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ON THE PROPAGATION OF ELASTIC-PLASTIC WAVE IN THE HALF-SPACE

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The problem of the propagation of elastic-plastic wave in the half-space occupied by an elastic-plastic medium has been studied. Treatments were carried out under the assumption of perpendicular load, on the boundary, which propagates with a constant speed D. The assumption further involved that displacements were in the direction of the load whereas the lateral displacements were neglected. The method of quadratic error was used throughout the solution of the wave equation. The theoretical calculation of space dependant stress were displayed with respect to the distance from the half-space boundary.

INTRODUCTION

Among various authors Rakhmatulin (1961), Shapiro (1946) and Sokolovskii (1948) have studied the problem of elastic-plastic wave propagation in the half-space.

The three-dimensional problem of elastic wave propagation has been solved by Fawze Shaban El-Dewik (1975 a). The problem has been studied when an instantaneous constant load acts on the boundary of the elastic half-space. The load is taken to act perpendicularly to the boundary and the lateral displacements were neglected. A similar study of this problem was carried in the case where the load is time-dependant (Fawze Shaban El-Dewik 1975b).

The two-dimensional problem was treated under the assumption that the propagation load acts perpendicularly to the boundary (Fawze Shaban El-Dewik 1977), and the solution was obtained taking into account the vertical displacements, whereas the lateral displacements were neglected.

Nevertheless, this problem was recently solved (Fawze Shaban El-Dewik 1976) taking into consideration the lateral displacements as well as the vertical ones.

In the present work the two-dimensional problem of elastic-plastic wave propagation has been studied, whereas the boundary load moves with a constant velocity.

1. BASIC EQUATION AND ITS SOLUTION

Let the semi infiinite elastic-plastic material occupy the domain $Z \geqslant 0, -\infty < x, y < \infty$ in the Cartesian coordinates.

Assume that a load exists perpendicular on the boundary of the half-space and we assume that this load propagates with a constant speed D. If we neglect the lateral displacement then, the vertical displacement W satisfies the wave equation :

$$\frac{\partial^2 W}{\partial t^2} = a^2(e)\left[\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial^2 y}\right] \quad \ldots(1.1)$$

where a is the velocity of the longitudinal wave which is a function of intensity deformation e.

The initial condition is

$$W(x, y, 0) = \frac{\partial}{\partial t} W(x, y, 0) = 0. \quad \ldots(1.2)$$

And the boundary conditions are

$$\left.\begin{array}{lll} \left(\frac{\partial W}{\partial y}\right)_{y=0} = \epsilon(x, t), & & 0 \leqslant |\chi| \leqslant Dt \\ \left(\frac{\partial W}{\partial y}\right)_{y=0} = 0, & & |\chi| > Dt. \end{array}\right\} \quad \ldots(1.3)$$

2. The Propagation of Elastic Wave in the Half-space

The solution of the problem in this case may be written in the form :-

$$W = \iint_\sigma \frac{C(\xi, \tau)\, d\xi\, d\tau}{[a^2(t-\tau)^2 - (x-\xi)^2 - y^2]^{1/2}} \quad \ldots(2.1)$$

where σ is the region bounded by

$$\tau = t - \frac{1}{a}\left[(x-\xi)^2 + y^2\right]^{1/2} \quad \ldots(2.2)$$

$$\xi = \pm M\tau, \quad M = D/a. \quad \ldots(2.3)$$

Consequently, one can prove that the expression (2.1) satisfies the wave equation (1.1) and then we get :

$$\left(\frac{\partial W}{\partial y}\right)_{y=0} = -\pi C(x, t). \quad \ldots(2.4)$$

Therefore, the expression (2.1) takes the form

$$W = -\frac{1}{\pi}\iint_\sigma \frac{\epsilon(\xi, \tau)\, d\xi\, d\tau}{[a^2(t-\tau)^2 - (x-\xi)^2 - y^2]^{1/2}}. \quad \ldots(2.5)$$

Using (2.2) and (2.3), then the bounds of the integration in (2.5) will be the following

$$W = -\frac{1}{\pi}\int_0^{\tau_c} d\tau \int_{-D\tau}^{D\tau} \frac{C(\xi, \tau)\, d\xi}{[a^2(t-\tau)^2 - (x-\xi)^2 - y^2]^{1/2}}$$

$$-\frac{1}{\pi}\int_{\tau_c}^{\tau_1} d\tau \int_{\xi_1}^{D\tau} \frac{C(\xi, \tau)\, d\xi}{[a^2(t-\tau)^2 - (x-\xi)^2 - y^2]^{1/2}}$$

$$-\frac{1}{\pi}\int_{\tau_1}^{\tau_m} d\tau \int_{\xi_1}^{\xi_2} \frac{C(\xi, \tau)\, d\xi}{[a^2(t-\tau)^2 - (x-\xi)^2 - y^2]^{1/2}}$$

where τ_c, τ_1, τ_m, ξ_1 and ξ_2 are defined from the equations

$$a^2 (t-\tau_c)^2 - (x-\xi)^2 = y^2, \quad \xi = - D \tau_c,$$

$$a^2 (t-\tau_1)^2 - (x-\xi)^2 = y^2, \quad \xi = - D\tau_1,$$

$$a^2 (t-\tau_m)^2 = y^2,$$

$$\xi_1 = x - [a^2(t-\tau)^2 - y^2]^{1/2} \quad \text{and}$$

$$\xi_2 = x + [a^2 (t-\tau)^2 - y^2]^{1/2}.$$

We shall discuss the problem when

$$\epsilon (\xi) = b_0 + b_1 \xi + b_2 \xi^2.$$

Then we have

$$W = - \frac{1}{\pi} (b_0 - b_1 x + b_2 x^2) \iint_\sigma \frac{d\xi \, d\tau}{[a^2 (t-\tau)^2 - (x-\xi)^2 - y^2]^{1/2}}$$

$$- \frac{1}{\pi} (\tfrac{1}{2} b_1 - b_2 x) \iint_\sigma \frac{2 (x-\xi) \, d\xi \, d\tau}{[a^2 (t-\tau)^2 - (x-\xi)^2 - y^2]^{1/2}}$$

$$- \frac{b_2}{\pi} \iint_\sigma \frac{(x-\xi)^2 \, d\xi \, d\tau}{[a^2 (t-\tau)^2 - (x-\xi)^2 - y^2]^{1/2}} \qquad \ldots(2.6)$$

i. e.

$$W = - \frac{1}{\pi} (b_0 - b_1 x + b_2 x^2) \left[\int_0^{\tau_0} d\tau \int_0^{D\tau} \frac{d\xi}{[a^2 (t-\tau)^2 - (x-\xi)^2 - y^2]^{1/2}} \right.$$

$$\left. - \int_{\tau_0}^{\tau_1} d\tau \int_{x-[a^2 (t-\tau)^2 + y^2]^{1/2}}^{D\tau} \frac{d\xi}{[a^2 (t-\tau)^2 - (x-\xi)^2 - y^2]^{1/2}} \right]$$

$$- \frac{1}{\pi} (\tfrac{1}{2} b_1 - b_2 x) \left[\int_0^{\tau_0} d\tau \int_0^{D\tau} \frac{2 (x-\xi) \, d\xi}{[a^2 (t-\tau)^2 - (x-\xi)^2 - y^2]^{1/2}} \right.$$

$$\left. - \int_{\tau_0}^{\tau_1} d\tau \int_{x-[a^2 (t-\tau)^2 + y^2]^{1/2}}^{D\tau} \frac{2 (x-\xi) \, d\xi}{[a^2 (t-\tau)^2 - (x-\xi)^2 - y^2]^{1/2}} \right]$$

$$- \frac{b_2}{\pi} \left[\int_0^{\tau_0} d\tau \int_0^{D\tau} \frac{(x-\xi)^2 \, d\xi}{[a^2 (t-\tau)^2 - (x-\xi)^2 - y^2]^{1/2}} \right.$$

$$\left. - \int_{\tau}^{\tau_1} d\tau \int_{x[a^2 (t-\tau)^2 + y^2]^{1/2}}^{D\tau} \frac{(x-\xi)^2 \, d\xi}{[a^2 (t-\tau)^2 - (x-\xi)^2 - y^2]^{1/2}} \right] \qquad \ldots(2.7)$$

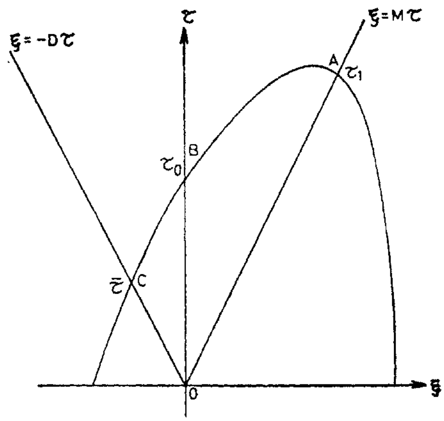
where τ_c may be obtained by solving (2.2) and the straight line $\xi = 0$,

Thus $\tau_0 = a\,t - [x^2 + y^2]^{1/2}$.

Also τ_1 — may be obtained by solving (2.2) and the two straight lines $\xi = \pm\, D\,\tau$

Thus $$\tau_1 = \frac{1}{(1-D^2)}\Big[-(Dx - a\,t) \pm [(Dx-a\,t)^2 - (1-D^2)\,(a^2\,t^2 - x^2 - y^2)^{1/2}\Big]$$

See Fig. 1



Differentiating (2.7) with respect to x and y respectively we get

$$\frac{\partial W}{\partial x} = -\frac{1}{\pi}\,(b_0 - b_1 x + b_2 x^2)\Bigg\{\Bigg[\cos^{-1}\frac{(x - D\,\tau_1)}{[a^2\,(t-\tau_1)^2 - y^2]^{1/2}}\,\frac{\partial \tau_1}{\partial x} - \cos^{-1}\frac{x}{[a^2(t-\tau)^2 - y^2]^{1/2}}\,\frac{\partial \tau_0}{\partial x}\Bigg]$$

$$+ \frac{1}{[D^2 - a^2]^{1/2}}\Bigg[\sin^{-1}\frac{(a^2-D^2)(t-\tau_1) - D(x-Dt)}{[a^2(x-Dt)^2 + (a^2-D^2)y^2]^{1/2}} - \sin^{-1}\frac{(a^2-D^2)\,t - D\,(x-D\,t)}{[a^2(x-Dt)^2[+a^2-D^2)y^2]^{1/2}}\Bigg]$$

$$-\frac{1}{a}\Bigg[\log\Bigg|(t-\tau_0) + \Big[(t-\tau_0)^2 - \frac{x^2+y^2}{a^2}\Big]^{1/2}\Bigg| - \log\Bigg|t + \Big[t^2 - \frac{x^2+y^2}{a^2}\Big|\Big]^{1/2}\Bigg]\Bigg\}$$

$$-\frac{1}{\pi}\Big(\frac{1}{2}b_1 - b_2 x\Big)\Bigg\{2\Big[a^2\,(t-\tau_1)^2 - (x - D\tau_1)^2 - y^2\Big]^{1/2}\frac{\partial \tau_1}{\partial x} - 2\Big[a^2(t-\tau_0)^2 - x^2 - y^2\Big]^{1/2}$$

$$\frac{\partial \tau_0}{\partial x}\;\frac{-2\,D}{(a^2-D^2)}\Bigg[\Bigg[(a^2-D^2)\,(t-\tau_1)^2 - 2D\,(x - D\,t)\,(t-\tau_1) - (x-Dt)^2 - y^2\Bigg]^{1/2}$$

$$-\Big[(a^2-D^2)\,t^2 - 2\,D(x-Dt)\,t - (x-Dt)^2 - y^2)\Big]^{1/2}\Bigg] - \frac{2x}{a}\Bigg[\log\Bigg|(t-\tau_0)$$

$$+\Big[(t-\tau_0)^2 - \frac{x^2+y^2}{a^2}\Big]^{1/2} - \log\Bigg|t + \Big[t^2 - \frac{x^2+y^2}{a^2}\Big]^{1/2}\Bigg|\Bigg] - \frac{2a^2\,(x - D\,t}{(a^2-D^2)^{3/2}}$$

$$\Bigg[\sin^{-1}\frac{(a^2-D^2)\,(t-\tau_1) - D\,(x - Dt)}{[a^2\,(x-Dt)^2 + (a^2-D^2)\,y^2]^{1/2}} - \sin^{-1}\frac{(a^2-D^2)\,t - D\,(x-Dt)}{[a^2(x-Dt)^2 + (a^2-D^2)y^2]^{1/2}}\Bigg]\Bigg\}$$

$$-\frac{b_2}{\pi}\left\{\left[\frac{1}{2}(x-D\tau_0)\left[a^2(t-\tau_0)^2-y^2(x-D\tau_0)^2\right]^{1/2}-x\left[a^2(t-\tau_0)^2-y^2-x^2\right]^{1/2}\right.\right.$$

$$\frac{\partial\tau_0}{\partial x}-(a^2(t-\tau_0)^2-y^2)\cos^{-1}\frac{x}{[a^2(t-\tau_0)^2-y^2]^{1/2}}\cdot\frac{\partial\tau_0}{\partial x}-\frac{x}{2}\left[a^2(t-\tau_1)^2-y^2-x^2\right]^{1/2}$$

$$\frac{\partial\tau_1}{\partial x}-\frac{1}{2}\left[a^2(t-\tau_0)^2-y^2\right]\cos^{-1}\frac{x-D\tau_0}{[a^2(t-\tau_0)^2-y^2]^{1/2}}\frac{\partial\tau_0}{\partial x}+\frac{1}{2}(t-\tau_0)\left[(t-\tau_0^2)-x^2-y^2\right]^{1/2}$$

$$-t\left[t^2-y^2-x^2)+\frac{1}{2}\frac{a^2}{[x^2+y^2]^{1/2}}\left(\sin^{-1}\frac{(t-\tau_0)\,x^2+y^2}{a}-\sin^{-1}\frac{t\,[x^2+y^2]^{1/2}}{a}\right)\right.$$

$$+\frac{(x^2+y^2)}{a}\left[\sin^{-1}\frac{a(t-\tau_0)}{[x^2+y^2]^{1/2}}-\sin^{-1}\frac{a\,t}{[x^2+y^2]^{1/2}}\right]+\left[\frac{a^2(t-\tau_1)^2-3D(x-Dt)}{2(a^2-D^2)}\right]$$

$$\left[\left[(a^2-D^2)(t-\tau_1)^2-2D(x-Dt)\,t-\tau_1)-(x-Dt)^2-y^2\right]^{1/2}\right]$$

$$-\left[\left[(a^2-D^2)\,t^2-2D(x-Dt)\,t-(x-Dt)^2-y^2\right]^{1/2}\right]\left(\frac{a^2\,t-3D(x-Dt)}{2(a^2-D^2)}\right)\frac{1}{[a^2-D^2]^{1/2}}$$

$$\left[y^2-\frac{3D(x-Dt)}{(a^2-D^2)}-\left(\sin^{-1}\frac{(a^2-D^2)(t-\tau_1)+D(x-Dt)}{[a^2(x-Dt)^2+y^2(a^2-D^2)]^{1/2}}\right.\right.$$

$$\left.\left.\left.-\sin^{-1}\frac{(a^2-D^2)\,t+D(x-Dt)}{[(a^2(x-Dt)^2+y^2(a^2-D^2)]^{1/2}}\right]\right\}\right) \qquad \ldots(2.8)$$

and

$$\frac{\partial W}{\partial y}=-\frac{1}{\pi}(b_0-b_1+b_2x^2)\left\{\cos^{-1}\frac{x-D\tau_1}{[a^2(t-\tau_1)^2-y^2]^{1/2}}\frac{\partial\tau_1}{\partial y}-\cos^{-1}\frac{x}{[a^2(t-\tau_0)^2-y^2]^{1/2}}\right.$$

$$\frac{\partial\tau_0}{\partial y}+\frac{1}{2a}\left[\sin^{-1}\frac{[a(t-\tau_1)-y][y(1-D^2)-D(x-Dt)]-[Dy-[Dy+(x-Dt)]^2}{[a(t-\tau_1)-y][y^2(1-D^2)+(x-Dt)^2]^{1/2}}\right.$$

$$\left.-\sin^{-1}\frac{(a\,t-y)[y(1-D^2)-D(x-Dt)]-(Dy+(x-Dt)]^2}{(a\,t-y)[y^2(1-D^2)-(x-Dt)^2]^{1/2}}\right]$$

$$+\frac{1}{2a}\left[\sin^{-1}\frac{[a(t-\tau_1)+y][y(1-D^2)+[D(x-Dt)]+[Dy-(x-Dt)]^2}{[a(t-\tau_1)-y][y^2(1-D^2)+(x-Dt)^2]^{1/2}}\right.$$

$$\left.-\sin^{-1}\frac{(a\,t+y)[y(1-D^2)+D(x-Dt)]+[Dy-(x-Dt)]^2}{(at+y)[y^2(1-D^2)+(x-Dt)^2]^{1/2}}\right]$$

$$+\left[\sin^{-1}\frac{a(t-\tau_1)\,y-y^2-x^2}{(a(t-\tau_1)-y)[x^2+y^2]^{1/2}}-\sin^{-1}\frac{a\,t\,y-y^2-x^2}{(a\,t-y)[x^2+y^2]^{1/2}}\right]$$

$$\left.-\left[\sin^{-1}\frac{a(t-\tau_1)+y^2-x^2}{[a(t-\tau_1)+y][x^2+y^2]^{1/2}}-\sin^{-1}\frac{a\,t\,y+y^2-x^2}{(a\,t+y)[x^2+y^2]^{1/2}}\right]\right\}$$

$$-\frac{1}{\pi}\left(\tfrac{1}{2}b_1-b_2x\right)\left\{2\left[a^2(t-\tau_1)^2-(x-D\tau_1)^2-y^2\right]^{1/2}\frac{\partial\tau_1}{\partial y}-2\left[a^2(t-\tau_0)^2x^2-y^2\right]^{1/2}\frac{\partial\tau_0}{\partial y}\right.$$

$$+\frac{2y}{[D^2-a^2]^{1/2}}\left[\sin^{-1}\frac{(a^2-D^2)(t-\tau_1)-D(x-Mt)}{[a^2(x-Dt)^2+(x-Dt)^2]^{1/2}}-\sin^{-1}\frac{(a^2-D^2)t-D(x-Dt)}{[a^2(x-Dt)^2+a^2-D^2)y^2]^{1/2}}\right]$$

(equation continued on p. 746)

$$
-\frac{2y}{a}\left[\log\left|(t-\tau_0)+\left[(t-\tau_0)^2-\left|\frac{x^2+y^2}{a^2}\right]^{1/2}-\log\left|t+\left[t^2-\frac{x^2+y^2}{a^2}\right]^{1/2}\right|\right|\right]\right\}
$$

$$
-\frac{b_2}{\pi}\left\{-\frac{1}{2}(x-D\tau_0)\left[a^2(t-\tau_0)^2-y^2(x-D\tau_0)^2.\right]^{1/2}\frac{\partial\tau_0}{\partial y}+x\left[a^2(t-\tau_0)^2-y^2-x^2\right]^{1/2}\right.
$$

$$
\frac{\partial\tau_0}{\partial y}-\frac{7}{2}[a^2(t-\tau_6)^2-y^2]\cos^{-1}\frac{x}{[a^2(t-\tau_0)^2-y^2]^{1/2}}\frac{\partial\tau_0}{\partial y}+\frac{3}{2}(a^2(t-\tau_0)^2-y^2)
$$

$$
\cos^{-1}\frac{x-D\tau_0}{[a^2(t-\tau_0)^2-y^2]^{1/2}}\frac{\partial\tau_0}{\partial y}+\frac{1}{2}(a^2(t-\tau_1)^2-y^2)\cos^{-1}\frac{x}{[a^2(t-\tau_1)^2-y^2]^{1/2}}\frac{\partial\tau_1}{\partial y}
$$

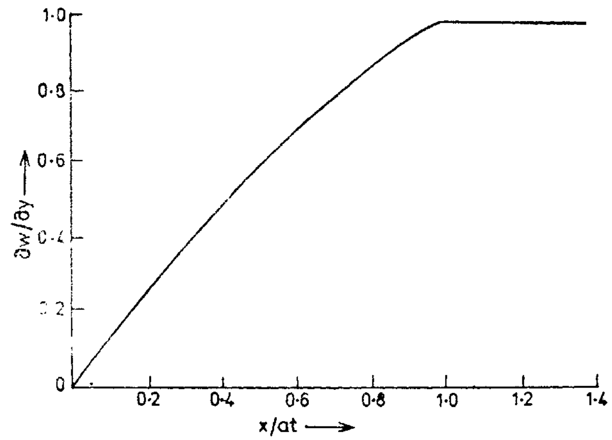
$$
+2y\left((\tau_1\cos^{-1}\frac{x}{[a^2(t-\tau_1)^2-y^2]^{1/2}}-\tau_0\cos^{-1}\frac{x-D\tau_0}{[a^2(t-\tau_0)^2-y^2]^{1/2}}\right)-\frac{(x-Dt)-y^2)}{[a^2-D^2]^{1/2}}
$$

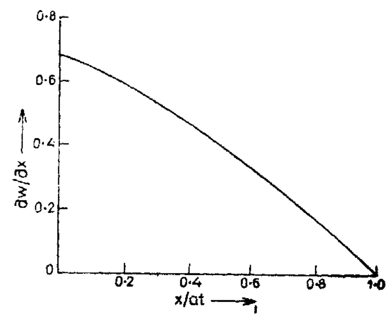
$$
\left[\sin^{-1}\frac{(a^2-D^2)(t-\tau_1)-D(x-Dt)}{[a(x-Dt)^2+y^2(a^2-D^2)]^{1/2}}-\sin^{-1}\frac{(a^2-D^2)t+D(x-Dt)}{[a^2(x-Dt)^2+y^2(a^2-D^2]^{1/2}}\right]
$$

$$
-\frac{x}{a}\left[\sin^{-1}\frac{a(t-\tau_0)}{[x^2+y^2]^{1/2}}-\sin^{-1}\frac{at}{[x^3+y^2]^{1/2}}\right]\right\} \quad \ldots(2.9)
$$

The numerical values $\frac{\partial W}{\partial y}$ and $\frac{\partial W}{\partial x}$ were calculated for different values of x.

The plots of $\frac{\partial W}{\partial y}$ and $\frac{\partial W}{\partial x}$ are represented in Figs. (2) and (3).





3. The Propagation of Elastic-Plastic wave in the Half-space

In this case if we take into consideration Prantel's diagram, then the elastic-plastic wave propagation may be illustrated as shown in Fig. 4. The region W_1 is the elastic medium, whereas W_2 is the plastic medium and the surface $ABCBA$ is the discontenious deformation interface. If the lateral displacements were neglected, then the vertical displacement W in the plastic medium satisfies the wave equation (1.1) with the boundary conditions (1.3) where

$$W = W_1 + W_2 \qquad ...(3.1)$$

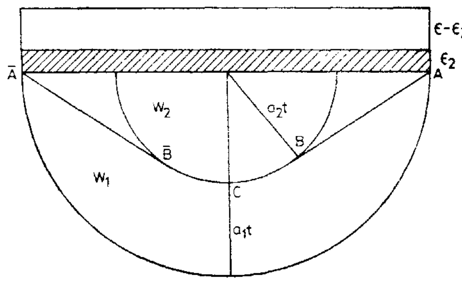
and W_1 and W_2 represent the vertical displacements in the elastic and plastic region respectively (See Fig. 4).

Suppose that the boundary condition for plastic region is :

$$\left(\frac{\partial W_2}{\partial y}\right)_{y=0} = \epsilon_2 \qquad ...(3.2)$$

then the boundary condition for elastic region will be :

$$\left(\frac{\partial W_1}{\partial y}\right)_{y=0} = \epsilon - \epsilon_2 \qquad ...(3.3)$$



and the solution of the problem may be written in the form :

$$W_2 = \iint\limits_{\sigma_2} \frac{C_2(\xi, \tau)\, d\xi\, d\tau}{[a_2^2(t-\tau)^2-(x-\xi)^2-y^2]^{1/2}} \qquad ...(3.4)$$

and

$$W_1 = \iint\limits_{\sigma_1} \frac{C_1(\xi, \tau)\, d\xi\, d\tau}{[a_1^2(t-\tau)^2-(x-\xi)^2-y^2]^{1/2}} \qquad ...(3.5)$$

where C_1, C_2 and ϵ_2 are unknown functions to be determined from the following relations :

$$\left(\frac{\partial W_1}{\partial y} + \frac{\partial W_2}{\partial y}\right)_{y=0} = -\pi C_1 - \pi C_2 = \epsilon \qquad ...(3.6)$$

$$\left(\frac{\partial W_1}{\partial y}\right)_{y=0} = -\pi C_1 = \epsilon - \epsilon_2 \qquad ...(3.7)$$

$$\Big[\epsilon_i\Big]_{ABC\bar{B}\bar{A}} = \epsilon_s \qquad ...(3.8)$$

where ϵ_s is the limit value of the deformation between the elastic and plastic regions and ϵ_i is the intensitive deformation.

In the present case ϵ_i reduced to the following form

$$\epsilon_i = \frac{2}{3}\left[(\frac{\partial W_1}{\partial y})^2 + \frac{3}{4}(\frac{\partial W_1}{\partial x})^2\right]^{1/2} \qquad ..(3.9)$$

then the relation (3.8) takes the form :

$$\epsilon_i^2 = \frac{4}{9}\left[(\frac{\partial W_1}{\partial y})^2 - \frac{3}{4}(\frac{\partial W_1}{\partial x})^2\right]_{AB\bar{C}\bar{B}\bar{A}} = \epsilon_s^2 \qquad ...(3.10)$$

the relation (3.6) and (3.7) and (3.10) are enough to determine C_1, C_2 and ϵ_s. Using Taylor's expansion, $\epsilon_2(\xi)$ may take the form :

$$\epsilon_2(\xi) = b_0 + b_1\xi + b_2\xi_2 \qquad ...(3.11)$$

Where b_0, b_1 and b_2 are unknown constants to be determined by using the method of quadratic error.

Let ϵ_i differs from ϵ_s by infinitisimal quantity e,

i.e. $\epsilon_i - \epsilon_s = e(b_0, b_1, b_2)$ ...(3.12)

Then $e^2 = \int_0^t (\epsilon_1 - \epsilon_s)^2 \, d\tau$ ...(3.13)

Using (3.9) we get

$$e^2 = \int_0^t \left[\frac{2}{3}\left[\left\{(\frac{\partial W_1}{\partial y})^2 + \frac{3}{4}(\frac{\partial W_1}{\partial x})^2\right\}\right]^{1/2}\epsilon_s\right]^2 d \qquad ...(3.14)$$

Where $\frac{\partial W_1}{\partial x}$ and $\frac{\partial W_1}{\partial y}$ are obtained in (2.8) and (2.9) by replacing a with $a(e)$. Making this error minimum, its first derivative with respect to b_0, b_1 and b_2 must to be equal to zero.

Thus $\frac{\partial e}{\partial b_0} = 0, \quad \frac{\partial e}{\partial b_1} = 0, \quad \frac{\partial e}{\partial b_2} = 0$ ...(3.15)

solving the above conditional relation (3.15) with respect to b_0, b_1, b_2 we get :

$b_0 = 1.08\,\epsilon_s$,

$b_1 = +\,0.29\,\frac{\epsilon_s}{at}$

$b_2 = 0.52\,\frac{\epsilon_s}{(at)^2}$

4. Discussion of Results and Conclusions

In the case of semi-infinite elastic medium the analytical solution for the vertical displacement W, $\frac{\partial w}{\partial y}$ and $\frac{\partial w}{\partial x}$ are obtained in (2.7), (2.8) and (2.9). The results for $y = 0.5$ and different values of x/at are shown in Figs. 2 and 3.

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