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A NOTE ON LINDELÖF SPACES

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It is proved that a product of Lindelof P-spaces is Lindelöf, when the product is paracompact and Hausdorff.

The object of this note is to show that a product of Lindelöf P-spaces is Lindelöf, provided the product is paracompact and T_2 (Theorem 9).

In what follows, X is a topological space, m, n are cardinal numbers, c is the cardinal number of the reals, w is the cardinal number of integers. For a set A, $|A|$ stands for the cardinal number of A.

Definition 1—X is said to be n-compact if every open cover u of X, with $|u| = m$ has a subcover v, with $|v|$ strictly less than n, for all $m \geqslant n$.

Note : X is C-compact if and only if X is Lindelöf.

Definition 2—X is said to be n-discrete if every point has a neighbourhood base u such that

$$v \subset u, |v| < n \Rightarrow \exists\, U \in u \ni U \subset \cap \{V \mid V \in v\}.$$

Definition 3—Let $\{X_i\}$ be a collection of topological spaces. Then the weak topological sum of $\{X_i\}$ is

$$w(\pi X_i) = \{x \in \pi x_i \mid x_i = a_i \text{ except for finitely many } i\}$$

where $a = (a_i) \in \pi x_i$.

Note : $w(\pi X_i)$ is a dense subspace of πX_i in the product topology.

Definition 4—X is said to be a P-space if every prime ideal in the ring $\mathscr{C}(X)$ of continuous functions is maximal.

Note : A completely regular T_1-space is a P-space if and only if it is c-discrete.

A proof of the following can be found in Vaughan (1972).

Theorem 5—If X_i is n-compact and n-discrete for $i \in I$ and if n is regular, then πX_i is n-compact.

The following can be found in Comfort (1975).

Theorem 6—Let n be regular and $\{X_i \mid i \in I\}$ be such that $\pi\{X_i \mid i \in J\}$ is n-compact for all $J \subset I$ with $|J| < n$. Then the weak topological sum $w(\pi X_i)$ is n-compact.

From the above two theorems, taking $n = c$, we have :

Theorem 7—The weak topological sum of a family of Lindelöf, c-discrete space is Lindelöf.

Now we prove a lemma which leads to our main result.

Lemma 8—Let X be a paracompact T_2-space with a dense n-compact subspace S. Then X is n-compact.

PROOF: Let u be an open cover of X. Since X is paracompact, we may assume that u is locally finite. Since $\overline{S} = X$, using the regularity of X, we can find a subcover v of u such that $| v | < n$.

From the lemma and Theorem 7, we have :

Theorem 9—Arbitrary product of Lindelöf P-spaces in Lindelöf, provided, the product is paracompact and T_2.

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