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INSTANTANEOUS HEAT SOURCES IN AN INFINITE ELASTIC SOLID WITH THERMAL RELAXATION

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The generalised dynamical theory of thermoelasticity is applied to solve the problem of determination of the distribution of temperature, deformation, stress and strain in an infinite isotropic elastic solid having distributed instantaneous heat sources. The solutions are derived by the use of Laplace transform on time and Fourier transform on space. Since the effects of relaxation time on thermoelastic interactions are short lived, wave fronts and small time approximations are considered and compared with the previous results deduced from classical coupled thermoelastic theory.

INTRODUCTION

Paria (1968) studied a problem of an infinite isotropic elastic solid having distributed instantaneous heat sources using the classical coupled thermoelastic theory. The solutions, arrived at, consisted of two parts—a wave part travelling with the speed of dilatational (elastic) wave and a part which is of diffused nature. His solutions for temperature and deformation were seen to be continuous at the dilatational wave front. In the present paper the authors study the same problem using the generalised dynamical theory of thermoelasticity of Lord and Shulman (1967). According to this theory the classical heat conduction equation is generalised by using the modified form of Fourier's law containing the thermal relaxation in time. The classical coupled heat equation then becomes fully hyperbolic and the paradox of infinite speed of heat propagation in classical theory is thereby removed. It is seen that the presence of thermal relaxation time makes the solutions non-diffusive in nature and the solutions are found to consist of two waves—a modified elastic wave propagating with speed v_1 and a thermal wave travelling with speed v_2 and that $v_1 < v_2$. In the absence of the thermal relaxation time $v_1 \to 1$ and $v_2 \to \infty$, corresponding to the classical coupled theory, which predicts an infinite speed of heat propagation. Since the effects of relaxation time on thermoelastic interactions are short lived, we concentrate our attention on the study of possible discontinuities at both the wave fronts and small-time approximations. It is seen that our solutions for deformation, temperature, stress and strain are discontinuous at both the wave fronts in contrast to Paria's result in classical case where the solutions for deformation and temperature are continuous at the elastic wave front. The magnitudes of these discontinuities at the wave fronts are studied, which are, in fact, valid for relatively short times. In the absence of thermal relaxation time, the discontinuity at the elastic wave front of deformation and temperature disappears which agrees well wiih Paria's result. In the absence of relaxation time, the discontinuities in strain and stress still exist at the elastic wave front and the amount of discontinuity decays exponentially with distance at the elastic wave front. In the absence of thermal coupling also, these discontinuities still exist and are uniform although the thermal field is continuous at the elastic wave front.

Though transient boundary value problems relating to elastic half space (Popov 1967) and surface waves in generalised thermoelasticity (Puri 1973, Agarwal 1978) have been studied, it is believed that this particular problem of an infinite solid having time-dependent distributed heat sources considering thermal relaxation time is not dealt with before. It may be mentioned that though very recently Bhatta (1981) has studied a similar problem, this particular problem under consideration is the extension of Paria's paper in generalised thermoelasticity.

BASIC EQUATIONS

The fundamental equations of linear dynamic thermoelasticity with thermal relaxation in time are as follows :

(a) Balance of linear momentum principle yields the equations of motion in absence of body forces

$$\tau_{ij,j} = \rho \ddot{u}_i \ (i, j = 1, 2, 3) \qquad \ldots(1)$$

(b) Local energy balance principle gives the linearised energy equation

$$- q_{i,i} + Q = \rho c_v \dot{T} + \beta T^* \dot{\Delta} \qquad \ldots(2)$$

(c) The modified Fourier's law of heat conduction taking into account the thermal relaxation in time (Lord and Shulman 1967)

$$q_i + \tau_0 \dot{q}_i = - K T_{,i} \qquad \ldots(3)$$

(d) Stress-strain-temperature relations in linear thermoelasticity are :

$$\tau_{ij} = (\lambda\Delta - \beta T)\, \delta_{ij} + 2\, \mu e_{ij} \qquad \ldots(4)$$

with $e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i})$; $(i, j = 1, 2, 3)$ ...(5)

where τ_{ij} = components or stress tensor

u_i = components of displacement vector

Δ = dilatation = $u_{i,i}$

q_i = components of heat flux vector

ρ = constant mass density

c_v = specific heat of the solid at constant strain

δ_{ij} = kronecker delta

T = increase of temperature above reference temperature

λ,μ = lame constants

β = $(3\lambda + 2\mu)\, \alpha_t$ where α_t = coefficient of linear thermal-expansion

τ_0 = thermal relaxation in time
K = coefficient of thermal conductivity of the solid
Q = heat source term.

Here 'dot' denotes time differentiation and a 'comma' means differentiation with respect to the space variable.

Eliminating τ_{ij} from eqns. (1), (4) and (5), we obtain the displacement equations of motion

$$\mu \nabla^2 u_i + (\lambda + \mu) \Delta_{,i} - (3\lambda + 2\mu) \alpha_t T_{,i} = \rho \ddot{u}_i \quad (i, = 1, 2, 3). \qquad \ldots(6)$$

Eliminating q_i between (2) and (3) we obtain (Dhaliwal and Singh 1980)

$$K T_{,ii} + Q + \tau_0 \dot{Q} = \rho c_v (\dot{T} + \tau_0 \ddot{T}) + (3\lambda + 2\mu) \alpha_t T^* (\dot{\Delta} + \tau_0 \ddot{\Delta}). \qquad \ldots(7)$$

Equations (6) and (7) constitute basic equations in generalised dynamical theory of thermoelasticity in the form of two coupled hyperbolic equations.

The Problem and its Transform Solution

We consider an infinite elastic solid unstrained and unstressed initially, but has a uniform reference temperature throughout. It is then subjected to instantaneous heat sources distributed over the plane $x = 0$. The problem is to find the subsequent distributions of deformation, temperature, strain and stress as well as the interaction between the thermal and deformation fields.

Let the solid occupy the whole space $-\infty < x < \infty$.

From symmetry consideration, it follows that the displacement vector $\mathbf{u} = [u(x, t), 0, 0]$ and the temperature increase $T = T(x, t)$ where x denotes the spatial coordinate and t, the time. Then basic equations (6) and (7) simplify to

$$(\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} = \rho \frac{\partial^2 u}{\partial t^2} + \beta \frac{\partial T}{\partial x} \qquad \ldots(8)$$

and $$K \frac{\partial^2 T}{\partial x^2} + Q(x, t) + \tau_0 \frac{\partial Q}{\partial t} = \rho c_v \left(\frac{\partial T}{\partial t} + \tau_0 \frac{\partial^2 T}{\partial t^2} \right) + \beta T^* \left(\frac{\partial^2 u}{\partial x \, \partial t} + \tau_0 \frac{\partial^3 u}{\partial x \, \partial t^2} \right). \qquad \ldots(9)$$

Introducing dimensionless quantities

$$\xi = \frac{ax}{\kappa}, \; U = \frac{a(\lambda + 2\mu) u}{\kappa \beta T^*}, \; \eta = a^2 t/\kappa$$

$$\theta = \frac{T}{T^*}, \; a^2 = (\lambda + 2\mu)/\rho, \; \frac{Q}{Q_0} = Q^*(\xi, \eta)$$

where $K = \kappa \rho c_v$, κ is the thermal diffusivity, the equation (9), in the case of distributed instantaneous heat sources on $\xi = 0$, reduces to

$$\frac{\partial^2 \theta}{\partial \xi^2} + K_0 \{ \delta(\xi) \delta(\eta) + \tau_0' \, \delta(\xi) \delta'(\eta) \}$$

(*equation continued on p.* 1343)

$$= \frac{\partial \theta}{\partial \eta} + \tau_0' \frac{\partial^2 \theta}{\partial \eta^2} + \epsilon \frac{\partial^2 U}{\partial \xi \partial \eta} + \epsilon \tau_0' \frac{\partial^3 U}{\partial \xi \partial \eta^2} . \qquad \text{...(9)}'$$

The equation (8) also reduces to

$$\frac{\partial^2 U}{\partial \xi^2} = \frac{\partial^2 U}{\partial \eta^2} + \frac{\partial \theta}{\partial \xi} . \qquad \text{...(8)}'$$

Here $\tau_0' = a^2 \tau_0 / \kappa$ is the thermal relaxation constant

$\epsilon = \frac{\beta^2 T^*}{\rho c_v (\lambda + 2\mu)}$ is the thermoelastic coupling constant

and $K_0 = \frac{\kappa Q_0}{\rho c_v T^* a^2}$ is a constant depending on the density and the thermal property of the medium.

We denote the Laplace transforms of $U(\xi, \eta)$, $\theta(\xi, \eta)$ by $\bar{U}(\xi, p)$, $\bar{\theta}(\xi, p)$ where p is Laplace parameter and the Fourier transforms of $\bar{U}(\xi, p)$, $\bar{\theta}(\xi, p)$ by $\bar{U}_1(\zeta, p)$, $\bar{\theta}_1(\zeta, p)$ where ζ is the Fourier transform parameter

Thus $\bar{U}(\xi, p) = \int_0^\infty U(\xi, \eta)\, e^{-p\eta}\, d\eta$

$$\bar{\theta}(\xi, p) = \int_0^\infty \theta(\xi, \eta)\, e^{-p\eta}\, d\eta$$

$$\bar{U}_1(\zeta, p) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \bar{U}(\xi, p)\, e^{i\zeta\xi}\, d\xi$$

$$\bar{\theta}_1(\zeta, p) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \bar{\theta}(\xi, p)\, e^{i\zeta\xi}\, d\xi .$$

First by applying Laplace transform to equations (8)′ and (9)′ and then applying Fourier transform we obtain

$$(\zeta^2 + p^2)\, \bar{U}_1 = i\zeta\, \bar{\theta}_1 \qquad \text{...(10)}$$

and $\left(\zeta^2 + p + \tau_0' p^2\right) \bar{\theta}_1 - \left(\epsilon p i \zeta + \epsilon \tau_0' p^2 i \zeta\right) \bar{U}_1 = \frac{K_0}{\sqrt{2\pi}} \left(1 + \tau_0' p\right).$ ...(11)

Solving for $\bar{U}_1$, $\bar{\theta}_1$

$$\left.\begin{aligned} \bar{U}_1(\zeta, p) &= \frac{K_0\, i\zeta\, (1 + \tau_0' p)}{\sqrt{2\pi}\, .\, M} \\ \bar{\theta}_1(\zeta, p) &= \frac{K_0\, (\zeta^2 + p^2)\, (1 + \tau_0' p)}{\sqrt{2\pi}\, .\, M} \end{aligned}\right\} \qquad \text{...(12)}$$

where $M = \zeta^4 + p\,\zeta^2 (K_1 + pK_2) + (p^3 + \tau_0' p^4) = (\zeta^2 + \zeta_3^2)(\zeta^2 + \zeta_4^2)$,

$K_1 = 1 + \epsilon,\ K_2 = 1 + \tau_0' + \epsilon\ \tau_0'$,

$\zeta_3^2 + \zeta_4^2 = p(K_1 + pK_2),\ \zeta_3^2\ \zeta_4^2 = p^3 + \tau_0'\ p^4$.

Then

$$\zeta_{3,4}^2 = \frac{p}{2}\left[(K_1 + p\,K_2) \pm \sqrt{R}\right]$$

$$R = (K_2^2 - 4\tau_0')\,p^2 + (2K_1 K_2 - 4)\,p + K_1^2 .$$

We write

$$\bar{\theta}_1\ (\zeta, p) = \frac{K_0\,(1 + \tau_0' p)}{\sqrt{2\pi}} \left[\frac{A}{\zeta^2 + \zeta_3^2} + \frac{B}{\zeta^2 + \zeta_4^2}\right]$$

where $A = \dfrac{p^2 - \zeta_3^2}{\zeta_4^2 - \zeta_3^2}$, $B = -\dfrac{p^2 - \zeta_4^2}{\zeta_4^2 - \zeta_3^2}$.

Similarly $$\bar{U}_1\ (\zeta, p) = \frac{K_0\ i}{\sqrt{2\pi}}\ \frac{(1 + \tau_0' p)}{(\zeta_4^2 - \zeta_3^2)} \left[\frac{\zeta}{\zeta^2 + \zeta_3^2} - \frac{\zeta}{\zeta^2 + \zeta_4^2}\right].$$

Inverse Fourier transforms then give

$$\bar{\theta}\ (\xi, p) = \frac{K_0(1 + \tau_0' p)}{2\,\zeta_3\,\zeta_4} \left[A\,\zeta_4\,e^{-\zeta_3 \xi} + B\,\zeta_3\,e^{-\zeta_4 \xi}\right] \text{ for } \xi > 0$$

$$\bar{U}\ (\xi, p) = \frac{K_0(1 + \tau_0' p)}{2\,(\zeta_4^2 - \zeta_3^2)} \left(e^{-\zeta_3 \xi} - e^{-\zeta_4 \xi}\right) \text{ for } \xi > 0.$$

As the effects of relaxation time on thermoelastic interactions are short lived Green and Lindsay (1972) we concentrate our attention on small-time approximations.

We make use of Abel's theorem $\lim\limits_{t \to 0} \bar{f}(t) = \lim\limits_{p \to \infty} \{p\,f(p)\}$ i. e. small values of the time correspond to large values of the parameter p.

Expanding $\bar{U}, \bar{\theta}$ in ascending powers of $1/p$ we obtain

$$\zeta_{3,4} = \frac{p}{v_{1,2}} + \beta_{1,2} + \frac{\phi_{1,2}}{p} + O\,(1/p^2)$$

where

$$\frac{1}{\nu_{1,2}} = \frac{1}{\sqrt{2}} \left(K_2 \pm \sqrt{(K_2^2 - 4\,\tau_0')} \right)^{1/2},$$

$$\beta_{1,2} = \frac{1}{2\sqrt{2}} \frac{\left(K_1 \pm \dfrac{(K_1 K_2 - 2)}{\sqrt{(K_2^2 - 4\,\tau_0')}} \right)}{\left(K_2 \pm \sqrt{(K_2^2 - 4\,\tau_0')} \right)^{\frac{1}{2}}}$$

$$\phi_{1,2} = \frac{1}{4\sqrt{2}} \frac{1}{\left(K_2 \pm \sqrt{(K_2^2 - 4\,\tau_0')} \right)^{\frac{1}{2}}} \left[\pm \frac{K_1^2}{\sqrt{(K_2^2 - 4\,\tau_0')}} \mp \frac{(K_1 K_2 - 2)^2}{(K_2^2 - 4\,\tau_0')^{3/2}} - \frac{1}{2} \frac{\left(K_1 \pm \dfrac{(K_1 K_2 - 2)}{\sqrt{(K_2^2 - 4\,\tau_0')}} \right)^2}{\left(K_2 \pm \sqrt{(K_2^2 - 4\,\tau_0')} \right)} \right]. \qquad \ldots(13)$$

Also $\dfrac{2}{\nu_{1,2}^2} = 1 + \epsilon\, \tau_0' + \tau_0' \pm \sqrt{\Gamma}$

where $\Gamma = (K_2^2 - 4\,\tau_0')$ ...(14)

$$= (1 + \epsilon\, \tau_0' - \tau_0')^2 + 4\,\epsilon\, \tau_0'^2$$

so that Γ is a positive quantity.

Also $\left(1 + \epsilon\, \tau_0' + \tau_0' \right)^2 > \Gamma$ so that $\dfrac{1}{\nu_1^2} > \dfrac{1}{\nu_2^2}$ or $\nu_1 < \nu_2$.

Thus ν_1 corresponds to the speed of the slowest wave and ν_2 corresponds to that of the fastest wave. As a consequence of this, the points of the solid for which $\xi > \eta\, \nu_2$ do not experience any disturbance. From (13) and (14) we see that as $\tau_0' \to 0$, $\nu_1 \to 1$ and $\nu_2 \to \infty$. But $\tau_0' = 0$ corresponds to the case of the classical coupled theory, which predicts an infinite speed of heat propagation. We conclude, therefore, that the wave propagating with speed ν_2 is the thermal wave and the other propagating with seed ν_1 must be the elastic wave influenced by the thermal field. Since $\nu_1 < \nu_2$, the elastic wave follows the thermal wave.

Now $$\frac{1}{\zeta_4^2 - \zeta_3^2} = -\frac{1}{p\, R^{1/2}} \simeq \left[-\frac{1}{\Gamma^{1/2}} \, \frac{1}{p^2} + \frac{(K_1 K_2 - 2)}{\Gamma^{3/2}} \cdot \frac{1}{p^3} + \frac{K_1^2}{2\,\Gamma^{3/2}} \cdot \frac{1}{p^4} + O(1/p^5) \right] \text{ for large } p.$$

Thus $$\bar{U}(\xi, p) \simeq \frac{K_0}{2}\left[-\frac{\tau_0'}{\Gamma^{1/2}}\cdot\frac{1}{p}+\left\{\frac{\tau_0'(K_1 K_2-2)}{\Gamma^{3/2}}-\frac{1}{\Gamma^{1/2}}\right\}\frac{1}{p^2}+O\left(\frac{1}{p^3}\right)\right]$$
$$\times\left(e^{-\left(\frac{p}{v_1}+\beta_1\right)\xi}-e^{-\left(\frac{p}{v_2}+\beta_2\right)\xi}\right).$$

Again

$$\frac{A}{\zeta_3} \simeq -\frac{(v_1^2-1)}{v_1\Gamma^{1/2}}\cdot\frac{1}{p}+\left\{\frac{2\beta_1}{\Gamma^{1/2}}+\frac{(v_1^2-1)(K_1K_2-2)}{v_1\Gamma^{3/2}}+\frac{\beta_1(v_1^2-1)}{\Gamma^{1/2}}\right\}\frac{1}{p^2}$$
$$+\left\{\frac{\beta_1^2 v_1+2\varphi_1}{\Gamma^{1/2}}-\frac{2\beta_1(K_1K_2-2)}{\Gamma^{3/2}}-\frac{2\beta_1^2 v_1}{\Gamma^{1/2}}\right.$$
$$\left.-\frac{\beta_1(v_1^2-1)(K_1K_2-2)}{\Gamma^{3/2}}-\frac{(v_1^2-1)(\beta_1^2 v_1-\varphi_1)}{\Gamma^{1/2}}\right\}\frac{1}{p^3}.$$

Similarly,

$$\frac{B}{\zeta_4} \simeq \frac{(v_2^2-1)}{v_2\Gamma^{1/2}}\cdot\frac{1}{p}-\left\{\frac{2\beta_2}{\Gamma^{1/2}}+\frac{(v_2^2-1)(K_1K_2-2)}{v_2\Gamma^{3/2}}+\frac{\beta_2(v_2^2-1)}{\Gamma^{1/2}}\right\}\frac{1}{p^2}.$$
$$-\left\{\frac{(\beta_2^2 v_2+2\varphi_2)}{\Gamma^{1/2}}-\frac{2\beta_2(K_1K_2-2)}{\Gamma^{3/2}}-\frac{2\beta_2^2 v_2}{\Gamma^{1/2}}\right.$$
$$\left.-\frac{(v_2^2-1)(K_1K_2-2)}{\Gamma^{3/2}}-\frac{(v_2^2-1)(\beta_2^2 v_2-\varphi_2)}{\Gamma^{1/2}}\right\}\frac{1}{p^3}.$$

Hence, $$\bar{\theta}(\xi, p) \simeq \frac{K_0}{2}\left[-\frac{\tau_0(v_1^2-1)}{v_1\Gamma^{1/2}}+\left\{\frac{2\beta_1\tau_0'}{\Gamma^{1/2}}+\frac{\tau_0'(v_1^2-1)(K_1K_2-2)}{v_1\Gamma^{3/2}}\right.\right.$$
$$\left.+\frac{\beta_1\tau_0'(v_1^2-1)}{\Gamma^{1/2}}\quad\frac{(v_1^2-1)}{v_1\Gamma^{1/2}}\right\}\frac{1}{p}+\left\{\frac{(\beta_1^2 v_1+2\varphi_1)\tau_0'}{\Gamma^{1/2}}\right.$$
$$-\frac{2\beta_1\tau_0'(K_1K_2-2)}{\Gamma^{3/2}}-\frac{2\beta_1^2 v_1\tau_0'}{\Gamma^{1/2}}-\frac{\beta_1\tau_0'(v_1^2-1)(K_1K_2-2)}{\Gamma^{3/2}}$$
$$-\frac{\tau_0'(v_1^2-1)(\beta_1^2 v_1-\varphi_1)}{\Gamma^{1/2}}+\left(\frac{2\beta_1}{\Gamma^{1/2}}+\frac{(v_1^2-1)(K_1K_2-2)}{v_1\Gamma^{3/2}}\right.$$
$$\left.\left.\left.+\frac{\beta_1(v_1^2-1)}{\Gamma^{1/2}}\right)\right\}\frac{1}{p^2}\right]e^{-(p/v_1+\beta_1)\xi}$$

(*equation continued on p.* 1347)

$$- \frac{K_0}{2}\Big[- \frac{\tau_0'(v_2^2-1)}{v_2 \Gamma^{1/2}} + \Big\{ \frac{2\beta_2 \tau_0'}{\Gamma^{1/2}} + \frac{\tau_0'(v_2^2-1)(K_1K_2-2)}{v_2 \Gamma^{3/2}}$$

$$+ \frac{\beta_2 \tau_0'(v_2^2-1)}{\Gamma^{1/2}} - \frac{(v_2^2-1)}{v_2 \Gamma^{1/2}} \Big\} \frac{1}{p} + \Big\{ \frac{(\beta_2^2 v_2 + 2\varphi_2)\tau_0'}{\Gamma^{1/2}}$$

$$- \frac{2\beta_2 \tau_0'(K_1K_2-2)}{\Gamma^{3/2}} - \frac{2\beta_2^2 v_2 \tau_0'}{\Gamma^{1/2}} - \frac{\beta_2 \tau_0'(v_2^2-1)(K_1K_2-2)}{\Gamma^{3/2}}$$

$$- \frac{\tau_0'(v_2^2-1)(\beta_2^2 v_2 - \varphi_2)}{\Gamma^{1/2}} + \Big(\frac{2\beta_2}{\Gamma^{1/2}} + \frac{(v_2^2-1)(K_1K_2-2)}{v_2 \Gamma^{3/2}}$$

$$+ \frac{\beta_2(v_2^2-1)}{\Gamma^{1/2}} \Big) \Big\} \frac{1}{p^2} \Big] e^{-(p/v_2+\beta_2)\xi}.$$

On making use of these expansions, we obtain for the dimensionless stress

$$\bar{\sigma}_{\xi\xi} = \frac{d\bar{U}}{d\xi} - \bar{\theta}$$

$$\simeq - \frac{K_0}{2}\Big[- \frac{v_1 \tau_0'}{\Gamma^{1/2}} + \Big\{ \frac{v_1^2 \beta_1 \tau_0'}{\Gamma^{1/2}} + \frac{v_1 \tau_0'(K_1K_2-2)}{\Gamma^{3/2}} - \frac{v_1}{\Gamma^{1/2}} \Big\} \frac{1}{p}$$

$$+ \Big\{ \frac{\tau_0' K_1^2}{2 v_1 \Gamma^{3/2}} + \frac{K_1K_2-2}{v_1 \Gamma^{3/2}} + \frac{\beta_1 \tau_0'(K_1K_2-2)}{\Gamma^{3/2}} - \frac{2\beta_1 \tau_0'(K_1K_2-2)}{\Gamma^{3/2}}$$

$$- \frac{\beta_1 \tau_0'(v_1^2-1)(K_1K_2-2)}{\Gamma^{3/2}} + \frac{(v_1^2-1)(K_1K_2-2)}{v_1 \Gamma^{3/2}} - \frac{\beta_1}{\Gamma^{1/2}} - \frac{\tau_0' \varphi_1}{\Gamma^{1/2}}$$

$$+ \frac{(\beta_1^2 v_1 + 2\varphi_1)\tau_0'}{\Gamma^{1/2}} - \frac{2\beta_1^2 v_1 \tau_0'}{\Gamma^{1/2}} - \frac{\tau_0'(v_1^2-1)(\beta_1^2 v_1 - \varphi_1)}{\Gamma^{1/2}}$$

$$+ \Big(\frac{2\beta_1}{\Gamma^{1/2}} + \frac{\beta_1(v_1^2-1)}{\Gamma^{1/2}} \Big) \Big\} \frac{1}{p^2} \Big] e^{-(p/v_1+\beta_1)\xi}$$

$$+ \frac{K_0}{2}\Big[- \frac{v_2 \tau_0'}{\Gamma^{1/2}} + \Big\{ \frac{v_2^2 \beta_2 \tau_0'}{\Gamma^{1/2}} + \frac{v_2 \tau_0'(K_1K_2-2)}{\Gamma^{3/2}} - \frac{v_2}{\Gamma^{1/2}} \Big\} \frac{1}{p}$$

$$+ \Big\{ \frac{\tau_0' K_1^2}{2 v_2 \Gamma^{3/2}} + \frac{(K_1K_2-2)}{v_2 \Gamma^{3/2}} + \frac{\beta_2 \tau_0'(K_1K_2-2)}{\Gamma^{3/2}} - \frac{2\beta_2 \tau_0'(K_1K_2-2)}{\Gamma^{3/2}}$$

$$- \frac{\beta_2 \tau_0'(v_2^2-1)(K_1K_2-2)}{\Gamma^{3/2}} + \frac{(v_2^2-1)(K_1K_2-2)}{v_2 \Gamma^{3/2}} - \frac{\beta_2}{\Gamma^{1/2}} - \frac{\tau_0' \varphi_2}{\Gamma^{1/2}}$$

(*equation continued on p.* 1348)

$$+\frac{\tau_0'(\beta_2^2 \nu_2+2\varphi_2)}{\Gamma^{1/2}}-\frac{2\beta_2^2\nu_2\tau_0'}{\Gamma^{1/2}}-\frac{\tau_0'(\nu_2^2-1)(\beta_2^2\nu_2-\varphi_2)}{\Gamma^{1/2}}$$

$$+\left(\frac{2\beta_2}{\Gamma^{1/2}}+\frac{\beta_2(\nu_2^2-1)}{\Gamma^{1/2}}\right)\Bigg\}\frac{1}{p^2}\Bigg]e^{-(p/\nu_2+\beta_2)\xi}.$$

We find for the strain

$$\bar{e}(\xi,p)=\frac{\beta T^*}{(\lambda+2\mu)}\cdot\frac{d\bar{V}}{d\xi}\simeq-\frac{\beta T^* K_0}{2(\lambda+2\mu)}\Bigg[-\frac{\tau_0'}{\nu_1\Gamma^{1/2}}+\Bigg\{\frac{\tau_0'(K_1K_2-2)}{\nu_1\Gamma^{3/2}}$$

$$-\frac{1}{\nu_1\Gamma^{1/2}}-\frac{\beta_1\tau_0'}{\Gamma^{1/2}}\Bigg\}\frac{1}{p}+\Bigg\{\frac{\tau_0'K_1^2}{2\nu_1\Gamma^{3/2}}+\frac{(K_1K_2-2)}{\nu_1\Gamma^{3/2}}$$

$$+\frac{\beta_1\tau_0'(K_1K_2-2)}{\Gamma^{3/2}}-\frac{\beta_1}{\Gamma^{1/2}}-\frac{\tau_0'\varphi_1}{\Gamma^{1/2}}\Bigg\}\frac{1}{p^2}\Bigg]e^{-(p/\nu_1+\beta_1)\xi}$$

$$+\frac{\beta T^* K_0}{2(\lambda+2\mu)}\Bigg[-\frac{\tau_0'}{\nu_2\Gamma^{1/2}}+\Bigg\{\frac{\tau_0'(K_1K_2-2)}{\nu_2\Gamma^{3/2}}-\frac{1}{\nu_2\Gamma^{1/2}}-\frac{\beta_2\tau_0'}{\Gamma^{1/2}}\Bigg\}\frac{1}{p}$$

$$+\Bigg\{\frac{\tau_0'K_1^2}{2\nu_2\Gamma^{3/2}}+\frac{(K_1K_2-2)}{\nu_2\Gamma^{3/2}}+\frac{\beta_2\tau_0'(K_1K_2-2)}{\Gamma^{3/2}}-\frac{\beta_2}{\Gamma^{1/2}}$$

$$-\frac{\tau_0'\varphi_2}{\Gamma^{1/2}}\Bigg\}\frac{1}{p^2}\Bigg]e^{-(p/\nu_2+\beta_2)\xi}.$$

Solution for Small Times

Inverting with respect to Laplace transform, we arrive at the following expressions for U, θ, e, $\sigma_{\xi\xi}$, valid for short times.

$$U(\xi,\eta)\simeq\frac{K_0}{2}\Bigg[-\frac{\tau_0'}{\Gamma^{1/2}}H\left(\eta-\frac{\xi}{\nu_1}\right)+\left\{\frac{\tau_0'(K_1K_2-2)}{\Gamma^{3/2}}-\frac{1}{\Gamma^{1/2}}\right\}$$

$$\times\left(\eta-\frac{\xi}{\nu_1}\right)H\left(\eta-\frac{\xi}{\nu_1}\right)\Bigg]e^{-\beta_1\xi}-\frac{K_0}{2}\Bigg[-\frac{\tau_0'}{\Gamma^{1/2}}H\left(\eta-\frac{\xi}{\nu_2}\right)$$

$$+\left\{\frac{\tau_0'(K_1K_2-2)}{\Gamma^{3/2}}-\frac{1}{\Gamma^{1/2}}\right\}\left(\eta-\frac{\xi}{\nu_2}\right)H\left(\eta-\frac{\xi}{\nu_2}\right)\Bigg]e^{-\beta_2\xi}$$

$$\theta(\xi,\eta)\simeq\frac{K_0}{2}\Bigg[-\frac{\tau_0'(\nu_1^2-1)}{\nu_1\Gamma^{1/2}}\delta\left(\eta-\frac{\xi}{\nu_1}\right)+\Bigg\{\frac{2\beta_1\tau_0'}{\Gamma^{1/2}}$$

$$+\frac{\tau_0'(\nu_1^2-1)(K_1K_2-2)}{\nu_1\Gamma^{3/2}}+\frac{\beta_1\tau_0'(\nu_1^2-1)}{\Gamma^{1/2}}-\frac{(\nu_1^2-1)}{\nu_1\Gamma^{1/2}}\Bigg\}H\left(\eta-\frac{\xi}{\nu_1}\right)$$

(equation continued on p. 1349)

$$+\left\{\frac{(\beta_1^2 \nu_1+2\,\varphi_1)\,\tau_0'}{\Gamma^{1/2}}-\frac{2\beta_1\,\tau_0'(K_1\,K_2-2)}{\Gamma^{3/2}}-\frac{2\,\beta_1^2\,\nu_1\,\tau_0'}{\Gamma^{1/2}}\right.$$

$$-\frac{\beta_1\,\tau_0'\,(\nu_1^2-1)\,(K_1\,K_2-2)}{\Gamma^{3/2}}-\frac{\tau_0'\,(\nu_1^2-1)\,(\beta_1^2\,\nu_1-\varphi_1)}{\Gamma^{1/2}}$$

$$\left.+\left(\frac{2\,\beta_1}{\Gamma^{1/2}}+\frac{(\nu_1^2-1)\,(K_1\,K_2-2)}{\nu_1\,\Gamma^{3/2}}+\frac{\beta_1\,(\nu_1^2-1)}{\Gamma^{1/2}}\right)\right\}$$

$$\left(\eta-\frac{\xi}{\nu_1}\right)H\left(\eta-\frac{\xi}{\nu_1}\right)\Big]\,{}^-e^{\beta_1\xi}$$

$$-\frac{K_0}{2}\left[-\frac{\tau_0'\,(\nu_2^2-1)}{\nu_2\,\Gamma^{1/2}}\,\delta\left(\eta-\frac{\xi}{\nu_2}\right)+\left\{\frac{2\beta_2\,\tau_0'}{\Gamma^{1/2}}+\frac{\tau_0'\,(\nu_2^2-1)\,(K_1\,K_2-2)}{\nu_2\,\Gamma^{3/2}}\right.\right.$$

$$\left.+\frac{\beta_2\,\tau_0'(\nu_2^2-1)}{\Gamma^{1/2}}-\frac{(\nu_2^2-1)}{\nu_2\,\Gamma^{1/2}}\right\}H\left(\eta-\frac{\xi}{\nu_2}\right)\left\{\frac{(\beta_2^2\,\nu_2+2\,\varphi_2)\,\tau_0'}{\Gamma^{1/2}}\right.$$

$$-\frac{2\beta_2\,\tau_0'(K_1\,K_2-2)}{\Gamma^{3/2}}-\frac{2\,\beta_2^2\,\nu_2\,\tau_0'}{\Gamma^{1/2}}-\frac{\beta_2\,\tau_0'\,(\nu_2^2-1)\,(K_1\,K_2-2)}{\Gamma^{3/2}}$$

$$-\frac{\tau_0'\,(\nu_2^2-1)\,(\beta_2^2\,\nu_2-\varphi_2)}{\Gamma^{1/2}}+\left(\frac{2\,\beta_2}{\Gamma^{1/2}}+\frac{(\nu_2^2-1)\,(K_1\,K_2-2)}{\nu_2\,\Gamma^{3/2}}\right.$$

$$\left.\left.+\frac{\beta_2\,(\nu_2^2-1)}{\Gamma^{1/2}}\right)\right\}\left(\eta-\frac{\xi}{\nu_2}\right)H\left(\eta-\frac{\xi}{\nu_2}\right)\Big]\,e^{-\beta_2\xi}$$

$$\sigma_{\xi\xi}(\xi,\eta)\simeq-\frac{K_0}{2}\left[-\frac{\nu_1\,\tau_0'}{\Gamma^{1/2}}\,\delta\left(\eta-\frac{\xi}{\nu_1}\right)+\left\{\frac{\nu_1^2\,\beta_1\,\tau_0'}{\Gamma^{1/2}}+\frac{\nu_1\,\tau_0'\,(K_1\,K_2-2)}{\Gamma^{3/2}}\right.\right.$$

$$\left.-\frac{\nu_1}{\Gamma^{1/2}}\right\}H\left(\eta-\frac{\xi}{\nu_1}\right)+\left\{\frac{\tau_0'\,K_1^2}{2\,\nu_1\,\Gamma^{3/2}}+\frac{(K_1K_2-2)}{\nu_1\Gamma^{3/2}}+\frac{\beta_1\,\tau_0'\,(K_1K_2-2)}{\Gamma^{3/2}}\right.$$

$$-\frac{2\,\beta_1\,\tau_0'\,(K_1K_2-2)}{\Gamma^{3/2}}-\frac{\beta_1\,\tau_0'\,(\nu_1^2-1)\,(K_1\,K_2-2)}{\Gamma^{3/2}}+\frac{(\nu_1^2-1)\,(K_1\,K_2-2)}{\nu_1\,\Gamma^{3/2}}$$

$$-\frac{\beta_1}{\Gamma^{1/2}}-\frac{\tau_0'\,\varphi_1}{\Gamma^{1/2}}+\frac{(\beta_1^2\,\nu_1+2\,\varphi_1)\,\tau_0'}{\Gamma^{1/2}}-\frac{2\,\beta_1^2\,\nu_1\,\tau_0'}{\Gamma^{1/2}}$$

$$\left.-\frac{\tau_0'\,(\nu_1^2-1)\,(\beta_1^2\,\nu_1-\varphi_1)}{\Gamma^{1/2}}+\left(\frac{2\,\beta_1}{\Gamma^{1/2}}+\frac{\beta_1\,(\nu_1^2-1)}{\Gamma^{1/2}}\right)\right\}$$

$$\left(\eta-\frac{\xi}{\nu_1}\right)H\left(\eta-\frac{\xi}{\nu_1}\right)\Big]\,e^{-\beta_1\xi}$$

(equation continued on p. 1350)

$$+\frac{K_0}{2}\left[-\frac{v_2\,\tau_0'}{\Gamma^{1/2}}\,\delta\left(\eta-\frac{\xi}{v_2}\right)+\left\{\frac{v_2^2\,\beta_2\,\tau_0'}{\Gamma^{1/2}}+\frac{v_2\,\tau_0'\,(K_1\,K_2-2)}{\Gamma^{3/2}}\right.\right.$$

$$\left.-\frac{v_2}{\Gamma^{1/2}}\right\}H\left(\eta-\frac{\xi}{v_2}\right)+\left\{\frac{\tau_0'\,K_1^2}{2\,v_2\,\Gamma^{3/2}}+\frac{(K_1\,K_2-2)}{v_2\,\Gamma^{3/2}}+\frac{\beta_2\,\tau_0'\,(K_1K_2-2)}{\Gamma^{3/2}}\right.$$

$$-\frac{2\beta_2\,\tau_0'\,(K_1\,K_2-2)}{\Gamma^{3/2}}-\frac{\beta_2\,\tau_0'\,(v_2^2-1)\,(K_1\,K_2-1)}{\Gamma^{3/2}}+\frac{(v_2^2-2)\,(K_1\,K_2-2)}{v_2\,\Gamma^{3/2}}$$

$$-\frac{\beta_2}{\Gamma^{1/2}}-\frac{\tau_0'\,\varphi_2}{\Gamma^{1/2}}+\frac{(\beta_2^2\,v_2+2\,\varphi_2)\,\tau_0'}{\Gamma^{1/2}}-\frac{2\,\beta_2^2\,v_2\,\tau_0'}{\Gamma^{1/2}}$$

$$\left.-\frac{\tau_0'\,(v_2^2-1)\,(\beta_2^2\,v_2-\varphi_2)}{\Gamma^{1/2}}+\left(\frac{2\,\beta_2}{\Gamma^{1/2}}+\frac{\beta_2\,(v_2^2-1)}{\Gamma^{1/2}}\right)\right\}$$

$$\left.\times\left(\eta-\frac{\xi}{v_2}\right)H\left(\eta-\frac{\xi}{v_2}\right)\right]e^{-\beta_2\xi}$$

$$e\,(\xi,\eta)\simeq-\frac{\beta\,T^*\,K_0}{2\,(\lambda+2\,\mu)}\left[-\frac{\tau_0'}{v_1\,\Gamma^{1/2}}\,\delta\left(\eta-\frac{\xi}{v_1}\right)+\left\{\frac{\tau_0'\,(K_1\,K_2-2)}{v_1\,\Gamma^{3/2}}\right.\right.$$

$$\left.-\frac{1}{v_1\,\Gamma^{1/2}}-\frac{\beta_1\,\tau_0'}{\Gamma^{1/2}}\right\}H\left(\eta-\frac{\xi}{v_1}\right)+\left\{\frac{\tau_0'\,K_1^2}{2\,v_1\,\Gamma^{3/2}}\right.$$

$$\left.+\frac{(K_1K_2-2)}{v_1\,\Gamma^{3/2}}+\frac{\beta_1\,\tau_0'\,(K_1K_2-2)}{\Gamma^{3/2}}-\frac{\beta_1}{\Gamma^{1/2}}-\frac{\tau_0'\,\varphi_1}{\Gamma^{1/2}}\right\}$$

$$\left.\times\left(\eta-\frac{\xi}{v_1}\right)H\left(\eta-\frac{\xi}{v_1}\right)\right]e^{-\beta_1\xi}$$

$$+\frac{\beta\,T^*\,K_0}{2\,(\lambda+2\,\mu)}\left[-\frac{\tau_0'}{v_2\,\Gamma^{1/2}}\,\delta\left(\eta-\frac{\xi}{v_2}\right)+\left\{\frac{\tau_0'\,(K_1K_2-2)}{v_2\,\Gamma^{3/2}}\right.\right.$$

$$\left.-\frac{1}{v_2\,\Gamma^{1/2}}-\frac{\beta_2\,\tau_0'}{\Gamma^{1/2}}\right\}H\left(\eta-\frac{\xi}{v_2}\right)+\left\{\frac{\tau_0'\,K_1^2}{2\,v_2\,\Gamma^{3/2}}+\frac{(K_1\,K_2-2)}{v_2\,\Gamma^{3/2}}\right.$$

$$\left.\left.+\frac{\beta_2\,\tau_0'\,(K_1K_2-2)}{\Gamma^{3/2}}-\frac{\beta_2}{\Gamma^{1/2}}-\frac{\tau_0'\,\varphi_2}{\Gamma^{1/2}}\right\}\left(\eta-\frac{\xi}{v_2}\right)H\left(\eta-\frac{\xi}{v_2}\right)\right]e^{-\beta_2\xi}.$$

Discussion

The above solutions, valid for relatively short times, reveal that they consist of two waves—one travelling with speed v_1 (dilatational wave) and the other with speed v_2 (thermal wave). The terms containing $H\left(\eta-\frac{\xi}{v_1}\right)$ represent the contribution of the elastic wave in the vicinity of its wave front $(\xi=v_1\,\eta)$ and the terms with $H\left(\eta-\frac{\xi}{v_2}\right)$ represent contribution of the thermal wave in the vicinity of its wave front $\xi=v_2\,\eta$. We readily observe that deformation, temperature, stress and strain all experience discontinuities at each of the wave fronts and the magnitudes of these discontinuities are given by

$$(U^+-U^-)_{\xi=\nu_1\eta} = \frac{K_0\,\tau_0'}{2\Gamma^{1/2}}\, e^{-\beta_1\xi}$$

$$(U^+-U^-)_{\xi=\nu_2\eta} = -\frac{K_0\,\tau_0'}{2\,\Gamma^{1/2}}\, e^{-\beta_2\xi}$$

$$(\theta^+-\theta^-)_{\xi=\nu_1\eta} = -\frac{K_0}{2}\left[\frac{2\,\tau_0'\,\beta_1}{\Gamma^{1/2}} + \frac{\tau_0'\,(\nu_1^2-1)\,(K_1\,K_2-2)}{\nu_1\,\Gamma^{3/2}} + \frac{\beta_1\,\tau_0'\,(\nu_1^2-1)}{\Gamma^{1/2}} - \frac{(\nu_1^2-1)}{\nu_1\,\Gamma^{1/2}}\right] e^{-\beta_1\xi}$$

$$(\theta^+-\theta^-)_{\xi=\nu_2\eta} = \frac{K_0}{2}\left[\frac{2\,\beta_2\tau_0'}{\Gamma^{1/2}} + \frac{\tau_0'\,(\nu_2^2-1)\,(K_1\,K_2-2)}{\nu_2\,\Gamma^{3/2}} + \frac{\beta_2\,\tau_0'\,(\nu_2^2-1)}{\nu_2\,\Gamma^{1/2}} - \frac{(\nu_2^2-1)}{\nu_2\,\Gamma^{1/2}}\right] e^{-\beta_2\xi}$$

$$(\sigma_{\xi\xi}^+ - \sigma_{\xi\xi}^-)_{\xi=\nu_1\eta} = \frac{K_0}{2}\left[\frac{\nu_1^2\,\beta_1\,\tau_0'}{\Gamma^{1/2}} + \frac{\nu_1\,\tau_0'\,(K_1\,K_2-2)}{\Gamma^{3/2}} - \frac{\nu_1}{\Gamma^{1/2}}\right] e^{-\beta_1\xi}$$

$$(\sigma_{\xi\xi}^+ - \sigma_{\xi\xi}^-)_{\xi=\nu_2\eta} = -\frac{K_0}{2}\left[\frac{\nu_2^2\,\beta_2\,\tau_0'}{\Gamma^{1/2}} + \frac{\nu_2\,\tau_0'\,(K_1\,K_2-2)}{\Gamma^{3/2}} - \frac{\nu_2}{\Gamma^{1/2}}\right] e^{-\beta_2\xi}$$

$$(e^+ - e^-)_{\xi=\nu_1\eta} = \frac{K_0\,\beta T^*}{2\,(\lambda+2\,\mu)}\left[\frac{\tau_0'\,(K_1\,K_2-2)}{\nu_1\,\Gamma^{3/2}} - \frac{1}{\nu_1\,\Gamma^{1/2}} - \frac{\beta_1\,\tau_0'}{\Gamma^{1/2}}\right] e^{-\beta_1\xi}$$

$$(e^+ - e^-)_{\xi=\nu_2\eta} = -\frac{K_0\,\beta\,T^*}{2\,(\lambda+2\mu)}\left[\frac{\tau_0'\,(K_1K_2-2)}{\nu_2\,\Gamma^{3/2}} - \frac{1}{\nu_2\,\Gamma^{1/2}} - \frac{\beta_2\,\tau_0'}{\Gamma^{1/2}}\right] e^{-\beta_2\xi}$$

We also note that $[\sigma_{\xi\xi}]_{\xi=\nu_{1,2}\eta} = \left[\frac{dV}{d\xi}\right]_{\xi=\nu_{1,2}\eta} - [\theta]_{\xi=\nu_{1,2}\eta}$

where $e = \frac{\beta\,T^*}{(\lambda+2\mu)}\frac{dV}{d\xi}$ and $[f] = (f^+ - f^-)$

= jump in f across a wave front.

We consider two particular cases :

Case (i) $\tau_0' = 0$: In this case Lord-Shulman theory reduces to the classical coupled theory. From (13) and (14), $\Gamma = 1$, $K_1 = 1+\epsilon$, $K_2 = 1$, $\nu_1 \rightarrow 1$, $\nu_2 \rightarrow \infty$, $\beta_1 = \epsilon/2$, $\beta_2 \rightarrow \infty$. We see that the deformation and the thermal field are both continuous at the elastic wave front, which agrees with Paria's (1968) result. The magnitudes of the discontinuities of the strain and stress at this wave front are given by

$$(\sigma_{\xi\xi}^+ - \sigma_{\xi\xi}^-)_{\xi=\eta} = -\frac{K_0}{2}\, e^{-(\epsilon/2)\xi}$$

(*equation continued on p.* 1352)

$$(e^+ - e^-)_{\xi=\eta} = -\frac{K_0 \beta T^*}{2(\lambda+2\mu)} e^{-(\epsilon/2)\xi}.$$

These discontinuities are not uniform but decay exponentially with distance at the elastic wave front. It is also verified that

$$[\sigma_{\xi\xi}]_{\xi=\eta} = \frac{(\lambda+2\mu)}{\beta T^*} [e]_{\xi=\eta} \text{ since } [\theta]_{\xi=\eta} = 0.$$

Case (ii) $\epsilon = 0$ (for weak thermoelastic coupling) : Then (14) yield $\nu_1 = 1$ and

$$\nu_2 = \frac{1}{\sqrt{\tau_0'}}$$

$$\beta_1 = 0, \beta_2 = \frac{1}{2\sqrt{\tau_0'}} = \frac{\nu_2}{2}, \sqrt{\Gamma} = 1 - \tau_0'$$

We see that the thermal field is continuous at the elastic wave front. The magnitudes of the discontinuity of the deformation, strain and stress at the elastic wave front are uniform and are given by

$$(e^+ - e^-)_{\xi=\eta} = -\frac{K_0 \beta T^*}{2(\lambda+2\mu)} \frac{1}{(1-\tau_0')^2}$$

$$(\sigma_{\xi\xi}^+ - \sigma_{\xi\xi}^-)_{\xi=\eta} = -\frac{K_0}{2} \frac{1}{(1-\tau_0')^2}$$

$$(U^+ - U^-)_{\xi=\eta} = \frac{K_0 \tau_0}{2(1-\tau_0')}.$$

It is verfied that $[\sigma_{\xi\xi}]_{\xi=\eta} = \frac{(\lambda+2\mu)}{\beta T^*} [e]_{\xi=\eta}$ since $[\theta] = 0$.

At the thermal wave front, these are not uniform and are given by

$$(e^+ - e^-)_{\xi = \frac{\eta}{\sqrt{\tau_0'}}} = \frac{K_0 \beta T^* \sqrt{\tau_0'}\,(3-\tau_0')}{4(\lambda+2\mu)(1-\tau_0')^2} e^{-\frac{\xi}{2\sqrt{\tau_0'}}}$$

$$(\sigma_{\xi\xi}^+ - \sigma_{\xi\xi}^-)_{\xi = \frac{\eta}{\sqrt{\tau_0'}}} = \frac{K_0(1+\tau_0')}{4\sqrt{\tau_0'}\,(1-\tau_0')^2} e^{-\frac{\xi}{2\sqrt{\tau_0'}}}$$

$$(U^+ - U^-)_{\xi = \frac{\eta}{\sqrt{\tau_0'}}} = -\frac{K_0 \tau_0'}{2(1-\tau_0')} e^{-\frac{\xi}{2\sqrt{\tau_0'}}}$$

$$(\theta^+ - \theta^-)_{\xi = \frac{\eta}{\sqrt{\tau_0'}}} = -\frac{K_0}{4\sqrt{\tau_0'}} e^{-\frac{\xi}{2\sqrt{\tau_0'}}}$$

It is also verified that $[\sigma_{\xi\xi}]_{\xi = \frac{\eta}{\sqrt{\tau_0'}}} = \left[\frac{dU}{d\xi}\right]_{\xi = \frac{\eta}{\sqrt{\tau_0'}}} - [\theta]_{\xi = \frac{\eta}{\sqrt{\tau_0'}}}$

where $e = \frac{\beta T^*}{(\lambda+2\mu)} \frac{dU}{d\xi}$ where as before the symbol $[f]$ indicates the jump in the function f across the corresponding wave front.

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