Indian J. pure appl. Math., 13(7): 769–771, July 1982

APPLICATIONS OF SOME THEOREMS OF SRIVASTAVA AND LAVOIE

S. K. BHARGAVA

Department of Mathematics, S. P. I. I. College, Jhansi 284001

(*Received* 21 *January* 1981 ; *after revision* 10 *September* 1981)

In this note we shall deduce a bilateral and a mixed multilateral generating functions for a class of polynomials by applying certain general theorems of Srivastava and Lavoie (1975) and Srivastava (1980).

1. INTRODUCTION

Recently, Chandel and Bhargava (1981) studied a general class of polynomials $G_n(a, k; h, g(x))$ defined by

$$G_n(a, k; h, g(x)) = e^{-hg(x)} \; T_{a,k}^{n} \left(e^{hg(x)}\right) \qquad ...(1.1)$$

where h is constant, $g(x)$ is a differentiable function of x and

$$T_{a,k} = x^k (a+xD), \;\; D \equiv \frac{d}{dx} \; . \qquad ...(1.2)$$

We also obtained a generating function

$$\sum_{n=0}^{\infty} G_{n+m}(a, k; h, g(x)) \frac{t^n}{n!} = (1-tkx^k)^{-a/k} \exp\left[h \left[g\left\{x(1-tkx^k)^{-1/k}\right\} - g(x) \right] \right] \times G_m(a, k; h, g\{x(1-tkx^k)-1/k\}), \qquad ...(1.3)$$

where m is an arbitrary non-negative integer.

In continuation of this study we shall deduce mainly two new results for $G_n(a, k; h, g(x))$ by applying the most useful theorems of Srivastava (1980) and Srivastava and Lavoie (1975).

Corresponding to every sequence of functions $\{ S_n(x) \mid n = 0, 1, 2, ... \}$ generated by [Singhal and Srivastava, 1972 p. 755, Equation (1)]

$$\sum_{n=0}^{\infty} A_{m,n} \, S_{m+n}(x) \, t^n = f(x,t) \, \{g(x,t)\}^{-m} \, S_m(h(x,t)) \qquad ...(1.4)$$

where $m \geqq 0$ is an integer, the $A_{m,n}$ are arbitrary constants, and f, g, h are suitable functions of x and t, Srivastava and Lavoie (1975) proved.

Theorem 1—[Srivastava and Lavoie (1975), p. 304, Theorem 1] If

$$F_q[x, t] = \sum_{n=0}^{\infty} a'_n \, S_{qn}(x) \, t^n, \qquad ...(1.5)$$

where q is an arbitrary positive integer and the $a'_n \neq 0$ are arbitrary constants, then

$$\sum_{n=0}^{\infty} S_n(x)\,\sigma_n^q(y)\,t^n = f(x,t)\,F_q[h(x,t),\,y\,\{t/g(x,t)\}^q] \qquad \ldots(1.6)$$

where $\sigma_n^q(y)$ is a polynomial of degree $[n/q]$ in y defined by

$$\sigma_n^q(y) = \sum_{k'=0}^{[n/q]} a'_{k'} A_{qk',n-qk'}\, y^{k'}. \qquad \ldots(1.7)$$

Very recently, Srivastava (1980) considered the functions $S_\mu^*(x)$ defined by

$$\sum_{n=0}^{\infty} A^*_{\mu,n}\, S^*_{\mu+n}(x)\, t^n = f^*(x,t)\,\{g(x,t)\}^{-\mu}\, S^*_\mu(h(x,t)) \qquad \ldots(1.8)$$

and obtained :

Theorem 2—[Srivastava (1980), p. 224, Theorem 2]—If

$$F^{p,\mu}_{q,\nu}\left[x;\, y_1,\ldots,y_s;\, t\right] = \sum_{n=0}^{\infty} C_n^{\mu,\nu} S^*_{\nu+qn}(x)\,\Omega_{\mu+pn}(y_1,\ldots,y_s)t^n, \qquad \ldots(1.9)$$

where (and in what follows) μ and ν are arbitrary complex numbers, p and q are positive integers, and $\Omega_\mu(y_1,\ldots,y_s)$ is a non-vanishing function of s variables $y_1,\ldots,y_s$; $s \geqq 1$,

then
$$\sum_{n=0}^{\infty} S^*_{\nu+n}(x)\, W^{p,\mu}_{n,q,\nu}(y_1,\ldots,y_s;z)\,t^n = f^*(x,t)\,\{g(x,t)\}^{-\nu}$$
$$\times\, F^{p,\mu}_{q,\nu}\left[h(x,t);\, y_1,\ldots,y_s;\, z\,\{t/g(x,t)\}^q\right] \qquad \ldots(1.10)$$

where $W^{p,\mu}_{n,q,\nu}(y_1,\ldots,y_s;z)$ is a polynomial of degree $[n/q]$ in z (with coefficients dependent on $y_1,\ldots,y_s$) defined by

$$W^{p,\mu}_{n,q,\nu}(y_1,\ldots,y_s;z) = \sum_{k'=0}^{[n/q]} A^*_{\nu+qk',n-qk'}\, C_{k'}^{\mu,\nu}\, \Omega_{\mu+pk'}(y_1,\ldots,y_s)\, z^{k'} \qquad \ldots(1.11)$$

in terms of the $A_{\mu,n}$ occurring in (1.8).

2. Bilateral and Mixed Multilateral Generating functions

It is quite clear that (1.3) is of the type (1.4) with $A_{m,n} = 1/n!$,

$f(x,t) = (1-t\,kx^k)^{-a/k} \exp[h\,[g\,\{x\,(1-tkx^k)^{-1/k}\} - g(x)]]$, $g(x,t) = 1$, $h(x,t) = \{x\,(1-tkx^k)^{-1/k}\}$.

Thus, using Theorem 1 of Srivastava and Lavoie (1975), we get for $G_n(a,k;h,g(x))$ a bilateral generating function given by

Theorem 3—If $F_q(x,t) = \sum_{i=0}^{\infty} \frac{1}{(qi)!} A'_i\, G_{qi}(a,k;h,g(x))\, t^i$. $\qquad \ldots(2.1)$

$$\sum_{n=0}^{\infty} G_n(a, k; h, g(x))\, \sigma_n^q(y) \frac{t^n}{n!} = (1 - tkx^k)^{-a/k} \times \exp[h[g\{x(1-tkx^k)^{-1/k}\} - g(x)]] \times F_q[x(1-tkx^k)^{-1/k}, yt^q) \quad \ldots(2.2)$$

where $$\sigma_n^q(y) = \sum_{i=0}^{[n/q]} \binom{n}{qi} A'_i\, y^i. \quad \ldots(2.3)$$

In this theorem, if we put $q = 1$, then a known bilateral generating function for the polynomials of Chandel and Bhargava [1981, p, 108, eqn. (3.8)] appears.

Now, using Theorem 2 of Srivastava (1980), we extend Theorem 3 in the form of mixed multilateral generating function given by :

Theorem 4—If $$\Lambda_{m,q}[x; y_1, \ldots, y_s; t] = \sum_{n=0}^{\infty} \frac{A'_n}{(qn)!} G_{m+qn}(a, k; h, g(x)) \times \Omega_{\mu+pn}(y_1, \ldots, y_s)\, t^n,\ A'_n \neq 0, \quad \ldots(2.4)$$

then $$\sum_{n=0}^{\infty} G_{m+n}(a,k;h,g(x))\, M_{n,q}^{p,\mu}(y_1, \ldots, y_s; z) \frac{t^n}{n!}$$

$$= (1-tkx^k)^{-a/k} \exp[h[g\{x(1-tkx^k)^{-1/k}\} - g(x)][\times \Lambda_{m,q}[x(1-tkx^k)^{-1/k} : y_1, \ldots, y_s; zt^q] \quad \ldots(2.5)$$

where $$M_{n,q}^{p,\mu}(y_1, \ldots, y_s; z) = \sum_{i=0}^{[n/q]} \binom{n}{qi} A'_i\, \Omega_{\mu+pi}(y_1, \ldots, y_s)\, z^i. \quad \ldots(2.6)$$

If we put $q = 1$ in the above theorem, we also get a multilateral generating function for $G_n(a, k; h, g(x)$ introducing $M_{n,1}^{p,\mu}(y_1, \ , y_s; z)$ which is a polynomial of degree n in z (with coefficients dependent on $(y_1, \ldots, y_s)$, while putting $a = 0$ and replacing k by $(k-1)$ we get a mixed multilateral generating function for $G_n(h, g, k)$ studied by Chandel (1974).

By giving suitable values to A'_i, we can obtain bilateral and mixed multilateral generating functions for a number of more familiar systems of polynomials.

ACKNOWLEDGEMENT

The author is thankful to the referee for his valuable suggestions.

REFERENCES

Chandel, R. C. Singh (1974). A further generalization of class of polynomials $T_n^{\alpha,k}(x, r, p)$. *Kyungpook Math. J.*, **14**, 45-54.

Chandel, R. C. Singh, and Bhargava, S. K. (1981). A generalization of certain classses of polynomials. *Indian J. pure appl. Math.*, **12**, 103-10.

Singhal. J. P., and Srivastava, H. M. (1972). A class of bilateral generating functions for certain classical polynomials. *Pacific J. Math.*, 42, 755-62.

Srivastava, H. M. (1980). Some bilateral generating functions for a certain class of special functions-I & II. *Nederl. Akad. Wetensch. Proc. Ser. A* **83**=*Indag. Math.*, **42**, 221-46.

Srivastava H. M., and Lavoie, J. L. (1975). A certain method of obtaining bilateral generating functions. *Nederl. Akad. Wetench. Proc. Ser. A* 78=*Indag. Math.* 37, 304-20.