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UNSTEADY FLOW OF A DUSTY GAS IN ELLIPTICAL AND CONFOCAL CHANNELS WITH MOVING BOUNDARIES

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The problem of a viscous incompressible dusty gas in an elliptical or confocal elliptical channel has been discussed. In this analysis it has been assumed that initially the liquid and the particles are at rest, whereas the boundaries move with arbitrary velocities dependent upon time. Exact expressions for both the velocities have been obtained; the case when the boundaries are moving with a constant velocity has been discussed in detail. It has been observed that the time of reaching the steady state decreases with the increase of ellipticity when the pressure gradient is con tant and the boundaries are at rest.

1. INTRODUCTION

The unsteady flow of a viscous incompressible fluid through channels under a time dependent pressure gradient has been considered by several investigators (Mithal 1960, 1972; Rawat 1970; Gupta 1964). In recent years a number of studies of fluid embedded with particles have appeared in the literature, Saffman (1962), Michael (1965), Michael and Miller (1966), Michael and Norvey (1968), Liu (1966, 1967), Healey and Young (1972), Rao (1969), Varma and Mathur (1972), Liu and Miller (1976), Gupta and Gupta (1976, 1977). The fluid flow embedded with particles is encountered in a wide variety of engineering problems concerned with atmospheric fall out, dust collection, nuclear reactor cooling, powder technology, acoustics, sedimentation, performance of solid fuel rock nozzles, batch settling, rainerosion and guided missiles and paint spraying etc.

Recently Gupta and Gupta (1979) have discussed flow of a dusty gas between two infinite coaxial cylinders under the influence of an arbitrary pressure gradient moving with arbitrary velocities. The present paper deals with the flow of a dusty gas through elliptical boundaries or confocal boundaries when the cylinders are moving with time dependent arbitrary velocities. Making use of the Mathieu transform explicit formula for the velocities have been obtained. The case when the cylinders are moving with constant velocities and the pressure gradient is also constant has been discussed in detail.

The graphs of the velocities have been drawn and it has been observed that the time reaching the steady state is decreased on the increase of the ellipticity. With the help of Mathieu transform some new doubly infinite series have been summed up.

The cases of circular pipe and a coaxial one are its particular cases.

EQUATIONS OF MOTION

The conservation equations of the flow of a gas with uniform distribution of small solid particles in the case when the gas may be considered incompressible have been formulated by Saffman (1962). In the present paper we have considered that the particles are spherical and uniform in size, and the bulk concentration (concentration by volume) of dust is very small. Following Saffman it is assumed that the steady Stokes resistance between the particles and the gas is applicable. However, the mass concentration of dust can be of order unity by allowing the ratio of the density of the dust and gas to be large.

Making use of the cartesian coordinate, and taking the gas and the particle velocities $u'(x', y', t')$, $v'(x', y', t')$ respectively in the direction of the axis of the pipe i. e. Z-axis and N_0 the number density particle to be constant throughout the motion the momentum equation from Saffman (1962) thus obtained appear in the form.

$$\frac{\partial u'}{\partial t'} = -\frac{1}{\rho}\frac{\partial p'}{\partial z'} + \mu\left(\frac{\partial^2 u'}{\partial x'^2} + \frac{\partial^2 u'}{\partial y'^2}\right) + \frac{KN_0}{\rho}(v' - u') \quad \text{...(1.1)}$$

and $$m\frac{\partial v'}{\partial t'} = K(u' - v') \quad \text{...(1.2)}$$

m being the mass of a particle, K the Stokes resistance coefficient, N_0 the number density of the particles, μ, ν and ρ are viscosity kinematic viscosity, and density of the gas respectively,

Introducing the following non-dimensional quantities

$$x = x'/a,\ y = y'/a,\ z = z'/a,\ t = \nu t'/a^2,\ u = u'a/\nu, v = v'a/\nu,\ p = p'a^2/\rho\nu^2.$$

Equations (1.1) and (1 2) are transformed into

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial z} + \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \beta(v - u) \quad \text{...(1.3)}$$

$$\frac{\partial v}{\partial t} = \gamma'(u - v) \quad \text{...(1.4)}$$

where $$\beta = f/\gamma = \frac{N_0 K a^2}{\rho\nu}\ f = \frac{mNo}{\rho},\ \gamma = \frac{m}{Ka^2},\ \gamma' = \gamma^{-1} \quad \text{...(1.5)}$$

are dimensionless constants. The above equations are to be solved under the following boundary conditions

$t \leqslant 0$ Initial conditions

$u = 0 = v$

Boundary conditions

$$\left.\begin{array}{l} u = \phi_1(t), \text{ one boundary} \\ u = \phi_2(t), \text{ on second boundary} \end{array}\right\} v = 0 \text{ on the boundaries} \quad \text{...(1.6)}$$

and the pressure gradient is an arbitrary function of time in exact form i.e. $\partial p/\partial z = +f(t)$

2. Elliptical Cylinder

In this section we shall discuss the flow of a dusty gas through an elliptical cylinder. The flow takes place from rest under the influence of the pressure gradient and also the boundary moves with an arbitrary velocity of time. The fundamental equations of motion in the non-dimensionalized form in this case are.

$$\frac{\partial u}{\partial t} = -f(t) + \frac{2}{h^2(\cosh 2\alpha - \cos 2\theta)}\left(\frac{\partial^2 u}{\partial \alpha^2} + \frac{\partial^2 u}{\partial^2 \theta^2}\right) + \beta(v-u) \qquad \text{...(2.1)}$$

$$\frac{\partial u}{\partial t} = \gamma'(u-v) \qquad \text{...(2.2)}$$

where $x = h \cosh \alpha \cos \theta$, $y = h \sinh \alpha \sin \theta$, $h = \sqrt{(1-(b/a)^2)} = e\, b/a = h \sinh \alpha_0$, $h \cosh \alpha_0 = 1$, and $\alpha = \alpha_0$ represents the boundary of the ellipse. The boundary conditions in elliptical coordinates are

$$\left.\begin{aligned} &u(\alpha_0,\theta,t) = \phi_1(t), \quad 0 \leqslant \theta \leqslant 2\pi, \quad \text{for } t>0, \\ &u(\alpha,\theta,0) = 0 \text{ for } t=+0 \text{ within the tube,} \\ &\frac{\partial u}{\partial \alpha} = 0; \ t=0, \\ &v(\alpha_0,\theta,t) = \phi_1(t), \ 0 \leqslant \theta \leqslant 2\pi, \\ &v(\alpha_0,\theta,0) = 0. \end{aligned}\right\} \qquad \text{...(2.3)}$$

Making use of the notation due to Morse and Feshbach (1953) and the technique given by Gupta (1964) we have

$$\int_0^{\alpha_0}\int_0^{2\pi} (u,v)(\cosh 2\alpha - \cos 2\theta)\, Je_{2n}(\alpha,q_{2n,m}) Se_{2n}(\theta,q_{2n,m})\, d\alpha\, d\theta = [\bar{u}(q_{2n,m}), \bar{v}(q_{2nm})] \qquad \text{...(2.4)}$$

where $q_{2n,m}$ is the mth root of the equation

$$Je_{2n}(\alpha_0,q) = 0 \qquad \text{...(2.5)}$$

and
$$Je_{2n}(\alpha,q) = \sum_{r=0}^{\infty} (-1)^n \sqrt{\frac{\pi}{2}}\, De^{2n}_{2r}(q)\, J_{2r}(\sqrt{q}\cosh \alpha) \qquad \text{...(2.6)}$$

$$Se_{2n}(\alpha,q) = \sum_{r=0}^{\infty} De^{2n}_{2r}(q) \cos 2r\theta\,. \qquad \text{...(2.7)}$$

Hence following Gupta (1964) the equations of motion after applying the finite Mathieu transform become

$$\int_0^{\alpha_0}\int_0^{2\pi}\left(\frac{\partial^2 u}{\partial \alpha^2} + \frac{\partial^2 u}{\partial \theta^2}\right) Je_{2n}(\alpha,q_{2n,m}) Se_{2n}(\theta,q_{2n,m})\, d\alpha\, d\theta$$

$$= -\tfrac{1}{2} q_{2n,m}\, \bar{u} - \phi_1(t)\, Je'_{2n}(\alpha_0, q_{2n,m}) 2\pi D_0^{2n}(q_{2n,m}) \qquad \text{...(2.8)}$$

which gives that the transformed equations become

$$\frac{d\bar{u}}{dt} = -\frac{q_{2n,m}}{h^2}\bar{u}+f(t)\,4\pi D_0^{2n}(q_{2n,m})\,Je'_{2n}(\alpha_0, q_{2n,m})/h^2$$
$$-\frac{4\pi}{h^2}D_0^{2n}(q_{2n,m})\phi_1(t)\,Je'_{2n}(\alpha_0, q_{2n,m}) + \beta(\bar{v}-\bar{u}) \quad \ldots(2.9)$$

$$\frac{d\bar{v}}{dt} = \gamma^{-1}(\bar{u}-\bar{v}) \quad \ldots(2.10)$$

which are to be solved u nder the following initial conditions.

$$\bar{u} = 0 = \bar{v}. \quad \ldots(2.11)$$

Applying the Laplace transform we find that

$$s\bar{u}_L=4\pi D_0^{2n}(q_{2n,m})\,Je'_{2n,m}(\alpha_0, q_{2n,m})\left[\frac{\bar{f}(s)}{q_{2n,m}}+\frac{\bar{\phi}_1(s)}{h^2}\right]-\frac{\bar{u}_L q_{2n,m}}{h^2}+\beta(\bar{v}_L-\bar{u}_L) \quad \ldots(2.12)$$

$$s\bar{v}_L = \gamma'(\bar{u}_L-\bar{v}_L). \quad \ldots(2.13)$$

Now to obtain u and v, first solve (2.12) and (2.13) and invert Laplace transform by convolution and finally by applying the inversion formula due to Gupta (1964) one obtains,

$$u=\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{\dfrac{\int_0^t \psi(t-z)\{(\lambda_1+\gamma')e^{\lambda_1 z}-(\lambda_2+\gamma')e^{\lambda_2 z}\}\,dz}{\lambda_1-\lambda_2}\,Je_{2n}(\alpha,q_{2n,m})Se_{2n}(\theta,q_{2n,m})}{\pi\int_0^{\alpha_0} Je_{2n}^2(\alpha,q_{2n,m})\,[Me_{2n}(q_{2n,m})\cosh 2\alpha-\theta_{2n,m}]\,d\alpha} \quad \ldots(2.14)$$

$$v=\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{(\gamma'\int_0^t (e^{\lambda_1 z}-e^{\lambda_2 z})\,\psi(t-z)dz)\,Je_{2n}(\alpha,q_{2n,m})\,Se_{2n}(\theta,q_{2n,m})}{(\lambda_1-\lambda_2)\,\pi\int_0^{\alpha_0} Je_{2n}^2(\alpha,q_{2n,m})\,[Me_{2n}(q_{2n,m})\cosh 2\alpha-\theta_{2n,m}]\,d\alpha} \quad \ldots(2.15)$$

where
$$\psi(s) = 4\pi\,D_0^{2n}(q_{2n,m})\,Je'_{2n}(\alpha_0,q_{2n,m})\left[\frac{\bar{f}(s)}{q_{2n,m}}+\frac{\bar{\phi}(s)}{h^2}\right] \quad \ldots(2.16)$$

and λ_1, λ_2 are the roots of

$$s^2+s(\gamma'+q_{2n,m}/h^2+\beta)+\gamma' q_{2n,m}/h^2\gamma^{-1}=0. \quad \ldots(2.17)$$

In case the pressure gradient is an absolute const. say C and $\phi_1(t) = u_0$, then (2.15) and (2.16) reduce to

$$u=-\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}K_1\left\{1-\frac{(\lambda_1+q_{2n,m}/h^2)\,e^{\lambda_2 t}-(\lambda_2+q_{2n,m}/h^2)e^{\lambda_1 t}}{\lambda_1-\lambda_2}\right\}$$
$$\times\frac{Je_{2n}(\alpha, q_{2n,m})\,Se_{2n}(\theta,q_{2n,m})}{\pi\int_0^{\alpha_0} Je_{2n}^2(\alpha,q_{2n,m})\,[Me_{2n}\cosh 2\alpha-\theta_{2n,m}]\,d\alpha.} \quad \ldots(2.17)$$

and

$$v = -\sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \gamma' K_1 \left[1 - \frac{\lambda_1 e^{\lambda_2 t} - \lambda_2 e^{\lambda_2 t}}{\lambda_1 - \lambda_2} \right]$$

$$\times \frac{Je_{2n}(\alpha, q_{2n,m}) Se_{2n}(\theta . q_{2n,m})}{\pi \int_0^{\alpha_0} Je^2_{2n}(\alpha,_{2n,m}) \left[Me_{2n}(q_{2n,m}) \cosh 2\alpha - \theta_{2n,m} \right] d\alpha} \quad \text{.. (2.18)}$$

where

$$K_1 = 2\pi \, D_0^{2n}(q_{2n,m}) \, Je'_{2n}(\alpha_0, q_{2n,m}) \left[\frac{C}{q_{2n,m}} + U_0 \right], \quad \text{...(2.19)}$$

Making use of the technique adopted by the author (1973) we have

$$U \cong \left[\frac{Ch^2}{8} \left\{ \cosh 2\alpha - \cosh 2\alpha_0 + \cosh 2\theta - \frac{\cosh 2\alpha \cos 2\theta}{\cosh 2\alpha_0} \right\} + u_0 \right]$$

$$\times \left[1 - \frac{(\lambda'_1 + \gamma^{2'}_{2n,m}) e^{\lambda'_2 t} - (\lambda'_2 + \gamma^{12}_{2n,m}) e^{\lambda'_2 t}}{\lambda'_1 - \lambda'_2} \right] \quad \text{...(2.20)}$$

$$V \cong \left[\frac{Ch^2}{8} \left\{ \cosh 2\alpha - \cosh 2\alpha_0 + \cosh 2\theta - \frac{\cosh 2\alpha \cos 2\theta}{\cosh 2\alpha_0} \right\} + u_0 \right]$$

$$\times \left[1 - \frac{\lambda'_1 e^{\lambda'_2 t} - \lambda'_2 e^{\lambda'_1 t}}{\lambda'_1 - \lambda'_2} \right] \quad \text{...(2.21)}$$

where λ'_1, λ'_2 are the roots of eqn. (2.17) after putting $q_{2n,m}/h^2$ by $q_{0,1}/h^2$.

The results due to Patreya (1973) are its particular cases and can be obtained by putting $\phi_1(t) = 0 = u_0$.

3. Confocal Elliptical Boundaries

In this section we have to solve (2.1) for the confocal boundaries subject to the following boundary conditions:

(1) Initial conditions

$$u = 0 = v$$

(2) Boundary conditions

$$\left.\begin{aligned}
&\text{(a) } u(\alpha_0, \theta, t) = \phi_1(t), && t > 0,\ 0 \leqslant \theta \leqslant 2\pi \\
&\text{(b) } u(\alpha_1, \theta, t) = \phi_2(t), && t > 0,\ 0 \leqslant \theta \leqslant 2\pi \\
&\text{(c) } v(\alpha_0, \theta, t) = 0 && t > 0,\ 0 \leqslant \theta \leqslant 2\pi \\
&\text{(d) } v(\alpha_1, \theta, t) = 0 && t > 0,\ 0 \leqslant \theta \leqslant 2\pi
\end{aligned}\right\}. \quad \text{...(3.1)}$$

Making use of the notation for Mathieu functions due to Morse and Feshbach (1953) and applying the finite Mathieu transform due to Gupta (1964) together with the Laplace transform we obtain

$$s(\bar{u}_L) = \left[\left\{ \frac{\bar{f}(s)}{q_{2n,m}} + \bar{\phi}_1(s) \right\} B'_{2n}(\alpha_0, q_{2n,m}) \left\{ \frac{f(\bar{s})}{q_{2n,m}} + \bar{\phi}_2(s) \right\} B'_{2n}(\alpha_1, q_{2n,m}) \right] 2\pi D^{2n}(q_{2n,m}) - \frac{1}{2} q_{2n,m} (\bar{u}_L) + \beta(-\bar{u}_L + \bar{V}_L) \quad \ldots(3.2)$$

$$s\bar{v}_L = \gamma' (\bar{u}_L - \bar{u}_L) \quad \ldots(3.3)$$

where

$$(\bar{u}, \bar{v}) = \int_{\alpha_0}^{\alpha_1} \int_0^{2\pi} (u, v) B_{2n}(\alpha, q_{2n,m}) Se_{2n}(\theta, q_{2n,m}) (\cosh 2\alpha - \cos 2\theta)\, d\alpha\, d\theta \quad \ldots(3.4)$$

$$(\bar{u}_L, \bar{v}_L) = \int_0^{\infty} (\bar{u}, \bar{v}) e^{-st} dt \quad \ldots(3.5)$$

where

$$B_{2n}(\alpha,q) = Ce_{2n}(\alpha_0, q_{2n,m}) Fey_{2n}(\alpha,q) - Fey_{2n}(\alpha_0,q) Ce_{2n}(\alpha,q) \quad \ldots(3.6)$$

and

$q_{2n,m}$ is the mth root of

$$B_{2n}(\alpha,q) = 0 \quad \ldots(3.7)$$

Now to obtain (u,v) first we solve (3.2) and (3.3) and then invert due to the technique used by Gupta (1964) and invert Laplace transform by convolution theorem we have

$$u = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{\dfrac{\int_0^t F(t-z)\left((\lambda_1+\gamma')e^{\lambda_1 z} - (\lambda_2+\gamma')e^{\lambda_2 z}\right)dz}{\lambda_1-\lambda_2}\, Be_{2n}(\alpha, q_{2n,m}) Se(\theta,q) 2n_{2n,m}}{\pi \int_{\alpha_0}^{\alpha_1} B^2_{2n}(\alpha, q_{2n,m}) [Me_{2n,m}(q_{2n,m}) \cosh 2\alpha - \theta_{2n,m}]\, d\alpha} \quad \ldots(3.8)$$

$$v = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{\gamma' \int_0^t F(t-z)(e^{\lambda_1 z} - e^{\lambda_2 z})dz\, B_{2n}(\alpha, q_{2n,m}) Se_{2n}(\theta, q_{2n,m})}{(\lambda_1-\lambda_2)\, \pi \int_{\alpha_0}^{\alpha_1} B^2_{2n}(\alpha, q_{2n,m}) [Me_{2n} \cosh 2\alpha - \theta_{2n,m}]\, d\alpha} \quad \ldots(3.9)$$

where

$$F(s) = 2\pi\, D_0^{2n}(q_{2n,m}) \left[\left\{ \frac{\bar{f}(s)}{q_{2n,m}} + \bar{\phi}_2(s) \right\} B'_{2n}(\alpha_1, q_{2n,m}) - \left\{ \frac{\bar{f}(s)}{q_{2n,m}} + \phi_1(s) \right\} B'_{2n}(\alpha_0, q_{2n,m}) \right]. \quad \ldots(3.10)$$

In case $\phi_1(t) = u_0$, $\phi_2(t) = u_1$, $f(t) = C$ then $\bar{F}(s)$ becomes

$$K_1/s = 2\pi D_0^{2n}(q_{2n,m}) \left[\left\{ C/q_{2n,m} + u_0 \right\} B'_{2n}(\alpha_1, q_{2n,m}) \left\{ \frac{C}{q_{2n,m}} + u_0 \right\} B'_{2n}(\alpha_0, q_{2n,m}) \right] \Big/ s. \quad \ldots(3.11)$$

Making use of eqn. (3.11) in (3.8) and (3.9) we find that

$$u = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} \frac{K_1 \, B_{2n}(\alpha, q_{2n,m}) \left\{ \frac{1 - (\lambda_1 + \gamma^2_{2n,m}) e^{\lambda_2 t} - (\lambda_2 + \gamma^2_{2n,m}) e^{\lambda_1 t}}{\lambda_1 - \lambda_2} \right\} Se_{2n}(\theta, q_{2n,m})}{\pi \int_{\alpha_0}^{\alpha} B^2_{2n}(\alpha, q_{2n,m}) \left[Me_{2n}(q_{2n,m}) \cosh 2\alpha - \theta_{2n,m} \right] d\alpha} \quad \ldots(3.12)$$

$$v = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{\gamma' \, K_1 B_{2n}(\alpha, q_{2n,m}) \; Se_{2n}(\theta, q_{2n,m}) \left[1 - \frac{\lambda_1 e^{\lambda_2 t} - \lambda_2 e^{\lambda_1 t}}{\lambda_1 - \lambda_2} \right]}{\pi \int_{\alpha_0}^{\alpha_1} B^2_{2n}(\alpha, q_{2n,m}) \left[Me(q_{2n,m}) \cosh 2\alpha - \theta_{2n,m} \right] d\alpha} \quad \ldots(3.13)$$

Where $K_1 = 2\pi \, D_0^{2n}(q_{2n,m}) \left[(C/q_{2n,m} + u_1) \, B'_{2n}(\alpha_1, q_{2n,m}) - \left\{ \frac{C}{q_{2n,m}} + u_0 \right\} B'_{2n}(\alpha_0 q_{2n,m}) \right.$

Making use of the same technique as done by the author in (1973) we have.

$$u \cong \left\{ \frac{Ch^2}{8} \left[\cos 2\theta \left\{ \frac{\sinh 2(\alpha - \alpha_0) + \sinh 2(\alpha_1 - \alpha)}{\sinh 2(\alpha_1 - \alpha_0)} \right\} - (\cosh 2\alpha + \cos 2\theta) \right. \right.$$

$$\times \left. \frac{(\alpha - \alpha_1) \cosh 2\alpha_0 = (\alpha_0 - \alpha) \cos h \, 2\alpha_1}{(\alpha_0 - \alpha_1)} \right] + \left. \frac{u_0 (\alpha - \alpha_1) + u_1 (\alpha_0 - \alpha)}{(\alpha_0 - \alpha_1)} \right\}$$

$$\times \left\{ 1 - \frac{\left(\lambda'_1 + \gamma^2_{2n,m} \right) e^{\lambda'_{2} t} - \left(\lambda'_2 + \gamma'^2_{2n,m} \right) e^{\lambda'_1 t}}{\lambda'_1 - \lambda'_2} \right\}$$

$$\ldots(3.14)$$

$$v \cong \left\{ Ch^2/8 \left[\cos 2\theta \left\{ \frac{\sinh 2(\alpha - \alpha_0) + \sinh 2(\alpha_1 - \alpha)}{\sinh 2(\alpha_1 - \alpha_0)} \right\} - (\cosh 2\alpha + \cos 2\theta) \right. \right.$$

$$+ \left. \frac{(\alpha - \alpha_1) \cos h \, 2\alpha_0 + (\alpha_0 - \alpha) \cos h \, 2\alpha_1}{(\alpha_0 - \alpha_1)} \right] + \frac{u_0 (\alpha - \alpha_1) + u_1 (\alpha_0 - \alpha)}{(\alpha_0 - \alpha_1)}$$

$$\times 1 - \left[\frac{\lambda'_1 e^{\lambda'_{2t}} - \lambda'_2 e^{\lambda'_{1t}}}{(\lambda'_1 - \lambda'_2)} \right] \quad \ldots(3.15)$$

where λ'_1, λ'_2 are the roots of (2.12) after replacing the first value of $\gamma_{2n,m} = q_{2n,m}/h^2$ where $q_{2n,m}$ is the mth root of (3.7)

If $u_1 = 0 = u_0 = \phi_1(t) = \phi_2(t)$ the results due to Patraya (1976) are its particular cases.

Appendix

In this appendix the following integrals are obtained which are the transforms of unity.

$$I_1 = \int_0^{2\pi} \int_0^{\alpha_0} (\cos h \, 2\alpha - \cos 2\theta) \, Je_{2n}(\alpha, q_{2n,m}) \, Se_{2n}(\theta, q_{2n,m}) \, d\alpha \, d\theta \quad \ldots(A.1)$$

$$I_2 = \int_0^{2\pi} \int_0^{\alpha_0} (\cos h \, 2\alpha - \cos 2\theta) \, Be_{2n}(\alpha, q_{2n,m}) \, Se_{2n}(\theta, q_{2n,m}) \, d\alpha \, d\theta \quad \ldots(A.2)$$

Now

$$I_1 = \int_0^{2\pi}\int_0^{\alpha} (\cosh 2\alpha - \cos 2\theta)\, Je_{2n}(\alpha, q_{2n,m})\, Se_{2n}(\theta, q_{2n,m})\, d\alpha\, d\theta$$

$$= -\frac{2}{q_{2n,m}} \int_0^{\alpha}\int_0^{2\pi} \left(\frac{\partial^2}{\partial\alpha^2} + \frac{\partial^2}{\partial\theta^2} \right) Je_{2n}(\alpha, q_{2n,m})\, Se_{2n}(\theta, q_{2n,m})\, d\alpha\, d\theta \qquad \text{...(A.3)}$$

since $Je_{2n}(\alpha, q_{2n,m})$ is the solution of

$$\left[\frac{\partial^2}{\partial\alpha^2} + \frac{\partial^2}{\partial\theta^2} + \frac{q_{2n,m}}{2} (\cosh 2\alpha - \cos 2\theta) \right] u = 0 \qquad \text{...(A.4)}$$

Making use of the Green's theorem to the right hand side of (A.3) we have.

$$I_1 = -\frac{2}{q_{2n,m}} \int_0^{2\pi} Je'_{n2}(\alpha_0, q_{2n,m})\, Se_{2n}(\theta, q_{2n,m})\, d\theta$$

$$= -\frac{4\pi}{q_{2n,m}} D_0^{2n}(q_{2n,m})\, Je'_{2n}(\alpha_0, q_{2n,m}) \qquad \text{...(A.5)}$$

and following the same technique as above we have.

$$I_2 = -\frac{4\pi D_0^{2n}(q_{2n,m})}{q_{2n,m}} \left[Be'_{2n}(\alpha_1, q_{2n,m}) - Be'_{2n}(\alpha_0, q_{2n,m}) \right] \qquad \text{.. (A.6)}$$

which proves that the r.h.s. of (A .5) and (A. 6) are the finite transform of unity in both the cases.

4. Discussion

Here in Figs 1 and 2 we have plotted the velocity ratio of gas and particle with that of steady velocity of gas and particle respectively for an elliptical pipe. In Figs. 3 and 4 the same has been depicted for confocal pipes and the following observations have been made:

(A) *Elliptical pipe*

(1) In the beginning the difference between the particle velocity and the gas velocity is more as the time increases the difference becomes smaller and smaller.

(2) On the increase of the ellipticity both the velocities increase and the time taken to reach the steady state is decreased.

(3) The time taken to reach the steady state is more than that of the fluid.

(B) *Confocal pipe*

Besides the above phenomenon the following observations are made:

On the increase of the ellipticity of the inner elliptical boundary and keeping the ellipticity of the outer ellipse fixed both the velocities increase and the time taken to reach the steady state becomes less.

In all the above cases $f = 0.2$ and $\gamma^{-1} = 0.6$.

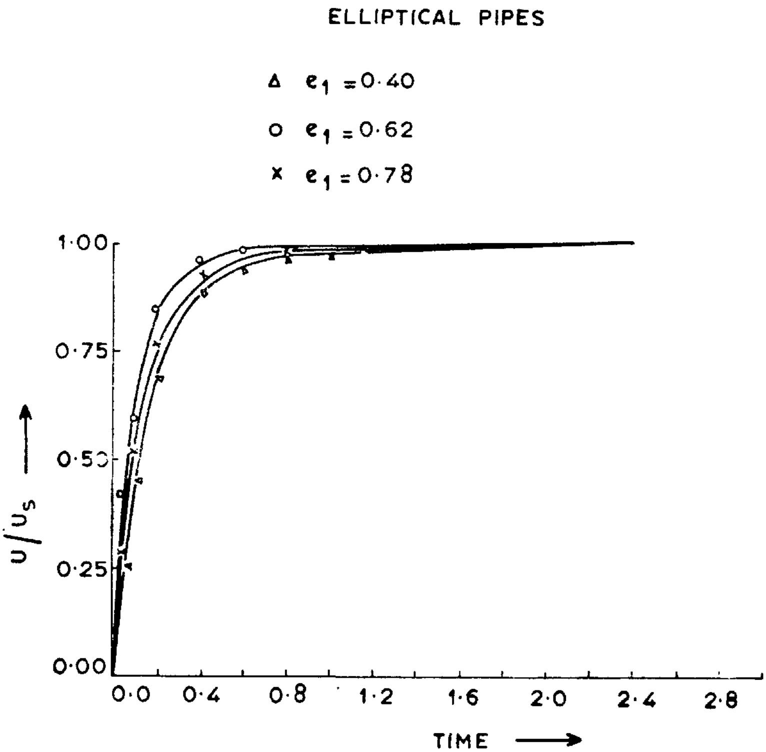

FIG. 1. Velocity ratio of gas against time for fixed ($f=0.2$, $\gamma^{-1}=0.6$).

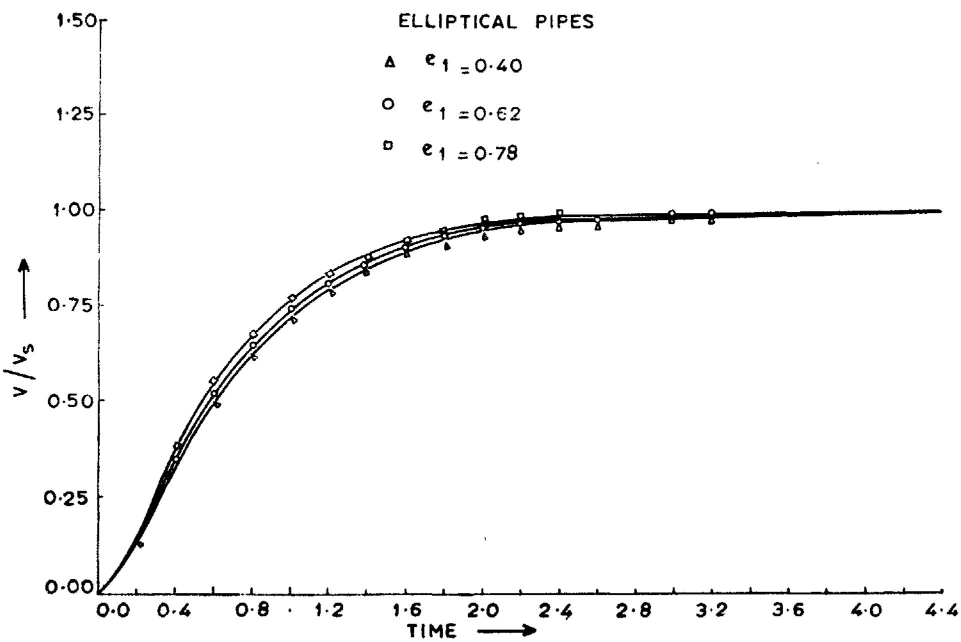

FIG. 2. Velocity ratio of dust particles against time for fixed ($f=0.2$, $r^{-1}=0.6$).

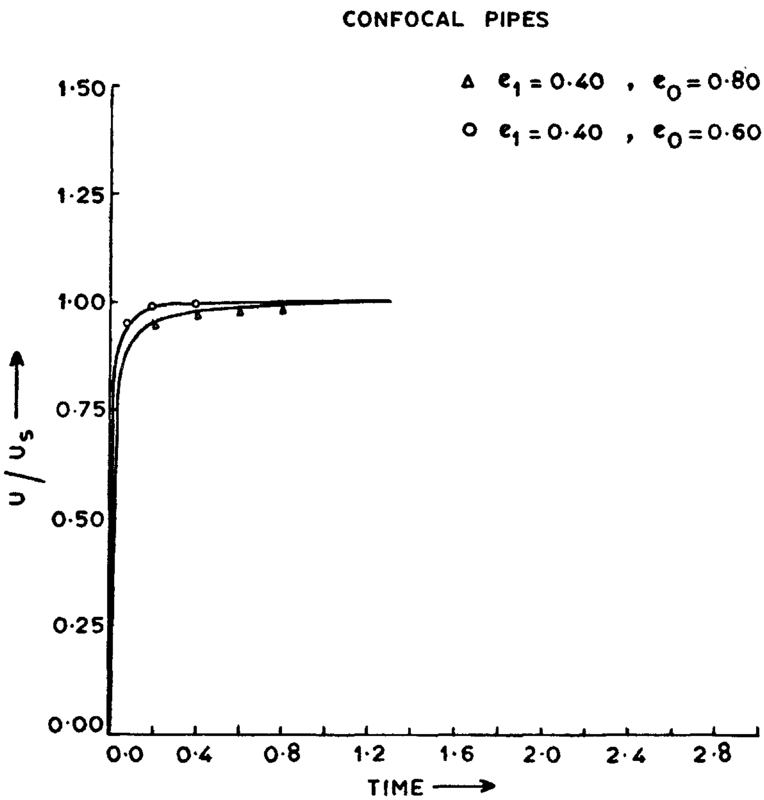

FIG. 3. Velocity ratio of gas against time for fixed ($f=0.2$, $\gamma^{-1}=0.6$)

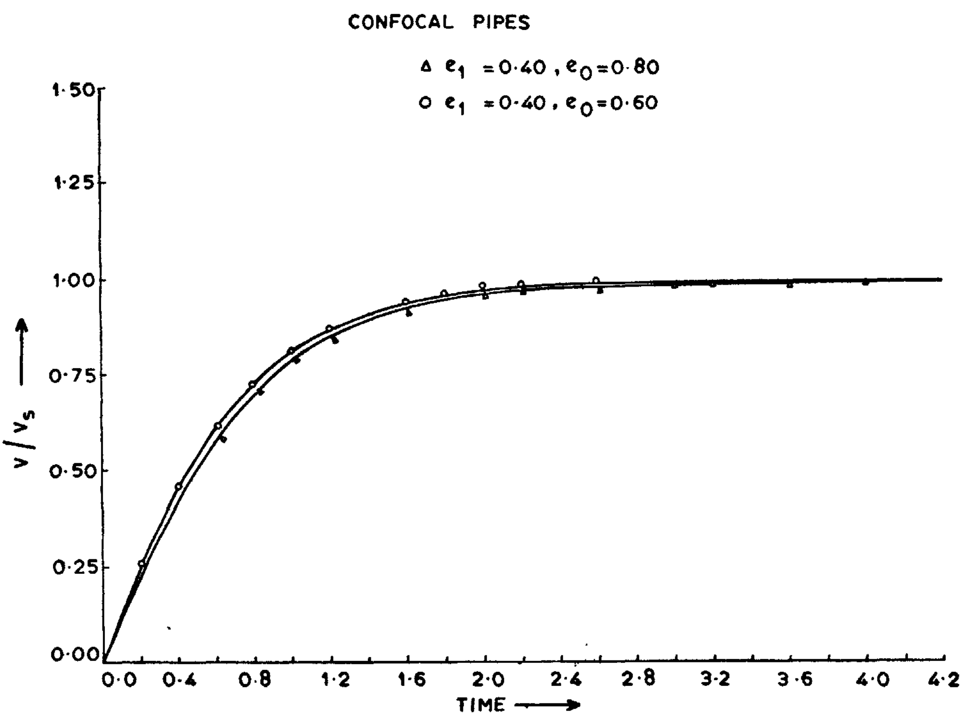

FIG. 4. Velocity ratio of dust particles against time for fixed ($f=0.2$, $\gamma^{-1}=0.6$)

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