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GENERATION OF LOVE WAVES UNDER INITIAL STRESS DUE TO A MOMENTARY POINT SOURCE

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In this paper the dispersion equation of Love waves in an initially stressed layer lying over an initially stressed semi-infinite medium due to a momentary point source is found by Green's function technique. It is shown that the displacement is very large when $h\left(k_1^2 - f_n^2\right)^{1/2} = m\pi$ and this may cause large scale fracture in the free surface.

1. INTRODUCTION

Displacement of Love waves generated by a two dimensional point source in a layered medium has been studied by Sezawa (1935) and Satô (1952) by the method of successive reflections at the boundaries.

Chattopadhyay (1978) has solved the same problem by using Green's function technique.

In this paper following Chattopadhyay (1978) the dispersion equation of Love waves in an initially stressed layer lying over an initially stressed material due to a momentary point source has been found by the Green's function technique. Graphs are drawn under the assumption that the rigidity in the lower substratum is greater than that in the crustal layer whereas $\beta_2 < \beta_1$.

2. SOLUTION OF THE PROBLEM

We take the origin on the interface of the upper layer of thickness h and the lower semi-infinite medium. The x-axis is taken along the interface and z-axis vertically downwards (Fig. 1). The source has been taken in the lower semi-infinite medium, but immediately near the point $(x_0, 0, 0)$. We consider both the upper layer and the lower semi-infinite medium to be homogeneous isotropic but initially stressed.

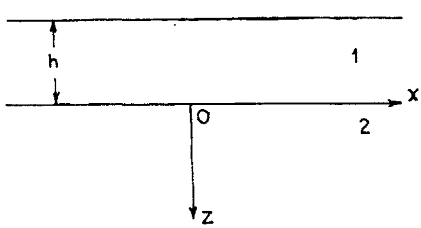

Fig. 1.

Let us assume that the upper layer and the lower semi-infinite medium are under initial compressive stresses of magnitude P_1 and P_2 along the x-axis. Then the components of initial stress in the upper layer are

$$\sigma_{11}^1 = -P_1$$

and

$$\sigma_{22}^1 = \sigma_{33}^1 = \sigma_{23}^1 = \sigma_{31}^1 = \sigma_{12}^1 = 0$$

and the dynamical equations of equilibrium are (Biot 1965),

$$\left.\begin{aligned}
&\frac{\partial s_{11}^1}{\partial x} + \frac{\partial s_{12}^1}{\partial y} + \frac{\partial s_{13}^1}{\partial z} - P_1 \frac{\partial \omega_z^1}{\partial y} + P_1 \frac{\partial \omega_y^1}{\partial z} = \rho_1 \frac{\partial^2 u_1}{\partial t^2} \\
&\frac{\partial s_{21}^1}{\partial x} + \frac{\partial s_{22}^1}{\partial y} + \frac{\partial s_{23}^1}{\partial z} - P_1 \frac{\partial \omega_z^1}{\partial x} = \rho_1 \frac{\partial^2 v_1}{\partial t^2} \\
&\frac{\partial s_{31}^1}{\partial x} + \frac{\partial s_{32}^1}{\partial y} + \frac{\partial s_{33}^1}{\partial x} - P_1 \frac{\partial \omega_y^1}{\partial x} = \rho_1 \frac{\partial^2 w_1}{\partial t^2}
\end{aligned}\right\} \qquad ...(1)$$

where u_1, v_1 and w_1 are the component of the displacement vector in the upper layer and

$$\omega_x^1 = \frac{1}{2}\left(\frac{\partial w_1}{\partial y} - \frac{\partial v_1}{\partial z}\right), \quad \omega_y^1 = \frac{1}{2}\left(\frac{\partial u_1}{\partial z} - \frac{\partial w_1}{\partial x}\right),$$

$\omega_z^1 = \frac{1}{2}\left(\frac{\partial v_1}{\partial x} - \frac{\partial u_1}{\partial y}\right)$ denote the rotational components s_{ij}^1 are the incremental stress components of the upper layer and ρ_1 is the density of the upper layer. Taking the orthotropic symmetry in incremental deformations, the relation between the strain and the incremental stress components are

$$\left.\begin{aligned}
s_{11}^1 &= B_{11}^1 e_{11}^1 + B_{12}^1 e_{22}^1 + B_{13}^1 e_{33}^1 \\
s_{22}^1 &= B_{21}^1 e_{11}^1 + B_{22}^1 e_{22}^1 + B_{23}^1 e_{33}^1 \\
s_{33}^1 &= B_{31}^1 e_{11}^1 + B_{32}^1 e_{22}^1 + B_{33}^1 e_{33}^1 \\
s_{23}^1 &= 2\,Q_1^1 e_{23}^1, \quad s_{31}^1 = 2\,Q_2^1 e_{31}^1, \\
s_{12}^1 &= 2\,Q_3^1 e_{12}^1
\end{aligned}\right\} \qquad ...(2)$$

where B_{ij}^1 and Q_i^1 are the incremental normal elastic coefficients and shear moduli respectively.

For SH waves propagating in the upper layer along the x-axis we have

$u_1 = w_1 = 0$ and $v_1 = v_2(x, z, t)$.

So,

$$\left.\begin{aligned}
&e_{11}^1 = e_{22}^1 = e_{33}^1 = e_{31}^1 = \omega_y^1 = 0 \\
&s_{11}^1 = s_{22}^1 = s_{33}^1 = s_{31}^1 = 0.
\end{aligned}\right\} \qquad ...(3)$$

By using (2) and (3) the first and third equations of the set (1) are identically satisfied. For isotropic material $Q_1^1 = Q_2^1 = Q_3^1 = \mu_1$. Therefore the only existing equation is

$$(\mu_1 - \tfrac{1}{2} P_1) \frac{\partial^2 v_1}{\partial x^2} + \mu_1 \frac{\partial^2 v_1}{\partial z^2} = \rho_1 \frac{\partial^2 v_1}{\partial t^2} \qquad \ldots(4)$$

which can be written as

$$\nabla_1^2 v_1 = \frac{\rho_1}{\mu_1 - \frac{1}{2} P_1} \frac{\partial^2 v_1}{\partial t^2} \qquad \ldots(5)$$

where

$$\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial z_1^2}$$

and

$$z = \left(\frac{\mu_1}{\mu_1 - \frac{1}{2} P_1} \right)^{\frac{1}{2}} z_1 = \frac{1}{M_1} z_1 \text{ (say)} \qquad \ldots(6)$$

so that

$$M_1 = \left(\frac{\mu_1 - \frac{1}{2} P_2}{\mu_1} \right)^{1/2} = \sqrt{1 - I_1}$$

where $I_1 = P_1/2\,\mu_1$.

In the same way the only existing equation of motion for the lower medium is

$$\nabla_2^2 v_2 = \frac{\rho_2}{\mu_2 - \frac{P_2}{2}} \frac{\partial^2 v_2}{\partial t^2} \qquad \ldots(7)$$

where P_2, ρ_2 and v_2 denote respectively the incremental compressive stress, density and the component along the y axis of the displacement vector of the lower medium, and

$$\nabla_2^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z_2^2}, \quad z = \left(\frac{\mu_2}{\mu_2 - \frac{1}{2} P_2} \right)^{\frac{1}{2}} z_2 = \frac{1}{M_2} z_2 \text{ (say)},$$

so that

$$M_2 = \left(\frac{\mu_2 - \frac{1}{2} P_2}{\mu_2} \right)^{\frac{1}{2}} = \sqrt{1 - I_2} \qquad \ldots(8)$$

where $I_2 = P_2/2\,\mu_2$.

Taking the time dependence of the displacement proportional to $e^{i\omega t}$, the equations for the upper and lower media are respectively

$$\nabla_1^2 V_1 + k_1^2 V_1 = 0 \qquad \ldots(9)$$

and

$$\nabla_2^2 V_2 + k_2^2 V_2 = 0 \qquad \ldots(10)$$

where

$$k_1^2 = \frac{\omega^2 \rho_1}{\mu_1 (1 - I_1)} \text{ and } k_2^2 = \frac{\omega^2 \rho_2}{\mu_2 (1 - I_2)} . \qquad \ldots(11)$$

The boundary conditions are

$$\left.\begin{aligned} \Delta f_y^1 &= 0 \quad \text{at} \quad z = -h \\ v_1 &= v_2 \quad \text{at} \quad z = 0 \\ \Delta f_y^1 &= \Delta f_y^2 \text{ at } z = 0 \end{aligned}\right\} \qquad \ldots(12)$$

where

$$\Delta f_y^r = s_{23}^r, \quad r = 1, 2. \qquad \ldots(13)$$

Alternatively (12) may be replaced by

$$\frac{\partial v_1}{\partial z} = 0 \quad \text{at } z = -h \qquad \ldots(14)$$

$$v_1 = v_2 \quad \text{at } z = 0 \qquad \ldots(15)$$

and

$$\mu_1 \frac{\partial v_1}{\partial z} = \mu_2 \frac{\partial v_2}{\partial z} \quad \text{at } z = 0. \qquad \ldots(16)$$

We propose to solve eqns. (9) and (10) under the prescribed boundary conditions (14), (15) and (16) by the Greens function technique. Covert (1958) indicated a method for finding the Green's function for composite bodies.

Let G_1 and G_2 be the green's functions for the upper and lower media under the boundary conditions $\frac{\partial G_1}{\partial n_1} = \frac{\partial G_2}{\partial n_2} = 0$ at the interface and $G_1 = 0$ at the free surface of the layer. n_1 and n_2 correspond to the normals drawn outwards from the upper and lower media respectively. Then we have

$$V_1(r) = \int G_1(r/r_0)\, \rho_1'(r_0)\, dv_1 + \frac{1}{4\pi}\int_{AB} G_1(r/r_s) \frac{\partial V_1(r_s)}{\partial n_1}\, ds_1 \qquad \ldots(17)$$

$$V_2(r) = \int G_2(r/r_0)\, \rho_2'(r_0)\, dv_2 + \frac{1}{4\pi}\int_{AB} G_2(r/r_s) \frac{\partial V_2(r_s)}{\partial n_2}\, ds_2 \qquad \ldots(18)$$

where ρ_1' and ρ_2' are source densities in the media 1 and 2. If the point source lies very near the origin but in the medium 2, that is, if $\rho_2' = \delta(r_0 - 0)$ and $\rho_1' = 0$, we have V_1 as Green's function for body 1 and V_2 as the Green's function for body 2 when the source is in the medium 2.

From (15), (17) and (18) we have

$$\int G_1\, \rho_1'(r_0)\, dv_1 + \frac{1}{4\pi}\int_{AB} G_1 \frac{\partial V_1}{\partial n_1} ds_1 = \int G_2\, \rho_2'(r_0)\, d\, v_2 + \frac{1}{4\pi}\int_{AB} G_2 \frac{\partial V_2}{\partial n_2}\, ds_2.$$

Using (16) and $\rho_1' = 0$, $\rho_2' = \delta(r_0 - 0)$, the above equation becomes

$$G_2(r/0) = \frac{1}{4\pi}\int_{AB} \left[G_1(r/r_0) + \frac{\mu_1}{\mu_2} G_2(r/r_0) \right] \frac{\partial G(0/r_0)}{\partial z}\, ds \qquad \ldots(19)$$

where the field point $r\ (x, y)$ and variable point $r_0\ (x_0, 0)$ both are on the interface AB and G is the proper Green's function for body 1 corresponding to the source in the medium 2.

Under the boundary condition of Green's function we have

$$G_2\ (r/r_0) = \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{\exp\{\alpha_2 z + if(x-x_0)\}}{\alpha_2}\, df \qquad \ldots(20)$$

where $\alpha_2^2 = f^2 - k_2^2.$ ...(21)

For the calculation of $G_1\ (r/r_0)$, we follow the reflection method. The reflected points are $(x_0, -2h)$, $(x_0, 2h)$, $(x_0, -4h)$, $(x_0, 4h)$, $(x_9 - 6h)$, $(x_0, 6h)$, ...
Then,

$$\begin{aligned} G_1\ (r/r_0) = \frac{2}{\pi} \int_{-\infty}^{\infty} & [\exp\{\alpha z + if\ (x-x_0)\} \\ & + \exp\{-\alpha\ (z+2h) + if\ (x-x_0)\} \\ & + \exp\{-\alpha\ (2h-z) + if\ (x-x_0)\} \\ & + \exp\{-\alpha\ (z+4h) + if\ x-x_0)\} \\ & + \exp\{-\alpha\ (4h-z) + if\ (x-x_0)\} \\ & + \exp\{-\alpha\ (z+6h) + if\ (x-x_0)\} + \ldots]\ \frac{df}{\alpha} \end{aligned} \qquad \ldots(22)$$

where $\alpha^2 = f^2 - k_1^2.$

Equation (22) can be written as,

$$G_1\ (r/r_0) = \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{\exp(\alpha\ z) + \exp\{-\alpha\ (z+2h)\}}{1-e^{-2h\alpha}}\ \frac{\exp\{\ if\ (x-x_0)\}}{\alpha} df. \qquad \ldots(23)$$

From (17) we have

$$G\ (r/0) = \frac{1}{4\pi} \int_{AB} G_1 \frac{\partial G}{\partial z}\ ds$$

which is the expression for Green's function for the body 1 corresponding to the source in the body 2. Taking the contour AB on the x axis from $-\infty$ to ∞, we have

$$G\ (r/0) = \frac{1}{4\pi} \int_{-\infty}^{\infty} G_1 \frac{\partial\ G\ (0/r_0)}{\partial\ z}\ d\ x_0. \qquad \ldots(24)$$

Using the relation (19), (20) and (23) we get

$$\frac{2}{\pi} \int_{-\infty}^{\infty} \frac{e^{ifx}}{\alpha_2} df = \frac{1}{4\pi} \int_{AB} \left[\frac{2}{\pi} \int_{-\infty}^{\infty} \frac{1+e^{-2h\alpha}}{1-e^{-2h\alpha}}\ \frac{e^{if\ (x-x_0)}}{\alpha}\ df \right.$$

(equation continued on p. 823)

$$+ \frac{2}{\pi} \frac{\mu_1}{\mu_2} \int_{-\infty}^{\infty} \frac{e^{if(x-x_0)}}{\alpha_2} df \Bigg] \frac{\partial G(0/r_0)}{\partial z} ds.$$

Writing the integral along AB as $-\infty$ to ∞,

$$\frac{1}{\alpha_2} = \frac{1}{4\pi} \int_{-\infty}^{\infty} \left[\frac{\mu_1}{\mu_2 \alpha_2} + \frac{1+e^{-2h\alpha}}{\alpha(1-e^{-2h\alpha})} \right] e^{-ifx_0} \frac{\partial G(0/r_0)}{\partial z} dx_0.$$

Now, by applying Fourier inversion, we have

$$\frac{\partial G(0/r_0)}{\partial z} = 2 \int_{-\infty}^{\infty} \frac{e^{ifx_0}}{\alpha_2 \left[\frac{\mu_1}{\mu_2 \alpha_2} + \frac{1+e^{-2h\alpha}}{\alpha(1-e^{-2h\alpha})} \right]} df. \quad \ldots(25)$$

Substituting the value of $\frac{\partial G(0/r_0)}{\partial z}$ and G_1 in eqn. (24), we have

$$G(r/0) = \frac{1}{\pi_2} \int_{-\infty}^{\infty} dx_0 \iint_{-\infty}^{\infty} \frac{\exp(\alpha z) + \exp\{-\alpha(z+2h)\}}{\alpha(1-e^{-2h\alpha})}$$

$$\times \frac{e^{if'x_0} e^{if(x-x_0)}}{\alpha_2' \left[\frac{\mu_1}{\mu_2 \alpha_2} + \frac{1+e^{-2h\alpha'}}{\alpha'(1-e^{-2h\alpha'})} \right]} df\, df'. \quad \ldots(26)$$

Put $f' - f = \eta$, therefore $df' = d\eta$ and using the result

$$\delta(f' - f) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i(f'-f)x_0} dx_0$$

and

$\int g(f')\,\delta(f'-f)\,df' = g(f)$, eqn. (26) becomes

$$G(r/0) = \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{\exp(\alpha z) + \exp\{-\alpha(z+2h)\}}{1-e^{-2h\alpha}}$$

$$\times \frac{e^{ifx}\, df}{\alpha \alpha_2 \left[\frac{\mu_1}{\mu_2 \alpha_2} + \frac{1+e^{-2h\alpha}}{\alpha(1-e^{-2h\alpha})} \right]}. \quad \ldots(27)$$

Equation (27) gives the SH displacement at a point when the point source is situated very nearly the origin in the lower medium. Integrals of this types given on the right-hand side of (27) have been calculated by Sezawa. For the evaluation of the integrals we choose the contour consisting of the real axis with indentations at the branch points $f = k_1$, k_2 and the infinite semi-circle in the upper half plane. Then the value can be expressed as the sum of the residues of the integrals and the two integrals along the branch lines corresponding to the branch points K_1 and K_2. Since the branch line integrals are of the order of $x^{-3/2}$ for large x, we neglect their contributions and evaluate the integrals, for large x, for the residue part only. We need consider only the contribution from the poles of $G\ (r/0)$ are given by the roots of

$$\frac{\mu_1}{\mu_2\,\alpha_2} + \frac{1+e^{-2h\alpha}}{\alpha\,(1-e^{-2h\alpha})} = 0. \qquad \ldots(28)$$

Since

$$\alpha = ik\left(\frac{c^2}{\beta_1^2\,(1-I_1)} - 1\right)^{\frac{1}{2}} \quad \text{and} \quad \alpha_2 = k\left(1 - \frac{c^2}{\beta_2^2\,(1-I_2)}\right)^{\frac{1}{2}}$$

then (28) becomes

$$\tan\left\{kh\left(\frac{c^2}{\beta_1^2\,(1-I_1)} - 1\right)^{\frac{1}{2}}\right\} = \frac{\mu_2}{\mu_1}\,\frac{\left(1 - \frac{c^2}{\beta_2^2\,(1-I_2)}\right)^{\frac{1}{2}}}{\left(\frac{c^2}{\beta_1^2\,(1-I_1)} - 1\right)^{\frac{1}{2}}} \qquad \ldots(29)$$

For SH waves propagation through the upper layer to be possible

$$\beta_1\sqrt{1-I_1} < c < \beta_2\sqrt{1-I_2}$$

must be statisfied.

Equation (27) can be written as

$$G\ (r/0) = \frac{2}{\pi}\int_{-\infty}^{\infty} \frac{\exp\{\alpha\,(z+h)\} + \exp\{-\alpha\,(z+h)\}}{e^{\alpha h} - e^{-\alpha h}}$$

$$\times \frac{e^{ifx}}{\alpha\left\{\frac{\mu_1}{\mu_2} + \frac{\alpha_2\,(1+e^{-2h\alpha})}{\alpha\,(1-e^{-2h\alpha})}\right\}}\,df. \qquad \ldots(30)$$

The contribution due to poles at $z = -h$, i.e., at the surface is

$$2\pi i\,\frac{2}{\pi}\sum\frac{1}{\alpha_n\,(e^{h\alpha_n} - e^{-h\alpha_n})\,F'\,(f_n)}$$

where

$F\ (f) = \dfrac{\mu_1}{\mu_2} + \dfrac{\alpha_2\,(1+e^{-2h\alpha})}{\alpha\,(1-e^{-2h\alpha})}$, f_n is a root of $F\,(f) = 0$ and α_n is the value of α in which f_n is put for f. Now in (30), when $e^{\alpha_n h} - e^{-\alpha_n h} \rightarrow 0$ where $\alpha_n = i\,(k_1^2 - f_n^2)^{1/2}$, we have

$$\sin\,(h\,(k_1^2 - f_n^2)^{1/2}) \rightarrow 0$$

i.e. $h\sqrt{k_1^2 - f_n^2} = m\,\pi$ and the amlitude expression of (27) is very large and this may lead to fracture.

3. Numerical Results

To show the nature of the motion we have computed the values of kh for different values of c/β_1 and different initial stresses in the two media. We have taken $\mu_2/\mu_1 = 1.8$ and $\beta_2/\beta_1 = 3.7/4.5$. For the purpose of graphical representation the computed values of kh for different values of c/β_1 and different initial stresses are given in the following tables :

(1) $I_2 = 0$

	I_1 \ c/β_1	1.03	1 05	1.07	1.09	1.11	1.13	1.15	1.17	1.19	1.21
	0.0	5.31	3.82	3.01	2.46	2.04	1.70	1.40	1.11	0.80	0.37
kh	0.2	2 10	1.85	1.64	1.45	1.27	1.10	0.93	0.76	0.56	0.26
	0.8	0.86	0.80	0.74	0.68	0.61	0.54	0 47	0.39	0.29	0.14

(2) (*i*) $I_1 = I_2 = 0.2$

c/β_1 :	0.91	0.93	0.95	0.97	0.99	1.01	1.03	1.05
kh :	7.37	4.47	3.30	2.60	2.11	1.72	1.38	1.06

(ii) $I_1 = I_2 = 0.4$

c/β_1 :	0.78	0.80	0.82	0.84	0.86	0.88	0.90	0.92
kh :	12.29	5.05	3.44	2.61	2.05	1.61	1.23	0.85

(iii) $I_1 = I_2 = 0.8$

c/β_1 :	0.45	0.46	0.47	0.48	0.49	0.50	0.52	0.53	0.54
kh :	13.06	5.50	3.80	2.92	2.34	1.91	1.22	0.89	0.45

(3) (i) $I_1 = 0.4$ and $I_2 = -0.2$

c/β_1 :	0.78	0.81	0.84	0.87	0.90	0.93	0.96	0.99	1.02	1.05
kh :	12.48	4.33	2.90	2.20	1.76	1.43	1.17	0.94	0.73	0.51

(ii) $I_1 = 0.8$ and $I_2 = 0.2$

c/β_1 :	0.45	0.50	0.55	0.60	0.65	0.75	0.85	0.95	1.05
kh :	13.44	2.53	1.59	1.15	0.89	0.57	0.37	0.23	0.10

(4) $I_1 = 0.2$ and $I_2 = 0.4$

c/β_1 :	0.895	0.900	0.905	0.910	0.915	0.920	0.925	0.930
kh :	42.10	12.19	8.24	6.34	5.12	4.21	3.46	2.77

4. Discussion

All the graphs are drawn on the assumption that the rigidity in the lower substratum is greater than that in the crustal layer whereas $\beta_2 < \beta_1$. The graphs are classified into four groups as mentioned below :

(1) The lower semi-infinite medium is free of any initial stress.

(2) Both the media possess equal initial stresses.

(3) Both the media possess initial stresses but the initial stress of the upper medium is greater than that in the lower medium.

(4) The initial stress in the upper medium is less than that in the lower medium.

In the cases (1), (2) and (3) we observe one interesting property—the phase velocity for a particular kh value diminishes with the increase of the initial stress in the upper medium, particularly in the low period range.

From a study of Fig. 2 we find that the phase velocity diminishes more rapidly with the increase of initial stress in the upper medium when the lower substratum is free of initial stress. Whereas Fig. 3 shows that when both the media are under equal initial stress the rate at which the phase velocity diminishes is comparatively low when the common value of the initial stress increases. Figure 4 indicates that, as in Fig. 2, in a low period range the rate of change of phase velocity is higher with the greater initial stress in the upper medium so long as I_2 remains the same. Figure 5 shows that when the initial stress in the upper medium is less than that in the lower

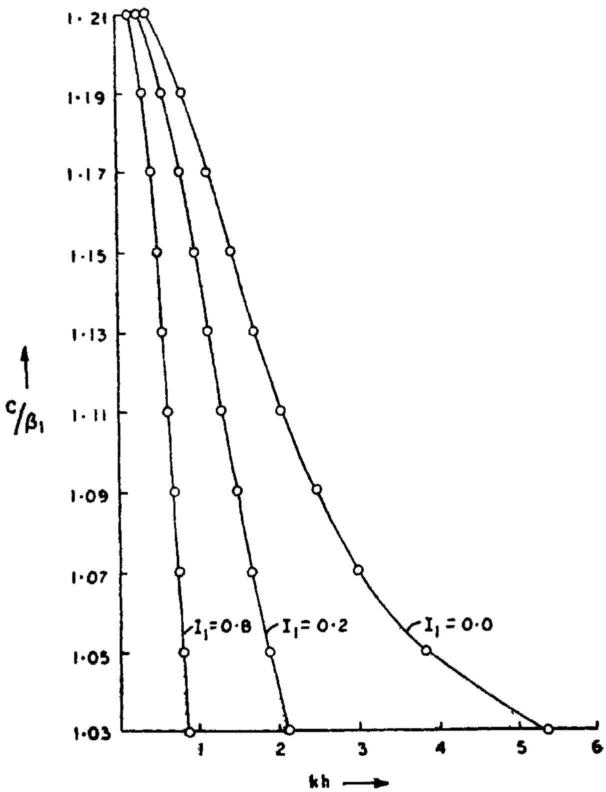

Fig. 2. Phase-velocity curves for Love-Type Waves in a Layered Medium when $I_2=0$.

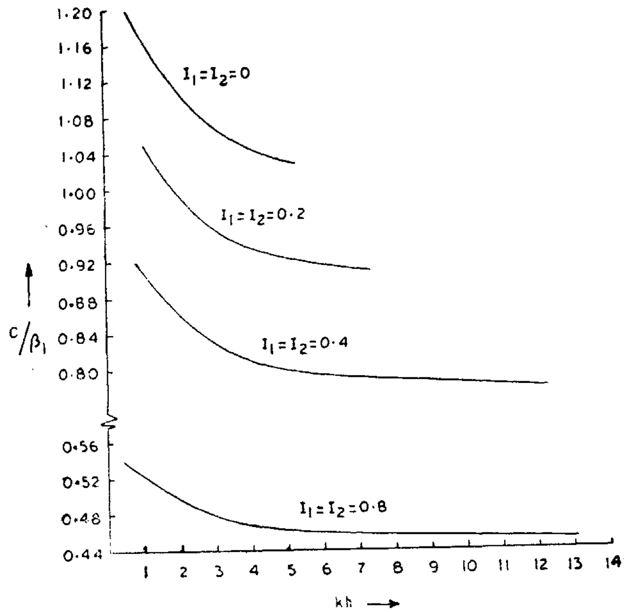

Fig. 3. Phase velocity curves for Love-Type Waves in a Layered Medium when $I_1=I_2$.

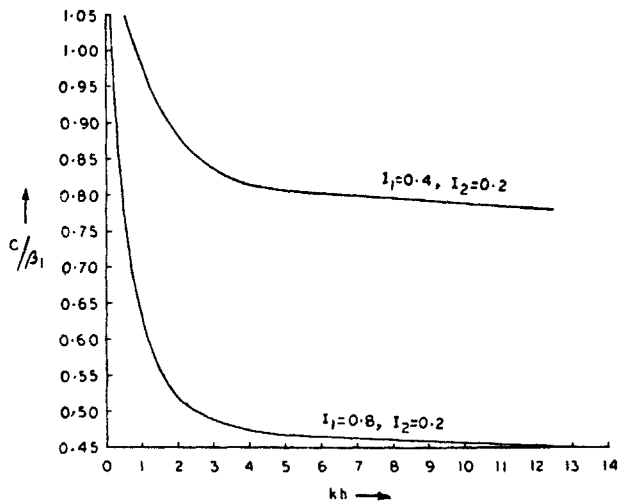

Fig. 4. Phase velocity curves for Love-Type waves in a Layered Medium when $I_1>I_2$.

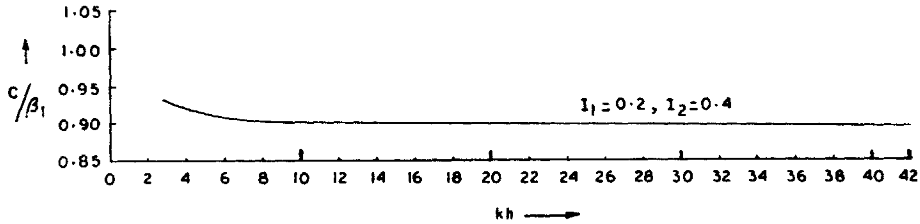

Fig. 5. Phase velocity curve for Love-Type wave in a Layered Medium when $I_1 < I_2$

substratum the phase velocity almost remains constant, particularly in a high period range.

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