac{1}{2} PnPd(1 - x^2) + a_3 \left(1 + \frac{2}{B_{im}} - x^2\right)$$
$$\times [C_{30} \exp(-m_{10}F_0) + C_{40} \exp(-m_{20}F_0)]$$
$$+\left[(a_4 + Pn)\left(1 + \frac{2}{B_{im}} - x^2\right) - (1 - x^2)\, Pn\right]$$
$$\times [C_{10} \exp(-m_{10}F_0) + C_{20} \exp(-m_{20}F_0)] \qquad \ldots(39)$$

where C_{k0} $(k = 1, 2, 3, 4)$ and $m_{i0} (i = 1, 2)$ are values of C_k and m_i for $\Gamma = 0$ respectively.

Comparison with exact solution : The exact solutions to this problem in case of an infinite plate are given by Luikov and Mikhaylov (1965) in the form:

$$\theta_1(x, F_0) = 1 - Pd\left[F_0 - \tfrac{1}{2}\left(1 - x^2 + \frac{2}{B_{iq}}\right)\right]$$
$$- \sum_{n=1}^{\infty} (C_{n2} \cos \nu_1\mu_n x - C_{n2} \cos \nu_2\mu_n x)$$
$$\times \exp(-\mu_n^2 F_0) \qquad \ldots(40)$$

and

$$\theta_2(x, F_0) = 1 + \tfrac{1}{2} PnPd(1 - x^2) - \sum_{n=1}^{\infty} (C^*_{n1}(1 - \nu_2^2)$$
$$\times \cos \nu_2 \mu_n x - C^*_{n2}(1 - \nu_1^2) \cos \nu_1\mu_n x)$$
$$\times \exp(-\mu_n^2 F_0). \qquad \ldots(41)$$

In the quasi-steady state. the distribution of potentials of heat and mass are exactly the same as (38) and (39) which are the approximate solutions found by the integral balance method.

(ii) *Infinite cylinder* — Putting $\Gamma = 1$ in eqns. (36) and (37), the transfer potentials of heat and mass in case of an infinite cylindrical porous body may be written in the form:

$$\theta_1(x, F_0) = 1 - Pd\left[F_0 - \tfrac{1}{4}\left(1 - x^2 + \frac{2}{B_{iq}}\right)\right]$$
$$+ (C_{11}x^2 + C_{21}) \exp(-m_{11}F_0)$$
$$+ (C_{21}x^2 + C_{41}) \exp(-m_{21}F_0) \qquad \ldots(42)$$

and

$$\theta_2(x, F_0) = 1 + \tfrac{1}{4}\, PnPd(1 - x^2) + a_3 \left(1 + \frac{2}{B_{im}} - x^2\right)$$
$$\times (C_{31} \exp(-m_{11}F_0) + C_{41} \exp(-m_{21}F_0))$$
$$+ \left[(a_4 + Pn)\left(1 + \frac{2}{B_{im}} - x^2\right) - (1 - x^2)\, Pn\right]$$
$$\times [C_{11} \exp(-m_{11}F_0) + C_{21} \exp(-m_{21}F_0)] \qquad ...(43)$$

where $C_{k1}(k = 1, 2, 3, 4)$ and $m_{i1}(i = 1, 2)$ are values of C_k and m_i for $\Gamma = 1$ respectively.

(iii) *Sphere* — Putting $\Gamma = 2$ in eqns. (36) and (37), the transfer potentials of heat and mass in case of an unidimensional spherical porous body may be written in the form:

$$\theta_1(x, F_0) = 1 - Pd\left[F_0 - \tfrac{1}{6}\left(1 - x^2 + \frac{2}{B_{iq}}\right)\right]$$
$$+ (C_{12}x^2 + C_{32}) \exp(-m_{12}F_0)$$
$$+ (C_{22}x^2 + C_{42}) \exp(-m_{22}F_0) \qquad ...(44)$$

and

$$\theta_2(x, F_0) = 1 + \tfrac{1}{6}\, PnPd\, (1 - x^2) + a_3 \left(1 + \frac{2}{B_{im}} - x^2\right)$$
$$\times (C_{32} \exp(-m_{12}F_0) + C_{42} \exp(-m_{22}F_0))$$
$$+ \left\{(a_4 + Pn)\left(1 + \frac{2}{B_{im}} - x^2\right) - (1 - x^2)\, Pn\right\}$$
$$\times \{C_{12} \exp(-m_{12}F_0) + C_{22} \exp(-m_{22}F_0)\} \qquad ...(45)$$

where $C_{k2}(k = 1, 2, 3, 4)$ and $m_{i2}(i = 1, 2)$ are values of C_k and m_i for $\Gamma = 2$ respectively.

Analysis of the Solution

The average values of the transfer potentials for heat and mass in a porous body of simple geometry like plate, cylinder and sphere are obtained from the relation

$$\langle \theta_i(x, F_0) \rangle = (1 + \Gamma) \int_0^1 x\theta_i(x, F_0)\, dx, \quad i = 1, 2. \qquad ...(46)$$

On using the expressions for $\theta_1(x, F_0)$ and $\theta_2(x, F_0)$ from eqns. (36) and (37) respectively, we obtain

$$\langle \theta_1(x, F_0) \rangle = 1 - Pd\left[F_0 - \frac{1}{(\Gamma + 1)}\left(\frac{1}{\Gamma + 3} + \frac{1}{B_{iq}}\right)\right]$$
$$+ \left(c_3 + \frac{\Gamma + 1}{\Gamma + 3} c_1\right) \exp(-m_1 F_0)$$
$$+ \left(c_4 + \frac{\Gamma + 1}{\Gamma + 3} c_2\right) \exp(-m_2 F_0) \qquad \ldots(47)$$

and

$$\langle \theta_2(x, F_0) \rangle = 1 + \frac{1}{(\Gamma + 1)(\Gamma + 3)} PnPd + 2a_3\left(\frac{1}{\Gamma + 3} + \frac{1}{B_{im}}\right)$$
$$\times (c_3 \exp(-m_1 F_0) + c_4 \exp(-m_2 F_0))$$
$$+ \left(2(a_4 + Pn)\left(\frac{1}{\Gamma + 3} + \frac{1}{B_{im}}\right) - \frac{2}{\Gamma + 3} Pn\right)$$
$$\times (c_1 \exp(-m_1 F_0) + c_2 \exp(-m_2 F_0)) \qquad \ldots(48)$$

in the light of (46).

The generalized time rate of transfer potentials for heat and mass can be obtained by differentiating equations (47) and (48) with respect to the generalized time F_0. Thus, we get

$$\frac{d\langle \theta_1 \rangle}{dF_0} = - Pd - m_1\left(c_3 + \frac{\Gamma + 1}{\Gamma + 3} c_1\right) \exp(-m_1 F_0)$$
$$- m_2\left(c_4 + \frac{\Gamma + 1}{\Gamma + 3} c_2\right) \exp(-m_2 F_0) \qquad \ldots(49)$$

and

$$\frac{d\langle \theta_2 \rangle}{dF_0} = - 2a_3\left(\frac{1}{\Gamma + 3} + \frac{1}{B_{im}}\right)(m_1 c_3 \exp(-m_1 F_0)$$
$$+ m_2 c_4 \exp(- m_2 F_0)) - \left(2(a_4 + Pn)\left(\frac{1}{\Gamma + 3} + \frac{1}{B_{im}}\right)\right.$$
$$\left. - \frac{2}{\Gamma + 3} Pn\right)(m_1 c_1 \exp(-m_1 F_0)$$
$$+ m_2 c_2 \exp(-m_2 F_0)). \qquad \ldots(50)$$

In cases where there is heat and mass transfer in an external medium with a time dependent temperature, the general solution will contain the Predvoditelev number. The latter characterizes the rate of change of temperature of the medium. Let us analyze the effect of the *Pd* number on average transfer potential of heat and mass and their generalized time rate in case of an unidimensional cylindrical porous body when the temperature of the external medium decreases linearly with time.

The dimensionless average transfer potentials of heat and mass and their generalized time rate are plotted against the generalized time F_0 for different value of *Pd* in Figs. 1 and 2 respectively. The dimensionless temperature reaches a maximum and then decreases. The higher the value of *Pd* the sooner the maximum temperature of the material is reached. Knowledge of the exact time at which the maximum temperature is reached, and of the absolute value of the maximum temperature, is important for the correct design of any drying process. In order to prevent condensation from the external medium on the material, the drying process should be discontinued at $F_{0\theta_{i\max}}$.

The generalized time rate of temperature decreases with increasing *Pd*. With increasing *Pd* the mass transfer potential increases while the generalized time rate of mass transfer potential decreases.

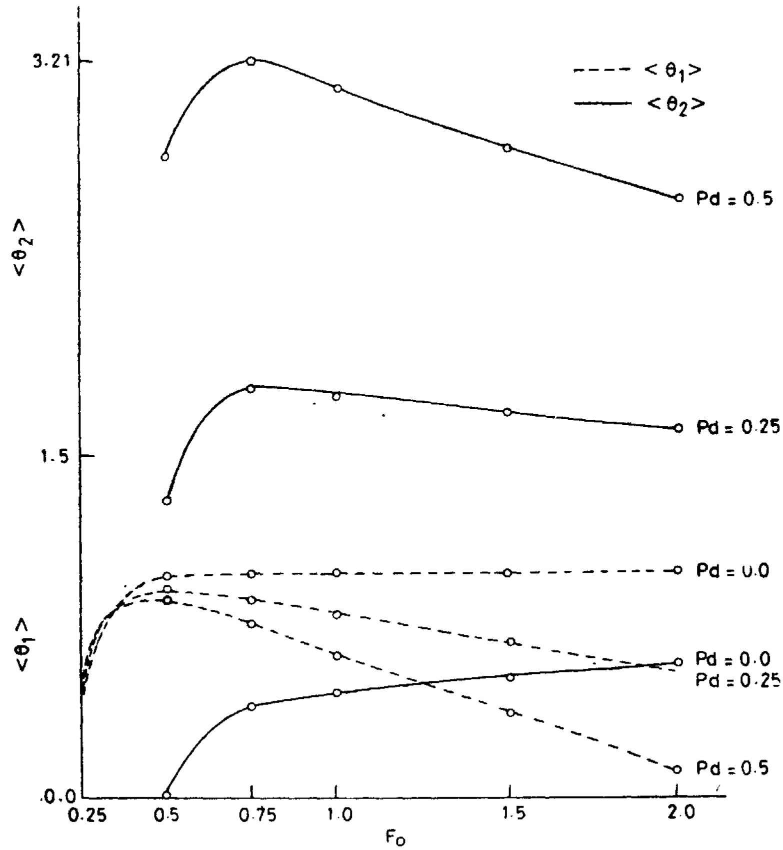

FIG. 1. Dimensionless average transfer potentials as a function of *Pd* and F_0 for $\epsilon = 0.5$, $Lu = 0.3$, $K_o = 1.2$, $Pn = 0.5$, $B_{im} = B_{iq} = 10$, $\Gamma = 1$.

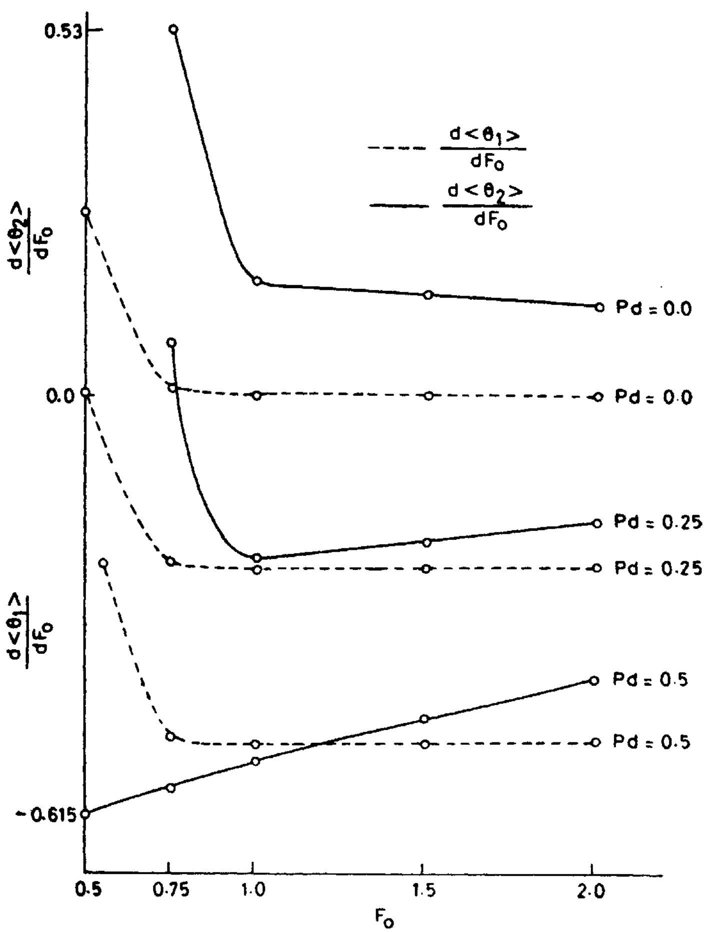

FIG. 2. Dimensionless generalized time rate of transfer potentials as a function of Pd and F_0 for $\epsilon = 0.5$, $Lu = 0.3$, $K_0 = 1.2$, $Pn = 0.5$, $B_{im} = B_{iq} = 10$, $\Gamma = 1$.

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