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RAYLEIGH WAVES ON A CONCAVE CYLINDRICAL POROELASTIC SURFACE

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A study is made of the phase velocity of Rayleigh waves on a concave cylindrical poroelastic surface for pervious and impervious surface. It is observed that Rayleigh Waves on this surface are dispersive unlike for the case of no curvature. The numerical results are presented graphically. The results of classical theory are recovered as particular case.

1. INTRODUCTION

The propagation Rayleigh Waves on a concave cylindrical Poroelastic surface is investigated by considering a circular cylindrical cavity in an infinite poroelastic medium using the dynamic equations of Biot (1956). Unlike for the case of no curvature, Rayleigh Waves are seen to be dispersive. The plot of phase velocity versus surface wave number (= 1/curvature) is presented. The investigation may be useful in oil/gas wellbore-logging. The results analogous to half space are obtained as a particular case. By neglecting liquid effects, after some rearrangement of terms, one can recover the stresses and wave velocity equation of classical theory due to Epstein (1976).

2. SOLUTION OF THE PROBLEM

Let r, θ, z be cylindrical polar co-ordinates. The equations of motion of a liquid-filled porous medium in presence of dissipation (b) are

$$\left.\begin{aligned} P\nabla^2\phi + Q\nabla^2\psi &= (\rho_{11}\ddot{\phi} + \rho_{12}\ddot{\psi}) + b(\dot{\phi} - \dot{\psi}) \\ Q\nabla^2\phi + R\nabla^2\psi &= (\rho_{12}\ddot{\phi} + \rho_{22}\ddot{\psi}) - b(\dot{\phi} - \dot{\psi}) \\ N\nabla^2 H &= (\rho_{11}\ddot{H} + \rho_{12}\ddot{G}) + b(\dot{H} - \dot{G}) \\ 0 &= (\rho_{12}\ddot{H} + \rho_{22}\ddot{G}) - b(\dot{H} - \dot{G}) \end{aligned}\right\} \quad \ldots(1)$$

where ∇^2 is the Laplacian, a dot over a quantity denotes partial differentiation w.r.t. time, t and the potential functions ϕ, ψ, H and G are connected with the solid displacement $\mathbf{u}$ (u_r, u_θ) and liquid displacement $\mathbf{U}$ (U_r, U_θ) by

$$\left.\begin{aligned} u_r &= D\phi + r^{-1}\,dH,\; u_\theta = r^{-1}\,d\,\phi - DH \\ U_r &= D\psi + r^{-1}\,dG,\; U_\theta = r^{-1}\,d\,\psi - DG \end{aligned}\right\} \quad \ldots(2)$$

and their relations with the relevant solid stresses σ_{rr}, $\sigma_{r\theta}$ and liquid pressure (s) by

$$\left.\begin{aligned} \sigma_{rr} &= 2N\,(D^2\phi + r^{-1}\,DdH - r^{-2}\,dH) + A\nabla^2\,\phi + Q\nabla^2\psi \\ \sigma_{r\theta} &= N\,(2\,r^{-1}\,Dd\,\phi + r^{-2}\,d^2H - 2\,r^{-2}\,d\phi - D^2H + r^{-1}\,DH) \\ s &= Q\,\nabla^2\,\phi + R\nabla^2\,\psi. \end{aligned}\right\} \quad \ldots(3)$$

In the above, the notations of Biot (1956) wherever possible are followed and $D = \partial/\partial r$, $d = \partial/\partial\theta$.

Considering a circular cylindrical cavity in an infinite poroelastic medium the bounded solutions of (1) for steadystate harmonic vibrations are seen to be

$$\left.\begin{aligned} \phi &= [\,C_1\,H_\alpha^{(2)}(x) + C_2\,H_\alpha^{(2)}(y)\,]\cos\alpha\theta\,\exp(ipt) \\ \psi &= -\,[C_1\,L_1\,H_\alpha^{(2)}(x) + C_2L_2H_\alpha^{(2)}(y)]\cos\alpha\theta\,\exp(ipt) \\ H &= H_\alpha^{(2)}(z)\,C_3\cos\alpha\theta\,\exp(ipt) \\ G &= -\frac{M_{12}\,H}{M_{22}} \end{aligned}\right\} \quad \ldots(4)$$

where

$$V_1\,x = V_2\,y = V_3\,z = pr$$

$$L_1 = \frac{V_1^2\,(RM_{11} - QM_{12}) - (PR - Q^2)}{V_1^2\,(RM_{12} - QM_{22})}$$

L_2 = Similar expression as L_1 with V_1 replaced by V_2,

$$M_{11} = \rho_{11} - \frac{ib}{p},\; M_{12} = \rho_{12} + \frac{ib}{p},\; M_{22} = \rho_{22} - \frac{ib}{p}\,. \quad \ldots(5)$$

V_1, V_2, V_3 are the wave velocities of dilatational wave of first and second kind and a shear wave respectively (Biot 1956). The angular wave number α is equal to ka; k is wave number, a is radius of cylinder and p is frequency of the wave. $H_\alpha^{(2)}$ is the Bessel function of third kind (Hankel function). The Hankel function tends to zero as the argument becomes large, although, the individual Bessel functions do not.

The stresses and pore-pressure are subsequently obtained from eqns. (3) and (4) which are

$$\begin{aligned} r^2\,\sigma_{rr} = \Big\langle &[2N\,\{x^2\;H_{\alpha-2}^{(2)}(x) - x\;(2\alpha-1)\;H_{\alpha-1}^{(2)}(x)\} \\ &+\{2N\alpha\,(\alpha+1) + x^2QL_1\;(1+L_1^{-1}\;p^{-2} - Q^{-1}\,p^{-2}\,R) \\ &-(P-2N)\,x^2\}H_\alpha^{(2)}(x)]\;C_1 + [2N\;\{y^2\;H_{\alpha-2}^{(2)}(y) - y\,(2\alpha-1)\,H_{\alpha-1}^{(2)}\;(y)\}) \\ &+\{2N\alpha\,(\alpha+1) + y^2\;QL_2\;(1+L_2^{-1}p^{-2} - Q^{-1}\;p^{-2}R) - (p-2N)y^2\}H_\alpha^{(2)}(y)]C_2 \\ &+2\,N^{i\alpha}\,[zH_{\alpha-1}^{(2)}(z) - (\alpha+1)\,H_\alpha^{(2)}\;(z)]\;\;C_3\Big\rangle\cos\alpha\,\theta\;e^{ipt} \end{aligned} \quad \ldots(6)$$

$$r^2 \sigma_{r\theta} = 2i\alpha [\{x H^{(2)}_{\alpha-1}(x) - (\alpha+1) H^{(2)}_{\alpha}(x)\} C_1$$
$$+ \{y H^{(2)}_{\alpha-1}(y) - (\alpha+1) H^{(2)}_{\alpha}(y)\} C_2$$
$$+ \{2\alpha z \; H^{(2)}_{\alpha-1}(z) - 2\alpha(\alpha+1) H^{(2)}_{\alpha}(z) - z^2 H^{(2)}_{\alpha-2}(z)\} C_3] \cos\alpha\theta \, e^{ipt} \qquad \dots(7)$$

$$s = [V_1^{-2}(Q - RL_1) \; H^{(2)}_{\alpha}(x) C_1 + V_2^{-2}(Q - RL_2) H^{(2)}_{\alpha}(y) C_2] \cos\alpha\theta \, e^{ipt}. \qquad \dots(8)$$

From the conditions of stress-free curved surface, the dispersion relation of wave velocity in non-dimensional form for pervious surface, after a long calculation, in absence of dissipation is

$$| A_{ij} | = 0, \; i, j = 1, 2, 3 \qquad \dots(9)$$

where

$$A_{11} = 2q_4^2 \alpha^2 a_3^2 C^2 \; H^{(2)}_{\alpha-2}(x_1) - 2\alpha C a_3 q_4^{3/2} (2\alpha - 1) \; H^{(2)}_{\alpha-1}(x_1)$$
$$+ \{2q_4 \alpha(\alpha+1) - (q_1 - 2q_4) \alpha^2 q_4 C^2 a_3^2 + q_2 q_4 \alpha^2 C^2 a_3^2$$
$$\times \; (a_1^2 \; a_3^2 (q_4 \alpha^2 C^2 + q_2^{-1} \; q_3) + \; a_2^2) - a_3^2 \; q_2 + a_2^2 \; a_3^2 \; q_3\} \; H^{(2)}_{\alpha}(x_1)$$

$$A_{13} = 2q_4 \; \alpha^2 C \sqrt{m_4} \; H^{(2)}_{\alpha-1} \; (z_1) - 2q_4 \alpha (\alpha+1) H^{(2)}_{\alpha}(z_1)$$

$$A_{21} = \alpha \sqrt{q_4} \, a_3 \, C H^{(2)}_{\alpha-1} \; (x_1) - (\alpha+1) \; H^{(2)}_{\alpha}(x_1)$$

$$A_{23} = 2^{-1} \alpha \, C^2 m_4 \; H^{(2)}_{\alpha-2}(z_1) - \alpha \, C \sqrt{m_4} \; H^{(2)}_{\alpha-1}(z_1) + (\alpha+1) \, H^{(2)}_{\alpha}(z_1)$$

$$A_{31} = a_3^2 (- \, q_2 + q_3 \, q_4 \, \alpha^2 \; a_3^2 \; C^2 \; a_1^2 + q_3 \; a_2^2) \; H^{(2)}_{\alpha}(x_1),$$

A_{12}, A_{22}, A_{32} = Similar expressions as A_{11}, A_{21}, A_{31} with a_3 and x_1 respectively replaced by a_4 and y_1,

$$A_{33} = 0. \qquad \dots(10)$$

In the determinant (9), we have

$$x_1 = \sqrt{q_4} \, \alpha \, a_3 \, C, \; y_1 = \sqrt{q_4} \, \alpha \, a_4 \, C, \; z_1 = \alpha \, C \sqrt{m_4},$$

$$a_1^2 = \frac{q_1 q_3 - q_2^2}{q_4 \alpha^2 C^2 (m_2 q_3 - q_2 m_3)}, \; a_2^2 = \frac{m_1 q_3 - q_2 m_2}{m_2 q_3 - q_2 m_3}$$

$$q_5 \; a_3^2, \; q_5 \; a_4^2 = q_6 \pm (q_6 - 4 \, q_5 \, m_3 \, m_4)^{\frac{1}{2}}$$

$$m_3 \, m_4 = m_1 \, m_3 - m_2^2, \; q_5 = q_1 \, q_3 - q_2^2, \; q_6 = q_1 \, m_3 - 2 \, q_2 \, m_2 + m_1 \, q_3$$

$$m_1 = \rho_{11}/\rho, \; m_2 = \rho_{12}/\rho \;\; m_3 = \rho_{22}/\rho$$

$$q_1 = P/H_1, \; q_2 = Q/H_1, \; q_3 = R/H_1, \; q_4 = N/H_1$$

with

$\rho = \rho_{11} + 2\,\rho_{12} + \rho_{22}$, $H_1 = P + 2Q + R$

C is dimensionless phase velocity.

The wave velocity equation for impervious surface is

$$| B_{ij} | = 0, \quad i, j = 1, 2, 3. \qquad \text{...(11)}$$

The elements of the above determinant are :

$B_{11}=A_{11}$, $B_{12}=A_{12}$, $B_{13}=A_{13}$; $B_{21}=A_{21}$ $B_{22}=A_{22}$, $B_{23}=A_{23}$,

$$B_{31} = a_3 A_{31} \{H^{(2)}_{\alpha-1}(x_1) - \alpha\, x_1^{-1} H^{(2)}_{\alpha}(x_1)\}/H^{(2)}_{\alpha}(x_1),$$

B_{32} = Similar expression as B_{31} with a_3 and x_1 replaced by a_4 and y_1 respectively,

$B_{33} = 0.$

By neglecting liquid effects in (6), (7) and (9), one can easily see, after some rearrangement of terms that these correspond to the stresses and wave velocity equation of classical theory of elasticity (Epstein 1976).

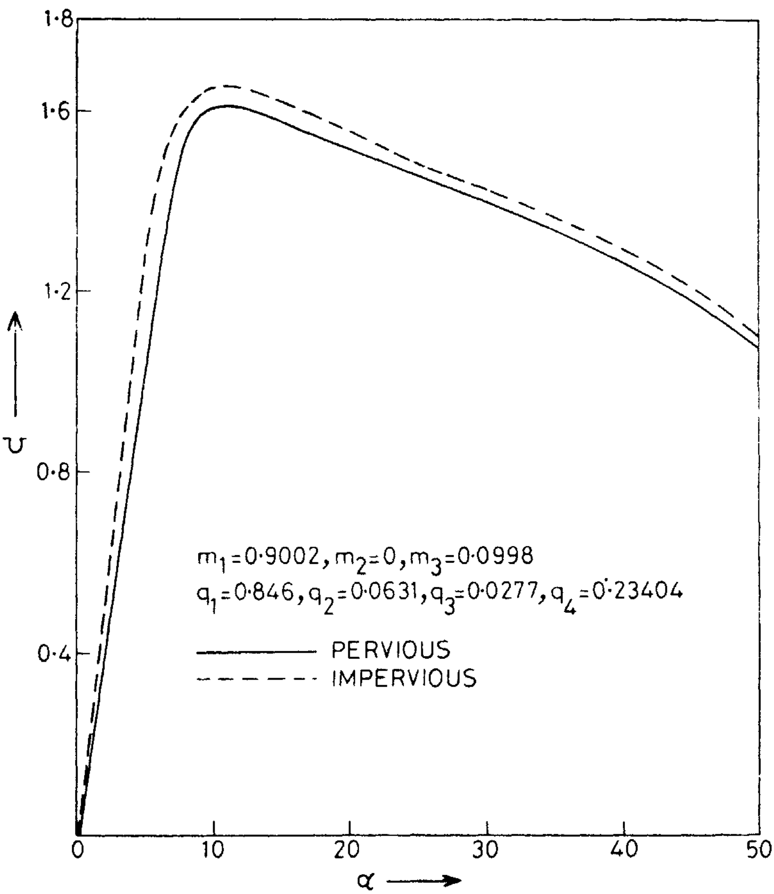

FIG. 1 Phase velocity versus wavenumber.

3. Discussions

It can be seen that in the limit $\alpha \rightarrow \infty$, the dispersion relation for Rayleigh waves on a concave cylindrical surface, eqns. (9) and (11), reduces to the equation for Rayleigh waves in the poroelastic half-space for pervious and impervious surface respectively. The phase velocity versus wave number is calculated for the material sand-stone saturated with kerosene (Fatt 1959) for two specific cases, namely pervious and impervious surface. These results are presented in the form of a graph. Rayleigh waves for this surface are dispersive unlike for a straight surface. Only one mode of propagation is observed to be existing. Further it can be seen from the Fig. 1 that phase velocity increases in the interval (0, 10) of wavenumber whereas in the remaining considered interval, it is decreasing. Moreover, phase velocity for impervious surface is greater than that of pervious surface.

References

Biot, M. A. (1956). Theory of propagation of elastic waves in fluid-saturated porous solid. *J. Acous. Soc. Am.*, **28**, 168–78.

Epstein, H. I. (1976). The effect of curvature on Stoneley waves. *J. Sound. Vib.*, **46**, 59–66.

Fatt, I. (1959). The Biot-Will's elastic coefficients for a sand Stone. *J. appl. Mech.*, **26**, 296–97.