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A REMARK ON "A NOTE ON THE SEMI-CONTINUITY PROPERTIES OF THE FARTHEST POINT MAP"

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Rao and Muthukumar (1980) proved the following four results concerning necessary and sufficient conditions for the farthest point map to be upper semi-continuous (u. s. c) and lower semi-continuous (l. s. c.).

Theorem 1 — If G if a remotal subspace of the normed linear space E, then $\oint_G$, the farthest point map, is u.s.c. if and only if for each closed subspace N of G, $N + \oint_G^{-1}(0)$ is closed.

Theorem 2 — Let G be a closed remotal subspace of the normed linear space E such that $E = G \oplus L$ (topological direct sum). Then $\oint_G$ is u.s.c. if for each compact subset $A \subset L$, the subset $\oint_G(A)$ is a compact subset of G.

Theorem 3 — If G is a remotal subspace of the normed linear space E, then $\oint_G$ is l.s.c. if and only if for each open set D of G, $D + \oint_G^{-1}(0)$ is open in E.

Theorem 4 — Let G be a closed remotal subspace of the normed linear space E such that $E = G \oplus L$ (topological direct sum). Then $\oint_G$ is l.s.c. if for each relatively compact subset A of L, $\overline{\oint_G(A)} = \oint_G(\overline{A})$.

In all these theorems G is taken to be a remotal subspace of the normed linear space E (as the proof of the theorems demand). Remotal subspace was defined by Rao and Muthukumar (1980) as follows :

Let E be a normed linear space and G be a subspace of E. Let $x \in E$. Then the point $g_0 \in G$ is said to be 'a farthest point' of x if

$$\| x - g_0 \| = \sup \{ \| x - g \| : g \in G \}.$$

The set of all farthest points of x in G is denoted by $\oint_G(x)$. If $\oint_G(x) \neq \phi$, for all $x \in E$, then G is called 'a remotal subspace' of E. If G is a remotal subspace, then the set-valued map $\oint_G : x \to \oint_G(x)$ is called the 'farthest point map'.

The definitions are valid if G is taken to be an arbitrary subset of the space E. It is easy to see that 'every remotal subset of a normed linear space is bounded'. Also it is well known (see e.g. Wilansky 1964, p. 178) that 'no non-zero linear subspace of a normed linear space is bounded (unless the space is trivially normed)'. Therefore the notion of 'a remotal subspace' of a normed linear space is meaningless i.e. no non-zero linear subspace of a normed linear space can be remotal. So the above four theorems do not have any significance as no such non trivial space exist.

REFERENCES

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