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HEAT-SOURCE PROBLEM OF THERMOELASTICITY IN A NON-SIMPLE ELASTIC MEDIUM

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This paper is an attempt to determine thermal stresses, in two different cases, developed due to two different time dependent heat sources, viz. (i) Ramp-type of heat-source and (ii) Point impulsive heat-source in a thin infinite rod made up of non-simple elastic material. Laplace transform method has been used to solve the problems. Finally, temperature fields, displacements and stresses in two different cases have been found out in terms of double infinite series which are found to be convergent. Numerical results are also obtained.

1. INTRODUCTION

Problems of determination of thermal stresses in a thin rod of simple material under various mechanical and thermal boundary conditions have been considered earlier by many researchers, viz. Sneddon (1958), Roy Chowdhuri (1970), Mahalanabis (1966), Nariboli and Nyayadish (1963) and many others. But none of the previous investigators solved such problems for a non-simple material. Chakraborty (1972) solved a problem of one-dimensional thermo-elastic wave propagation due to an instantaneous heat-source in a non-simple elastic material. The present author also has made an attempt, in this paper, to solve problems of thermal stresses in a non-simple elastic medium.

In classification of real materials into simple and non-simple materials Chen and Gurtin (1968) proposed a theory of non-simple materials for which thermodynamic and conductive temperatures are not identical unlike simple materials for which they are identical. This theory was further extended to deformable bodies by Chen *et al.* (1969). They have shown that the equation of heat conduction for such materials contains an additional term involving the time derivative of the Laplacian of the conductive temperature; the equation of motion also includes terms involving the space derivatives of the Laplacian of the conductive temperature. Considering isotropy and linearity, for such materials, they have shown that the two temperatures are related by

$$\phi = T - a\nabla^2 T, \ a \geqslant 0$$

where ϕ is the thermodynamic temperature, T is the conductive temperature and a is the temperature discrepancy factor.

In this paper the author has considered two problems in a thin finite rod made up of non-simple elastic material when (i) it is subjected to a time dependent heat-source continuously distributed over a finite portion of the rod varying with time according to ramp-type function and (ii) when it is subjected to a point-impulsive heat-source. In both the cases, one end of the rod is fixed, the other end being stress-free and both ends are kept at zero temperature. The aim of this paper is to determine the thermal stresses, in these cases, in non-simple medium and to compare the results with those of simple medium. Laplace transform technique has been used as mathematical tool to solve the problems. Finally the solutions, have been obtained in the terms of double infinite series which are found to be convergent. It is also observed that when temperature discrepancy factor $a \to 0$, the results agree with those of simple elastic medium, vide, Roy Chowdhuri (1972). Finally numerical results for temperature and displacement in the second case have been shown graphically for different values of time.

2. Fundamental Equations and Boundary Conditions

We consider a thin elastic finite rod of length l of non-simple elastic material occupying the region $0 \leqslant x \leqslant l$. The rod is heated over the portion $x_0 < x < x_1$. The ends of the rod are maintained at zero temperature over the absolute temperature, T_0 in a state of zero stress and strain.

Now using the dimensionless quantities

$$\xi = x/l,\ \tau = K_1 t/l^2,\ \theta = T/T_0,\ U = u/l$$

the modified equations of heat conduction for the Ramp-type and impulsive heat-sources are respectively obtained as

$$\left(\frac{\partial^2}{\partial \xi^2} + b\,\frac{\partial^2}{\partial \xi^2}\frac{\partial}{\partial \tau} - \frac{\partial}{\partial \tau} \right)\theta$$

$$= \begin{cases} -q_0 \Phi(\tau)\{H(\xi - \xi_0) - H(\xi - \xi_1)\},\ \xi_0 < \xi < \xi_1 & \quad ...(1) \\ -q_0 \delta(\tau)\,\delta(\xi - \xi'),\ \xi_0 < \xi' < \xi_1 & \quad ...(2) \end{cases}$$

where

$$q_0 = Q_0 l^2/k_1 T_0,\ \xi_0 = x_0/l,\ \xi_1 = x_1/l,\ b = a/l^2$$

with

$$\Phi(\tau) = \begin{cases} (\phi_0/T_0)\,\tau,\ 0 \leqslant \tau \leqslant \tau_0 \\ \phi_0,\ \tau \geqslant \tau_0 \end{cases} \quad ...(3)$$

and the modified expressions for the normal stress and the equation of motion in the absence of the body force are respectively obtained as

$$\frac{\sigma(\xi,\tau)}{E} = \frac{\partial U}{\partial \xi} - \alpha T_0\left(1 - b\frac{\partial^2}{\partial \xi^2}\right)\theta(\xi,\tau) \quad ...(4)$$

and

$$\frac{\partial^2 U}{\partial \xi^2} = \left(K_1^2/v_1^2 l^2 \right)\left(\frac{\partial^2 U}{\partial \tau^2} \right) + \alpha T_0 \frac{\partial \theta}{\partial \xi} - b\alpha T_0 \frac{\partial^3 \theta}{\partial \xi^3} \quad ...(5)$$

where Q_0 and ϕ_0 are constants and other have their usual representations, K_1 is the thermal diffusivity, $v_1 = (E/\rho)^{1/2}$.

Now eqns. (1), (2) and (5) are to be solved subject to the following initial and boundary conditions:

$$\left.\begin{array}{l}\text{(i)}\ \ \theta(\xi, 0) = U(\xi, 0) = U,_{\tau}(\xi, 0) = 0 \\ \text{(ii)}\ \ \sigma(\xi, 0) = \sigma,_{\tau}(\xi, 0) = 0 \\ \text{(iii)}\ \ \theta(0, \tau) = \theta(l, \tau) = 0 \\ \text{(iv)}\ \ U(0, \tau) = U,\ (l, \tau) = 0.\end{array}\right\} \quad \ldots(6)$$

3. Solution of the Problem

Applying Laplace transform to equations (1)—(2) and (5)—(6) respectively, we obtain

$$\frac{\partial^2\bar{\theta}}{\partial\xi^2} - \frac{p}{1+bp}\bar{}$$

$$= -\frac{q_0\phi_0}{\tau_0 p^2(1+bp)}\left(1 - e^{-\tau_0 p}\right)\left\{H(\xi-\xi_0)-H(\xi-\xi_1)\right\} \quad \ldots(7)$$

$$= -\frac{q_0\delta(\xi-\xi')}{(1+bp)} \quad \ldots(8)$$

and

$$\frac{\partial^2\bar{U}}{\partial\xi^2} - \frac{K_1^2p^2}{v_1^2l^2}\bar{U} = \alpha T_0\frac{\partial}{\partial\xi}\left(1 - b\frac{\partial^2}{\partial\xi^2}\right)\bar{T} \quad \ldots(9)$$

with

$$\left.\begin{array}{l}\text{(i)}\ \ \bar{\theta}(0, p) = \bar{\theta}(l, p) = 0 \\ \text{(ii)}\ \ \bar{U}(0, p) = \bar{U},_{\xi}(l, p) = 0\end{array}\right\} \quad \ldots(10)$$

Case I : *Ramp-type Heat-source*

As a solution of (7) satisfying conditions (10) we take

$$\bar{\theta}(\xi, p) = \sum_{n=1}^{\infty} C_n^{(1)} \sin(n\pi\xi) \quad \ldots(11)$$

where $C_n^{(1)}$ is independent of ξ.

We assume

$$H(\xi-\xi_0) - H(\xi-\xi_1) = \sum_{n=1}^{\infty} C_n^{(2)} \sin(n\pi\xi) \quad \ldots(12)$$

where $C_n^{(2)}$ is independent of ξ and then $C_n^{(2)}$ is obtained as

$$C_n^{(2)} = \frac{2}{n\pi}[\cos(n\pi\xi_0) - \cos(n\pi\xi_1)]. \quad \ldots(13)$$

The equation (7) using (11) and (12) gives a relation between $C_n^{(1)}$ and $C_n^{(2)}$ as

$$C_n^{(1)} = \frac{q_0\phi_0}{\tau_0 p^2}\left(1 - e^{-\tau_0 p}\right)\frac{1}{n^2\pi^2 + (1+n^2\pi^2 b)p}\,C_n^{(2)}. \quad \ldots(14)$$

Equation (11) with the aid of (13)–(14), yields

$$\bar{\theta}(\xi, p) = \frac{2q_0\phi_0}{\tau_0}\sum_{n=1}^{\infty}\frac{\sin(n\pi\xi)}{n^3\pi^3}\left\{\cos(n\pi\xi_0) - \cos(n\pi\xi_1)\right\}$$

$$\times\left(1 - e^{-\tau_0 p}\right)\left\{\frac{1}{p^2} - \frac{1+n^2\pi^2 b}{n^2\pi^2}\left(\frac{1}{p} - \frac{1}{n^2\pi^2+(1+n^2\pi^2 b)p}\right)\right\}. \quad \ldots(15)$$

Inverting (15) by Laplace inversion theorem and using the standard results, vide, Carslaw and Jaeger (1963) the temperature field is finally obtained as

$$\theta(\xi, \tau) = \frac{2q_0\phi_0}{\tau_0}\sum_{n=1}^{\infty}\frac{\sin(n\pi\xi)}{n\pi}\left\{\cos(n\pi\xi_0) - \cos(n\pi\xi_1)\right\}$$

$$\times\left\{f_n(\tau) - f_n(\tau - \tau_0)H(\tau - \tau_0)\right\} \quad \ldots(16)$$

where

$$f_n(\tau) = \frac{1}{n^2\pi^2}\left\{\tau - \frac{1+n^2\pi^2 b}{n^2\pi^2}\left(1 - \frac{1}{1+n^2\pi^2 b}\,e^{-\frac{n^2\pi^2}{1+n^2\pi^2 b}\tau}\right)\right\}.$$

Next to solve the elastic part of the problem the equation (9) using (15) can be written as

$$\frac{\partial^2\bar{U}}{\partial\xi^2} - \frac{K_1^2 p^2}{v_1^2 l^2}\bar{U} = \frac{2q_0\phi_0\alpha T_0}{\tau_0}\sum_{n=1}^{\infty}(1+n^2\pi^2 b)\cos(n\pi\xi)$$

$$\times\left\{\cos(n\pi\xi_0) - \cos(n\pi\xi_1)\right\}\frac{1-e^{-\tau_0 p}}{p^2\{n^2\pi^2+(1+n^2\pi^2 b)p\}}. \quad \ldots(17)$$

The solution of equation (17) is obtained as

$$\bar{U}(\xi, p) = B_1 e^{(K_1 p/v_1 l)\xi} + B_2 e^{-(K_1 p/v_1 l)\xi}$$

$$-\frac{2q_0\phi_0\alpha T_0}{\tau_0}\frac{v_1^2 l^2}{K_1^2}\sum_{n=1}^{\infty}(1 + n^2\pi^2 b)\cos(n\pi\xi)$$

$$\times\frac{1}{p^2+\frac{n^2\pi^2 v_1^2 l^2}{K_1^2}}\left\{\cos(n\pi\xi_0) - \cos(n\pi\xi_1)\right\}\frac{1-e^{-\tau_0 p}}{p^2\{n^2\pi^2+(1+n^2\pi^2 b)p\}} \quad \ldots(18)$$

where the arbitrary constants B_1 and B_2 are obtained from the boundary conditions (10) as $B_2 = B_1\, e^{2K_1 p/v_1 l}$, where

$$B_1 = \frac{2q_0\phi_0\alpha T_0}{\tau_0}\frac{v_1^2 l^2}{K_1^2}\sum_{n=1}^{\infty}(1+n^2\pi^2 b)\,\{\cos(n\pi\xi_0)-\cos(n\pi\xi_1)\}$$

$$\times \frac{e^{-K_1 p/v_1 l} \qquad 1-e^{-\tau_0 p}}{\cosh(K_1 p/v_1 l)\, p^2\left\{n^2\pi^2+(1+n^2\pi^2 b)p\right\}\left\{p^2+\dfrac{n^2\pi^2 v_1^2 l^2}{K_1^2}\right\}}.$$

Inverting (18) by Laplace's Inversion Theorem, we obtain

$$V(\xi,\tau)= \frac{2q_0\phi_0\alpha T_0}{\tau_0}\frac{v_1^2 l^2}{K_1^2}\sum_{n=1}^{\infty}(1+n^2\pi^2 b)\{\cos(n\pi\xi_0)$$

$$-\cos(n\pi\xi_1)\}\{\eta_n(\xi,\tau)-\eta_n(\xi,\tau-\tau_0)H(\tau-\tau_0)\}$$

$$-\frac{2q_0\phi_0\alpha T_0}{\tau_0}\frac{v_1^2 l^2}{K_1^2}\sum_{n=1}^{\infty}(1+n^2\pi^2 b)\cos(n\pi\xi)$$

$$\times\left\{\cos(n\pi\xi_0)-\cos(n\pi\xi_1)\right\}\frac{K_1}{n^2\pi^2 v_1^2 l^2}$$

$$\times\left\{f_n(\tau)-f_n(\tau-\tau_0)H(\tau-\tau_0)-\psi_n(\tau)+\psi_n(\tau-\tau_0)H(\tau-\tau_0)\right\} \qquad \ldots(19)$$

where

$$L^{-1}\left[\frac{1-e^{-\tau_0 p}}{p^2\left\{n^2\pi^2+(1+n^2\pi^2 b)p\right\}\left(p^2+\dfrac{n^2\pi^2 v_1^2 l^2}{K_1^2}\right)}\right]$$

$$=\frac{K_1^2}{n^2\pi^2 v_1^2 l^2}\Big[f_n(\tau)\quad f_n(\tau-\tau_0)H(\tau-\tau_0)-\psi_n(\tau)+\psi_n(\tau-\tau_0)H(\tau-\tau_0)\Big]$$

$$\psi_n(\tau)=L^{-1}\left[\frac{1}{\left\{n^2\pi^2+(1+n^2\pi^2 b)p\right\}\left\{p^2+\dfrac{n^2\pi^2 v_1^2 l^2}{K_1^2}\right\}}\right]$$

$$=\frac{1}{1+n^2\pi^2 b}\,\frac{1}{\dfrac{n^4\pi^4}{(1+n^2\pi^4 b)^2}+\dfrac{n^2\pi^2 v_1^2 l^2}{K_1^2}}\left[\gamma\sin\left(\frac{n\pi v_1 l}{K_1}\tau-\beta\right)\right.$$

$$\left.+\,e^{-\frac{n^2\pi^2}{1+n^2\pi^2 b}\tau}\right]$$

$$\gamma\cos\beta=\frac{n\pi K_1}{v_1 l(1+n^2\pi^2 b)}\,,\quad \gamma\sin\beta=1$$

$$\eta_n(\xi,\tau)=L^{-1}\left[\frac{\cosh\{(K_1/v_1 l)(1-\xi)p\}}{p^2\cosh(K_1 p/v_1 l)\{n^2\pi^2+(1+n^2\pi^2 b)p\}\{p^2+n^2\pi^2 v_1^2 l^2/K_1^2\}}\right]$$

(*equation continued on p.* 1389)

$$= \frac{K_1\gamma}{v_1 l(1+n^2\pi^2 b)n\pi\left\{\frac{n^4\pi^4}{(1+n^2\pi^2 b)^2}+\frac{n^2\pi^2 v_1^2 l^2}{K_1^2}\right\}}\left[\tau\cos\beta\right.$$

$$\left. - \frac{K_1}{n\pi v_1 l}\left\{\sin\left(\frac{n\pi v_1 l}{K_1}\tau-\beta\right)+\sin\beta\right\}\right]$$

$$+ \frac{1}{n^2\pi^2\left\{\frac{n^4\pi^4}{(1+n^2\pi^2 b)^2}+\frac{n^2\pi^2 v_1^2 l^2}{K_1^2}\right\}(1+n^2\pi^2 b)}$$

$$\times\left\{\tau-\frac{1+n^2\pi^2 b}{n^2\pi^2}\left(1-e^{-\frac{n^2\pi^2}{1+n^2\pi^2 b}\tau}\right)\right\}$$

$$-\frac{8}{n^2\pi^4}\sum_{m=1}^{\infty}\frac{\sin\{[(2m-1)/2]\pi\xi\}}{(2m-1)^2}\frac{K_1}{v_1 l(1+n^2\pi^2 b)\left\{\frac{n^2\pi^2}{(1+n^2\pi^2 b)^2}+\frac{v_1^2 l^2}{K_1^2}\right\}}$$

$$\times\frac{1}{\left\{\frac{n^4\pi^4}{(1+n^2\pi^2 b)^2}+\frac{(2m-1)^2\pi^2 v_1^2 l^2}{4K_1^2}\right\}^{1/2}}\times\left\{\sin\left(\frac{(2m-1)\pi v_1 l}{2K_1}\tau\right.\right.$$

$$\left.-\tan^{-1}\left(\frac{(2m-1)v_1 l(1+n^2\pi^2 b)}{2K_1 n^2\pi}\right)\right)+e^{-\frac{n^2\pi^2}{1+n^2\pi^2 b}\tau}$$

$$\left.\times\sin\left(\tan^{-1}\frac{(2m-1)v_1 l(1+n^2\pi^2 b)}{2K_1 n^2\pi}\right)\right\}-\frac{4\gamma K_1}{v_1 l\pi^3}\sum_{m=1}^{\infty}\frac{\sin\{[(2m-1)/2]\pi\zeta\}}{(2m-1)^2}$$

$$\times\frac{1}{(1+n^2\pi^2 b)\left\{\frac{n^4\pi^4}{(1+n^2\pi^2 b)^2}+\frac{n^2\pi^2 v_1^2 l^2}{K_1^2}\right\}}$$

$$\times\left[\left\{\frac{\sin[(n\pi v_1 l/K_1)\tau-\beta]+\sin[\beta+\{(2m-1)\pi v_1 l/2K_1\}\tau]}{n+(2m-1)/2}\right\}\right.$$

$$\left.-\left\{\frac{\sin[(n\pi v_1 l/K_1)\tau-\beta]+\sin[\beta-\{(2m-1)\pi v_1 l/2K_1\}\tau]}{n-(2m-1)/2}\right\}\right].$$

Stress $\sigma(x, \tau)$ is now obtained from (4) using (16) and (19) and are given by

$$\frac{\sigma(\xi_1\tau)}{E}=\frac{2q_0\phi_0\alpha T_0}{\tau_0}\frac{v_1^2 l^2}{K_1^2}\sum_{n=1}^{\infty}(1+n^2\pi^2 b)\{\cos(n\pi\xi_0)-\cos(n\pi\xi_1)\}$$

$$\times\left\{\frac{\partial\eta_n(\xi_1\tau)}{\partial\xi}-\frac{\partial\eta_n(\xi_1\tau-\tau_0)}{\partial\xi}H(\tau-\tau_0)\right\}$$

$$-\frac{2q_0 f_0\alpha T_0}{\pi\tau_0}\sum_{n=1}^{\infty}(1+n^2\pi^2 b)\{\cos(n\pi\xi_0)-\cos(n\pi\xi_1)\}$$

$$\times\frac{\sin(n\pi\xi)}{n}\left\{\psi_n(\tau)-\psi_n(\tau-\tau_0)H(\tau-\tau_0)\right\}. \qquad \ldots(20)$$

Thus the temperature field, displacement and normal stress are obtained and are respectively given by the equations (16), (19) and (20). If the temperature discrepancy factor $a \to 0$ i.e. $b \to 0$, the corresponding expressions for simple elastic medium are obtained and agree with those of Roy Chowdhuri (1972).

Case II: Point Impulsive Heat Source

Proceeding, as in case (I), the solutions in this case are also obtained from eqns. (8)–(9) with the aid of (10) as

$$\theta(\xi, \tau) = 2q_0 \sum_{n=1}^{\infty} \frac{\sin(n\pi\xi')\sin(n\pi\xi)}{(1+n^2\pi^2 b)} e^{-\frac{n^2\pi^2}{1+n^2\pi^2 b}\tau} \quad \ldots(21)$$

$$U(\xi, \tau) = 2\alpha q_0 T_0 \frac{v_1^2 l^2}{K_1^2} \sum_{n=1}^{\infty} (n\pi + n^3\pi^3 b)\sin(n\pi\xi')\chi_n(\xi,\tau)$$

$$- 2\alpha q_0 T_0 \frac{v_1^2 l^2}{K_1^2} \sum_{n=1}^{\infty} (n\pi + n^3\pi^3 b)\sin(n\pi\xi')\cos(n\pi\xi)\psi_n(\tau) \quad \ldots(22)$$

where

$$\chi_n(\xi, \tau) = -\frac{2}{n^2\pi^2} \sum_{m=1}^{\infty} \sin\{[(2m-1)/2]\pi\xi\} \frac{v_1 l}{K_1(1+n^2\pi^2 b)}$$

$$\times \frac{1}{\left\{\frac{n^2\pi^2}{(1+n^2\pi^2 b)^2} + \frac{v_1^2 l^2}{K_1^2}\right\}\left[\frac{n^4\pi^4}{(1+n^2\pi^2 b)^2} + \frac{(2m-1)^2\pi^2 v_1^2 l^2}{4K_1^2}\right]^{1/2}}$$

$$\times \left\{\sin\left(\frac{(2m-1)\pi v_1 l}{2K_1}\tau - \tan^{-1}\frac{(2m-1)v_1 l(1+n^2\pi^2 b)}{2K_1 n^2\pi}\right)\right.$$

$$\left. + e^{-\frac{n^2\pi^2}{1+n^2\pi^2 b}\tau} \sin\left[\tan^{-1}\frac{(2m-1)v_1 l(1+n^2\pi^2 b)}{2K_1 n^2\pi}\right]\right\}$$

$$- \frac{v_1 l}{K_1} \sum_{m=1}^{\infty} \frac{\sin\{[(2m-1)/2]\pi\xi\}}{\left(\frac{n^4\pi^4}{(1+n^2\pi^2 b)^2} + \frac{n^2\pi^2 v_1^2 l^2}{K_1^2}\right)^{1/2}}$$

$$\times \left[\frac{\left\{\sin\left[\left(\frac{n\pi v_1 l}{K_1}\right)\tau - \beta\right] + \sin\left[\beta + \left\{(2m-1)\frac{\pi v_1 l}{2K_1}\right\}\tau\right]\right\}}{n+(2m-1)/2}\right.$$

$$\left. - \frac{\left\{\sin\left[\left(\frac{n\pi v_1 l}{K_1}\right)\tau - \beta\right] + \sin\left[\beta - \left\{(2m-1)\frac{\pi v_1 l}{2K_1}\right\}\tau\right]\right\}}{n-(2m-1)/2}\right]$$

and the normal stress $\sigma(x, \tau)$ is obtained from (4) using (22) and (23) as

$$\frac{\sigma(\xi,\tau)}{E} = 2\alpha q_0 T_0 \frac{v_1 l^2}{K_1^2} \sum_{n=1}^{\infty} \Big\{ (n\pi + n^3\pi^3 b) \sin(n\pi\xi') \times \{\sin(n\pi\xi)\psi_n(\tau)$$

$$+ \frac{\partial x_n(\xi,\tau)}{\partial \xi} \Big\} - 2\alpha T_0 q_0 \sum_{n=1}^{\infty} \sin(n\pi\xi')\sin(n\pi\xi) e^{-\frac{n^2\pi^2}{1+n^2\pi^2 b}\tau}. \quad ...(23)$$

The corresponding expressions for simple elastic case are obtained when the temperature discrepancy factor $b \to 0$ and these results also agree with those of Roy Chowdhuri (1972).

The final expressions for the temperature, the displacement and the normal stress, for both the cases are obtained in terms of double infinite series which are found to be convergent.

Numerical Calculations for Point-Impulsive Heat Source

For numerical calculations we consider only the first two terms of infinite series for the temperature field and displacement for small values of time and we choose the numerical values of the material constants for Copper and numerical values of the other constants as follows:

$l = 10$ cm, $\xi_0 = 1.0$ cm, $\xi_1 = 8.0$ cm, $\xi = 6.0$ cm,

$\xi' = 4.0$ cm, $b = 0.1$, $K_1 = 1.16$ cm^2/^{0}C.

The numerical calculations have been carried out with the help of Burrough 6700 Computer at Regional Computer Centre, Calcutta.

The numerical results are shown in Table I and II

TABLE I

Time sec.	Temperature $\theta(\xi,\tau)$°C Non-Simple case	Temperature $\theta(\xi,\tau)$°C Simple case
0.00	0.407330310840	1.301092887950
0.01	0.382820919929	0.994883638230
0.02	0.359871849003	0.773748491040
0.03	0.338378812072	0.612613064740
0.04	0.318244764163	0.493927996517
0.05	0.299379384440	0.405398783117
0.06	0.281698596908	0.338401247926
0.07	0.265124125963	0.286876348138
0.08	0.249583084216	0.246557915265
0.09	0.235007592070	0.214434152615
0.10	0.221334413814	0.188371423263

TABLE II

Time sec.	Displacement $U(\xi,\tau)$ cm Non-Simple case $\times 10^{-6}$	Displacement $U(\xi,\tau)$ C Simple Case $\times 10^{-6}$
0.00	0.0	0.0
0.01	1.5610335101	0.4142322157
0.02	−13.2553643777	−3.4891175157
0.03	17.0219267767	4.4821979103
0.04	−17.9444582004	−4.7282437193
0.05	32.5903808269	8.6963790172
0.06	−30.6584059149	−8 2103804512
0.07	26.7435949108	7.2004150498
0.08	−21.0801123174	−5.9639142679
0.09	17.5686252102	4.8158456976
0.10	−14.5558969636	−4.0308489844

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