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SOME FURTHER TRIPLE FOURIER-LAGUERRE SERIES EQUATIONS

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The present note provides an investigations of the solution of certain triple equations involving Fourier-Laguerre series. It exhibits equivalence of the solutions obtained earlier [see Dwivedi *et al.* (1974, 1980)]. The corresponding discussions for certain dual Fourier-Laguerre series equations were presented, almost a decade ago, by Srivastava (1973).

1. INTRODUCTION

In recent papers (Dwivedi *et al.* 1974, 1980) the solution of the triple series equations involving Laguerre polynomials have been obtained. In this note first of all we discuss the equivalence of the solutions obtained by Dwivedi *et al.* (1974, 1980). Then we proceed to show how the solution of the triple equations (1) to (6) of Dwivedi and Trivedi (1974) when $\beta = \alpha$ (instead $\nu = \alpha$) can be obtained easily from that of equations (1) to (6) of Dwivedi and Sharma (1980). Finally, we conclude our observations regarding the solvability of equations (1) to (6) of (1974) under various conditions on parameters α, β, ν and σ. We closely follow the lines of Srivastava (1973) who presented the corresponding discussions for certain dual equations involving series of Laguerre polynomials.

2. DISCUSSIONS AND RESULTS

On comparing the solutions (13) of the triple series equations of the first kind (1) to (3) of Dwivedi *et al.* (1974, 1980), observe that these solutions are identical if $\beta+1 = \alpha+\beta'$, $\sigma = \alpha = -\beta$ and then reducing β' to β with $p = 0$. Similarly, the solutions of the triple series equations of the second kind (4) to (6) of Dwivedi *et al.* (1974, 1980) are identical essentially under the same conditions.

Next we consider the triple series equations of first kind

$$\sum_{n=0}^{\infty} \frac{C_n}{\Gamma(\alpha+n+1)} L_n^{(\sigma)}(x) = g_1(x), \quad 0 \leqslant x < a \qquad \ldots(2.1)$$

$$= h_1(x), \quad b < x < \infty$$

$$\sum_{n=0}^{\infty} \frac{C_n}{\Gamma(\alpha+n+1)} L_n^{(\nu)}(x) = f(x), \quad a < x < b \qquad \ldots(2.2)$$

which correspond to the special case $\beta = \alpha$ of the triple equations (1) to (3) of Dwivedi and Trivedi (1974). On setting

$$C_n = \Gamma(\alpha+n+1)/\Gamma(\nu+n+1)\, D_n, \quad n = 0, 1, 2 \qquad ...(2.3)$$

these equations will reduce to

$$\sum_{n=0}^{\infty} \frac{D_n}{\Gamma(\nu+n+1)} L_n^{(\sigma)}(x) = g_1(x), \quad 0 \leqslant x < a$$

$$= h_1(x), \quad b < x < \infty \qquad ...(2.4)$$

$$\sum_{n=0}^{\infty} \frac{D_n}{\Gamma(\nu+n+1)} L_n^{(\nu)}(x) = f(x), \quad a < x < b. \qquad ...(2.5)$$

It is interesting to note that the solution of the triple equations (2.4) and (2.5) is contained in that of (1) to (3) of Dwivedi and Sharma (1980). If in (1) to (3) of that paper, we take $p=0$, $\alpha+\beta = \nu+1$ and $\alpha = \nu$, we shall at once be led to equation (2.4) and (2.5) respectively. Therefore, on using (13) of that paper in conjunction with (2.3), we obtain the solution of the triple series equations (2.1) and (2.2).

Similarly, we consider the triple equations of the second kind which correspond to the same special case $\beta=\alpha$ of the equations (4) to (6) of Dwivedi and Trivedi (1974). If in (4) to (6) of Dwivedi and Sharma (1980), we take $p = 0$, $\alpha+\beta=\nu+1$ and then $\alpha = \nu$, we obtain equations (2.4) and (2.5) with ν and σ interchanged. Therefore, on using (29) of that paper in conjunction with (2.3), we obtain the solution of these triple series equations of second kind.

Finally, we remark that the equations (1) to (3) of Dwivedi and Trivedi (1974) cannot be solved by other known method when all the parameters α, β, ν and σ are free. They can only be solved if at most three of these parameters remain free. Out of six such cases, the case $\alpha = \beta$ is trivial and is contained in the general case $\nu = \alpha$ In other four cases extra conditions on the parameters would be necessary to obtain the solution of the resulting problem from equations (1) to (6) of Dwivedi and Sharma (1980).

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