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MHD COUETTE FLOW AND HEAT TRANSFER IN A ROTATING SYSTEM

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MHD Couette flow and heat transfer of an electrically conducting fluid in a rotating system is investigated considering the effect of polarization of the electric field when the plates are insulated and the fluid is permeated by a transverse magnetic field. Exact solution of the governing equations is obtained in closed form. The asymptotic behaviour of the solution is analysed for both small and large rotation parameter K^2 and the magnetic parameter M^2. Heat transfer characteristics are also studied taking account of viscous and Joule dissipations. It is observed that the temperature field is independent of rotation for small values of K^2 and M^2. It is also found that the rates of heat transfer at both the plates are considerably influenced by K^2 as well as M^2.

1. INTRODUCTION

Theoretical study on MHD flow through a channel or tube is of great interest in general due to its simple geometry and wide applications in designing generators, accelerators, pumps and flow meters. At first Lehnert (1952), Bleviss (1958 a, b) and Pai (1962) investigated the Couette flow and heat transfer in the presence of an applied magnetic field. Recently another field of interest on magnetohydrodynamic Couette flow and heat transfer came out in literature in rotating frame of reference. Jana and Datta (1977) considered the Couette flow and heat transfer in a rotating frame of reference. Subsequently Jana *et al.* (1977) studied this problem in the presence of an applied transverse magnetic field considering the stationary plate of the channel perfectly conducting and the moving plate to be an insulator. However, the effect of polarization of electric field is absent in their work due to the consideration of the stationary plate as perfectly conducting which implies that the applied electric field has no effect on the velocity field as well as on the magnetic field. Thus, further consideration of this problem is required to bring out certain essential features of MHD flows in a rotating system.

The aim of the present paper is to study the hydromagnetic Couette flow and heat transfer in a rotating frame of reference taking into account of the effect of polarization of electric field when the plates are insulated. Exact solution of the governing equations is obtained in closed form. It is found that the applied electric field has significant effect on the flow-field and heat transfer characteristics. The asymptotic behaviour of the solution (which is not studied by Jana *et al.* 1977) is analysed for both small and large values of rotation parameter K^2 and the magnetic parameter M^2. For large values of K^2 and M^2, near the moving plate of the channel, there arise thin boundary layers of thicknesses $O\,[K + (M^2/4K)]^{-1}$ and $O\,(M)^{-1}$ respectively and for small values of K^2 and M^2 the primary flow is independent of rotation while the secondary flow is unaffected by the magnetic field. It is worthy to note that there exists an incipient flow reversal near the stationary plate in the direction of movement of the plate with the increase in rotation parameter K^2 for fixed M^2. It also reveals that, at the moving plate, the resultant shear stress first increases, attains a maximum and then decreases with the increase in either K^2 or M^2 while it increases with the increase in K^2 for fixed $M^2 \geqslant 8$. In the case of Jana *et al.* the resultant shear stress at the moving plate increases with the increase in either K^2 or M^2. Heat transfer characteristics are also studied taking into account of viscous and Joule dissipations. It is interesting to note that the rates of heat transfer at both the plates have considerale effects of magnetic field as well as rotation while Jana *et al.* observed that the rate of heat transfer at the stationary plate is unaffected by the magnetie field as well as rotation. It is found that the rates of heat transfer at both the plates decrease with the increase in K^2 for fixed M^2 while, for fixed K^2, at the stationary plate it increases with the increase in M^2, and at the moving plate it first decreases, attains a minimum and then increases with the increase in M^2 which implies that the magnetic field helps in cooling the system (Rohsenhow and Hartnett 1973) but it is observed by Jana *et al.* (1977) that the rates of heat transfer at both the plates decrease with the increase in M^2. This result may be of interest in the cooling of turbine blades, MHD generators, flow meters, pumps, accelerators, centrifugal machines and space-vehicles etc.

2. Formulation of the Problem and its Solution

Consider the steady hydromagnetic Couette flow of an electrically couducting incompressible viscous fluid, confined between two parallel infinite plates distant h apart, rotating with uniform angular velocity Ω about y-axis, perpendicular to their planes, and is subjected to the action of a uniform magnetic flux density B_0 parallel to the axis of rotation. The xz-plane coincides with the stationary plate $y = 0$ while the plate $y = h$ moves with a uniform velocity U_0 in the x-direction.

Since the plates are infinite along x and z-directions, all physical quantities except pressure will be the functions of y only. The equation of continuity $\nabla.\mathbf{q} = 0$ and the solenoidal relation for the magnetic field $\nabla.\,\mathbf{B} = 0$ give $\mathbf{q} \equiv (u^*, 0, w^*)$ and $\mathbf{B} \equiv (B_x^*, B_0, B_z^*)$. The velocity components are consistent with the fact that a secondary flow $w^*(y)$ is induced by rotation in z-direction (Batchelor 1967). The equation of conservation of electric charge $\nabla.\mathbf{J} = 0$ gives $J_y^* =$ constant. This constant is zero since $J_y^* = 0$ at the plates which are electrically non-conducting. Thus $J_y^* = 0$ everywhere in the flow. Hence $\mathbf{J} \equiv (J_x^*,\ 0,\ J_z^*)$. Further since in the steady state we have $\nabla \times \mathbf{E} = 0$ which implies that E_x^* and E_z^* are constant everywhere in the flow where electric field $\mathbf{E} \equiv (E_x^*, E_y^*, E_z^*)$.

Now in a rotating frame of reference, the hydromagnetic equations of motion are given by

$$0 = \nu \frac{d^2 u^*}{dy^2} - 2\,\Omega\, w^* - \frac{B_0}{\rho} J_z^* \qquad \text{...(1)}$$

$$0 = -\frac{\partial p^*}{\partial y} + (B_x^* J_z^* - B_z^* J_x^*) \qquad \text{...(2)}$$

$$0 = \nu \frac{d^2 w^*}{dy^2} + 2\,\Omega\, u^* + \frac{B_0}{\rho} J_x^* \qquad \text{...(3)}$$

$$\mu_e J_x^* = \frac{d B_z^*}{d y} \qquad \text{...(4)}$$

$$\mu_e J_z^* = -\frac{d B_x^*}{d y} \qquad \text{...(5)}$$

$$J_x^* = \sigma (E_x^* - B_0 w^*) \qquad \text{...(6)}$$

$$0 = E_y^* + (B_x^* w^* - B_z^* u^*) \qquad \text{...(7)}$$

$$J_z^* = \sigma (E_z^* + B_0 u^*) \qquad \text{...(8)}$$

where ν, ρ, μ_e, σ and p^* are the kinematic coefficient of viscosity, the fluid density, the magnetic permeability, the electrical conductivity of the fluid and the modified pressure including the centrifugal force respectively. Equation (2) shows the constancy of the magneto-fluid-dynamic pressure along the axis of rotation. The absence of $\partial p^*/\partial z$ in eqn. (3) implies that there is a net cross flow along z-direction and since the motion of the fluid is induced by the movement of the upper plate in x-direction so $\partial p^*/\partial x$ is not taken into account in eqn. (1).

Introducing non-dimensional variables

$$\eta = y/h,\ p = p^*/\rho U_0^2$$

$$(u, 0, w) = (u^*, 0, w^*)/U_0$$

$$(J_x, 0, J_z) = (J_x^*, 0, J_z^*)/\sigma U_0 B_0 \qquad \text{...(9)}$$

$$(b_x, 1, b_z) = (B_x^*, B_0, B_z^*)/B_0$$

$$(E_x, E_y, E_z) = (E_x^*, E_y^*, E_z^*)/U_0 B_0$$

eqns. (1) to (8) become

$$\frac{d^2 u}{d\eta^2} - 2 K^2 w - M^2 J_z = 0 \qquad \ldots(10)$$

$$R_e \frac{\partial p}{\partial \eta} - M^2 (b_x J_z - b_z J_x) = 0 \qquad \ldots(11)$$

$$\frac{d^2 w}{d \eta^2} + 2 K^2 u + M^2 J_x = 0 \qquad \ldots(12)$$

$$R_m J_x = \frac{d b_z}{d \eta} \qquad \ldots(13)$$

$$R_m J_z = - \frac{d b_x}{d \eta} \qquad \ldots(14)$$

$$J_x = E_x - w \qquad \ldots(15)$$

$$0 = E_y + (b_x w - u b_z) \qquad \ldots(16)$$

$$J_z = E_z + u \qquad \ldots(17)$$

where $M = B_0 h (\sigma/ \rho\nu)^{1/2}$ is the Hartmann number, $R_e = U_0 h/\nu$ is the Reynolds number, $R_m = \sigma \mu_e U_0 h$ is the magnetic Reynolds number, and $K = (\Omega h^2/\nu)^{1/2}$ is the reciprocal of the Ekman number.

The boundary conditions to be satisfied are

$$\left. \begin{array}{l} u(0) = w(0) = 0,\ u(1) = 1,\ w(1) = 0 \\ b_x(0) = b_z(0) = 0,\ b_x(1) = b_z(1) = 0. \end{array} \right\} \qquad \ldots(18)$$

Assuming

$$\left. \begin{array}{l} F = u + i w,\ J = J_z - i J_x \\ E_0 = E_z - i E_x,\ b = b_x + i b_z \end{array} \right\} \qquad \ldots(19)$$

and making use of it in eqns. (10) to (17), we have

$$\frac{d^2 F}{d \eta^2} + 2 i K^2 F - M^2 J = 0 \qquad \ldots(20)$$

$$R_m J = - \frac{d b}{d \eta} \qquad \ldots(21)$$

$$J = E_0 + F. \qquad \ldots(22)$$

Substitution of eqn. (22) in eqn. 20 and (21) gives

$$\frac{d^2 F}{d \eta^2} - (M^2 - 2 i K^2) F = M^2 E_0 \qquad \ldots(23)$$

$$R_m (E_0 + F) = - \frac{d b}{d \eta}. \qquad \ldots(24)$$

On using eqn. (19), the boundary conditions (18) become

$$\left. \begin{array}{l} F(0) = 0 \text{ and } F(1) = 1 \\ b(0) = 0 \text{ and } b(1) = 0. \end{array} \right\} \qquad \ldots(25)$$

Equations (23) and (24) together with boundary conditions (25) can be easily solved and the solution for $F(\eta)$ and $b(\eta)$ are

$$F(\eta) = \frac{\sinh(\alpha - i\beta)\eta}{\sinh(\alpha - i\beta)} + \frac{M^2 E_0}{(\alpha - i\beta)^2}\left[\{1 - \cosh(\alpha - i\beta)\}\frac{\sinh(\alpha - i\beta)\eta}{\sinh(\alpha - i\beta)} + \cosh(\alpha - i\beta)\eta - 1\right] \quad ...(26)$$

$$\frac{b(\eta)}{R_m} = \frac{1 - \cosh(\alpha - i\beta)\eta}{(\alpha - i\beta)\sinh(\alpha - i\beta)} - E_0\left[\eta + \frac{M^2}{(\alpha - i\beta)^2}\left\{\frac{[1 - \cosh(\alpha - i\beta)][\cosh(\alpha - i\beta)\eta - 1]}{(\alpha - i\beta)\sinh(\alpha - i\beta)} + \frac{\sinh(\alpha - i\beta)\eta}{(\alpha - i\beta)} - \eta\right\}\right] \quad ...(27)$$

where

$$E_0 = \left[\frac{\{\cosh(\alpha - i\beta) - 1\}(\alpha - i\beta)^2}{2iK^2(\alpha - i\beta)\sinh(\alpha - i\beta) + 2M^2\{1 - \cosh(\alpha - i\beta)\}}\right] \quad ...(28)$$

$$\alpha = \frac{1}{\sqrt{2}}\left\{(M^4 + 4K^4)^{1/2} + M^2\right\}^{1/2} \quad ...(29)$$

$$\beta = \frac{1}{\sqrt{2}}\left\{(M^4 + 4K^4)^{1/2} - M^1\right\}^{1/2}. \quad ...(30)$$

The solutions (26) and (27) exhibit the effect of polarization of the electric field on the flow-field as well as on the induced magnetic field which is absent in the case of Jana *et al.* (1977). On using (26) and (27), the pressure p, the current density $(J_x, 0, J_z)$ and the electric field E_y can easily be obtained from eqns. (11), (15), (17) and (16) respectively.

The non-dimensional resultant shear stress τ_0 at $\eta = 0$ and τ_1 at $\eta = 1$ with their respective angles of inclination θ_0 and θ_1 to the x-axis are given by

$$\tau_0 = \{(c_1 + c_2)/(\alpha^2 + \beta^2)\}^{1/2} \quad ...(31)$$

$$\tau_1 = [\{c_1 \cosh 2\alpha + c_2 \cos 2\beta - a_9 \sinh 2\alpha + b_9 \sin 2\beta\}/(\alpha^2 + \beta^2)]^{1/2} \quad ...(32)$$

$$\tan\theta_0 = (\alpha b_8 + \beta a_8)/(\alpha a_8 - \beta b_8) \quad ...(33)$$

$$\tan\theta_1 = (A/B) \quad ...(34)$$

where c_1, c_2, a_8, b_8, a_9, b_9, A and B, the known functions of K^2 and M^2, are given in appendix.

We shall now discuss a few particular cases of interest.

Case I: $M^2 \ll 1$ *and* $K^2 \ll 1$.

Since both M^2 and K^2 are very small, neglecting squares and higher powers of M^2 and K^2 in eqns. (26) and (27) we obtain the components of velocity and induced magnetic field as

$$u = \left\{1 + \frac{M^2}{12}(1 - 3\eta + 2\eta^2)\right\}\eta + \dots \quad ...(35)$$

$$w = \frac{K^2\eta}{3}(1 - \eta^2) + \dots, \quad ...(36)$$

$$b_x/R_m = \frac{1}{2}\eta(1-\eta) - \frac{M^2\eta^2}{24}(1-2\eta+\eta^2) + \dots, \qquad \dots(37)$$

$$b_z/R_m = \frac{K^2\eta}{12}(1-2\eta+\eta^3)\dots. \qquad \dots(38)$$

The expressions (35) to (38) show that in a slowly rotating system when the conductivity of the fluid is low and the applied magnetic field is weak, the effect of the magnetic field on the secondary flow and on the induced magnetic field b_z/R_m is negligible while the primary flow and the induced magnetic field b_x/R_m are unaffected by rotation. It may also be noted that in the absence of the magnetic field and rotation, the problem reduces to the classical Couette flow (Schlichting 1968).

Case 2 : $K^2 \gg 1$ *and* $M^2 \sim O(1)$

When K^2 is large and M^2 is of small order of magnitude, the highest order terms in eqns. (10) and (12) are multiplied by $1/K^2$ and so we can expect boundary layer type flow. For the boundary layer at the moving plate $\eta = 1$, writing $(1-\eta) = \zeta$, we obtain

$$u = \frac{M^2}{4K^3} + \exp\left\{-K\left(1+\frac{M^2}{4K^2}\right)\zeta\right\}\left[\cos\left\{K\left(1-\frac{M^2}{4K^2}\right)\zeta\right\} - \frac{M^2}{4K^3}\left(\cos\left\{K\left(1-\frac{M^2}{4K^2}\right)\zeta\right\} + \sin\left\{K\left(1-\frac{M^2}{4K^2}\right)\zeta\right\}\right)\right] \qquad \dots(39)$$

$$w = -\frac{M^2}{4K^3} + \exp\left\{-K\left(1+\frac{M^2}{4K^2}\right)\zeta\right\}\left[\sin\left\{K\left(1-\frac{M^2}{4K^2}\right)\zeta\right\} - \frac{M^2}{4K^3}\left(\sin\left\{K\left(1-\frac{M^2}{4K^2}\right)\zeta\right\} - \cos\left\{K\left(1-\frac{M^2}{4K^2}\right)\zeta\right\}\right)\right] \qquad \dots(40)$$

$$\frac{b_x}{R_m} = \frac{1}{2K}\left[\left(1+\frac{M^2}{4K^2}\right)\eta - \exp\left\{-K\left(1+\frac{M^2}{4K^2}\right)\zeta\right\}\left(\left(1+\frac{M^2}{4K^2}\right)\cos\left\{K\left(1-\frac{M^2}{4K^2}\right)\zeta\right\} - \left(1-\frac{M^2}{4K^2}\right)\sin\left\{K\left(1-\frac{M^2}{4K^2}\right)\zeta\right\}\right)\right], \qquad \dots(41)$$

$$\frac{b_z}{R_m} = \frac{1}{2K}\left[\left(1-\frac{M^2}{4K^2}\right)\eta - \exp\left\{-K\left(1+\frac{M^2}{4K^2}\right)\zeta\right\}\left(\left(1+\frac{M^2}{4K^2}\right)\sin\left\{K\left(1-\frac{M^2}{4K^2}\right)\zeta\right\} + \left(1-\frac{M^2}{4K^2}\right)\cos\left\{K\left(1-\frac{M^2}{4K^2}\right)\zeta\right\}\right)\right]. \qquad \dots(42)$$

The expressions (39) to (42) demonstrate the existence of a thin boundary layer of thickness of $O\{K[1+(M^2/4K^2)]\}^{-1}$ near the moving plate. This layer may be identified as the modified Ekman-Hartmann layer (Nanda and Mohanty 1971). In addition, it can be easilly seen from the expressions (39) to (42) that the boundary layer solution exhibits spatial oscillations in the flow field.

Now the exponential terms in the expressions (39) to (42) damp out quickly as ζ increases, i.e. when $\zeta \gg 1/K[1+(M^2/4K^2)]$, we have

$$u \approx M^2/4K^3, \quad w \approx -M^2/4K^3 \qquad \dots(43)$$

$$b_x/R_m \approx \frac{1}{2K}\left(1 + \frac{M^2}{4K^2}\right)\eta, \quad b_z/R_m \approx \frac{1}{2K}\left(1 - \frac{M^2}{4K^2}\right)\eta. \quad ...(44)$$

Expression (43) reveals that in a certain core, given by $\zeta \gg 1/K\,(1 + M^2/4K^2)$, the primary and the secondary flows are very weak and of same order of magnitude $(M^2/4K^3)$ but in opposite direction. The induced magnetic field, given by (44), in the main body of the fluid, i.e. out side the boundary layer, varies linearly with η.

Case 3 : $M^2 \gg 1$ *and* $K^2 \sim O(1)$.

In this case also flow is of boundary layer type and we obtain

$$u = \frac{1}{2}\left[1 + \exp(-M\zeta)\left\{\cos\left(\frac{K^2}{M}\zeta\right) + \frac{K^2}{M}\sin\left(\frac{K^2}{M}\zeta\right)\right\}\right] \quad ...(45)$$

$$w = \frac{1}{2}\left[\frac{K^2}{M} + \exp(-M\zeta)\left\{\sin\left(\frac{K^2}{M}\zeta\right) - \frac{K^2}{M}\cos\left(\frac{K^2}{M}\zeta\right)\right\}\right] \quad ...(46)$$

$$\frac{b_x}{R_m} = \frac{1}{2M}\left[1 - \frac{1}{M}\exp(-M\zeta)\left(\left\{M\cos\left(\frac{K^2}{M}\zeta\right) - \frac{K^2}{M}\sin\left(\frac{K^2}{M}\zeta\right)\right\}\right.\right.$$
$$\left.\left. + \frac{K^2}{M}\left\{M\sin\left(\frac{K^2}{M}\zeta\right) + \frac{K^2}{M}\cos\left(\frac{K^2}{M}\zeta\right)\right\}\right)\right] \quad ...(47)$$

$$\frac{b_z}{R_m} = \frac{1}{2M^2}\left[\left(K^2 + \frac{K^2}{M}\right) - 2K^2\eta - \exp(-M\zeta)\left(\left\{M\sin\left(\frac{K^2}{M}\zeta\right)\right.\right.\right.$$
$$\left.\left.\left. + \frac{K^2}{M}\cos\left(\frac{K^2}{M}\zeta\right)\right\} - \frac{K^2}{M}\left\{M\cos\left(\frac{K^2}{M}\zeta\right) - \frac{K^2}{M}\sin\left(\frac{K^2}{M}\zeta\right)\right\}\right)\right]. \quad ...(48)$$

The expressions (45) to (48) demonstrate the existence of a thin boundary layer of thickness of $O(M)^{-1}$ near the moving plate. This layer may be identified as the Hartmann layer (Nanda and Mohanty 1971, Seth and Jana 1980).

When $\zeta \gg 1/M$, i.e. outside the boundary layer, we have

$$u \approx 1/2, \quad w \approx K^2/2M, \quad ...(49)$$

$$b_x/R_m \approx 1/2M, \quad b_z/R_m \approx \{(K^2 + K^2/M) - 2K^2\eta\}/2M^2. \quad ...(50)$$

It is interesting to note from the eqn. (49) that in the main body of the fluid, the primary flow is independent of the magnetic field as well as rotation while the secondary flow has considerable effects of these parameters. It is evident from (50) that the induced magnetic field component b_x/R_m is independent of rotation and has considerable effect of magnetic field. Also the induced magnetic filed component b_z/R_m varies linearly with η.

3. Heat Transfer Characteristics

We now consider the heat transfer characteristics in the MHD Couette flow when the upper and lower plates are maintained at uniform temperatures T_1 and T_0 respectively.

The energy equation for the steady fully developed hydromagnetic Couette flow is given by

$$0 = \alpha^* \frac{d^2 T}{d y^2} + \frac{\mu}{\rho c_p} \left[\left(\frac{du}{dy}\right)^2 + \left(\frac{dw}{dy}\right)^2 \right] + \frac{1}{\mu_e^2 \rho \sigma c_p} \left[\left(\frac{dB_x}{dy}\right)^2 + \left(\frac{dB_z}{dy}\right)^2 \right] \qquad ...(51)$$

where α^* is the thermal conductivity, T is the temperature of the fluid taken as a function of y only and c_p is the specific heat at constant pressure. The last two terms within the parentheses are the viscous and Joule dissipation terms respectively.

The boundary conditions for T are

$$T = T_0 \text{ at } y = 0 \text{ and } T = T_1 \text{ at } y = h, \; (T_0 < T < T_1). \qquad ...(52)$$

Introducing non-dimensional quantities

$$\theta(\eta) = (T - T_0)/(T_1 - T_0), \; E \text{ (Eckert number) } = \frac{U_0^2}{c_p (T_1 - T_0)}$$

$$P_r \text{ (Prandtl number) } = \frac{\nu}{\alpha^*}$$

and using (9), the eqn. (51) reduces to

$$\frac{d^2 \theta}{d\eta^2} + P_r E \left[\left(\frac{du}{d\eta}\right)^2 + \left(\frac{dw}{d\eta}\right)^2 + \frac{M^2}{R_m} \left\{ \left(\frac{db_x}{d\eta}\right)^2 + \left(\frac{db_z}{d\eta}\right)^2 \right\} \right] = 0. \qquad ...(53)$$

The boundary conditions (52) become

$$\theta(0) = 0 \text{ and } \theta(1) = 1. \qquad ...(54)$$

Substituting the values of $u(\eta)$, $w(\eta)$, $b_x(\eta)$, and $b_z(\eta)$ from eqns. (26) and (27) in eqn. (53) and solving the resultant differential equation subject to the boundary conditions (54) we obtain

$$\theta(\eta) = \eta - \frac{P_r E}{(\alpha^2 + \beta^2)} \Big[d_2 \cosh 2\alpha\eta + d_3 \sinh 2\alpha\eta - d_4 \cos 2\beta\eta - d_5 \sin 2\beta\eta + 2d_1 (d_6 \cosh \alpha\eta \cos \beta\eta - d_7 \sinh \alpha\eta \sin \beta\eta + d_8 \sinh \alpha\eta \cos \beta\eta - d_9 \cosh \alpha\eta \sin \beta\eta) + d_{10} \eta^2 - d_{12}\eta - d_{11} \Big] \qquad ...(55)$$

where d_i ($i = 1, 2, ..., 12$), the known functions of M^2 and K^2, are given in appendix.

When M^2 and K^2 are very small, i.e. $M^2 \ll 1$ and $K^2 \ll 1$, neglecting squares and higher powers of M^2 and K^2 in eqn. (55) we have

$$\theta(\eta) = \eta + \frac{1}{2} P_r E \left\{ \eta(1-\eta) + \frac{M^2}{12} (\eta - 5\eta^2 + 8\eta^3 - 4\eta^4) + ... \right\}. \qquad ...(56)$$

It may be noted from eqn. (56) that in a slowly rotating system when the conductivity of the fluid is low and the applied magnetic field is weak, the temperature field is unaffected by the rotation. In the absence of the magnetic field, eqn. (56) reduces to

$$\theta(\eta) = \eta + \frac{1}{2} P_r E \eta (1 - \eta) \qquad ...(57)$$

which represents the temperature distribution in classical Couette flow (Schlichting 1968).

The rates of heat transfer at the plates $\eta = 0$ and $\eta = 1$ are given by

$$\left.\frac{d\theta}{d\eta}\right|_{\eta=0} = 1 - \frac{P_r E}{(\alpha^2+\beta^2)}\left[2\alpha d_3 - 2\beta d_5 + 2d_1(\alpha d_8 - \beta d_9) - d_{12}\right] \qquad \ldots(58)$$

$$\left.\frac{d\theta}{d\eta}\right|_{\eta=1} = 1 - \frac{P_r E}{(\alpha^2+\beta^2)} D, \qquad \ldots(59)$$

where
$$D = [2\alpha(d_2 \sinh 2\alpha + d_3 \cosh 2\alpha) + 2\beta(d_4 \sin 2\beta - d_5 \cos 2\beta) + 2d_1\{(\alpha d_6 - \beta d_7)\sinh\alpha\cos\beta - (\beta d_6 + \alpha d_7)\cosh\alpha\sin\beta + (\alpha d_8 - \beta d_9)\cosh\alpha\cos\beta - (\beta d_8 + \alpha d_9)\sinh\alpha\sin\beta\} + 2d_{10} - d_{12}].$$

It is worthy to note from eqns. (58) and (59) that the rates of heat transfer at both the plates have considerable effects of magnetic field as well as rotation while in the case of Jana *et al.* (1977) the rate of heat transfer at the stationary plate is unaffected by the magnetic field. The critical Eckert number, for which there is no flow of heat either from plate to the fluid or from fluid to the plate, is obtained from eqn. (59) as

$$E_c = (\alpha^2 + \beta^2)/P_r D. \qquad \ldots(60)$$

Equations (59) and (60) reveal that the heat will flow from upper plate to the fluid if $\left.\frac{d\theta}{d\eta}\right|_{\eta=1} > 0$ i.e. when $E < E_c$ while the heat will flow from fluid to the plate if $\left.\frac{d\theta}{d\eta}\right|_{\eta=1} < 0$ i.e. when $E > E_c$. Physically this heat flow reversal is justified if there is significant viscous and Joule dissipations near the plate in which case the fluid temperature in the vicinity of the plate may exceed the plate temperature. This will cause the flow of heat from the fluid to the plate even if the plate temperature is higher than the ambient temperature.

4. Discussions

The effects of magnetic and rotation parameters on the flow field and induced magnetic field are depicted in Figs. 1 to 4 for various values of M^2 and K^2. It is evident from Fig. 1 that, with increase in M^2 for fixed K^2, the velocity components u and w increase near the stationary plate while these components decrease near the moving plate. Figure 2 shows that the velocity components u and w decrease with the increase in K^2 for fixed M^2. However near the moving plate the secondary velocity w changes its characteristics. It is worthy to note that there exist an incipient flow reversal near the stationary plate in x-direction as K^2 increases. Figures 3 and 4 show that the induced magnetic field components b_x/R_m and b_z/R_m decrease with increase in M^2 for fixed K^2 while for fixed M^2 wiih increase in K^2 the induced magnetic field b_z/R_m increases but b_x/R_m decreases. However near the moving plate the behaviour of b_x/R_m is changed and the magnetic field b_z/R_m changes its direction.

The resultant shear stresses τ_1 at $\eta=1$ and τ_0 at $\eta=0$ are shown in Figs. 5 and 6 respectively for different values of K^2 against M^2. It is found from Fig. 5 that the

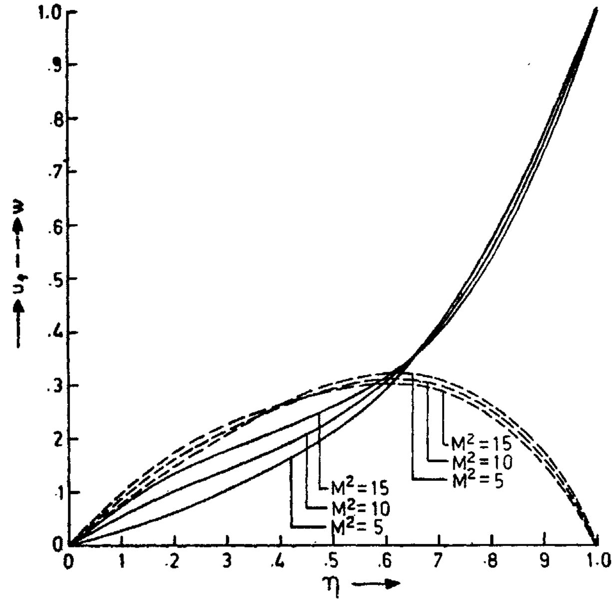

Fig. 1. Velocity profiles ($-u$, $---w$) for $K^2=5.0$.

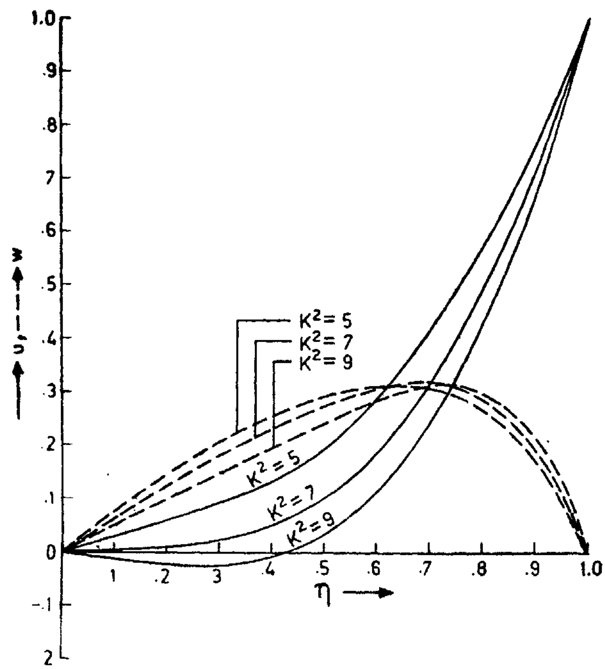

Fig. 2. Velocity profiles ($-u$, $---w$) for $M^2=4.0$.

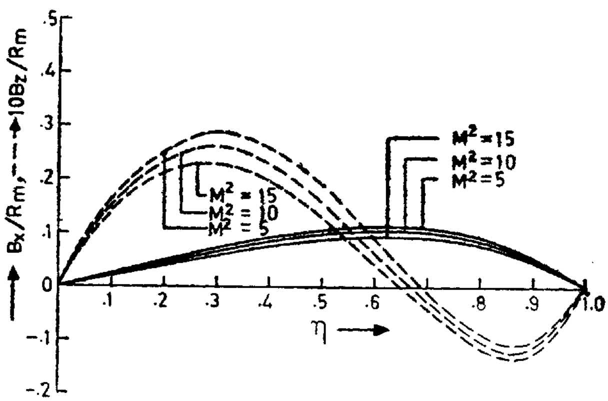

Fig. 3. Induced magnetic field profiles ($-B_x/R_m$, ——— 10 B_z/R_m) for $K^2=5.0$.

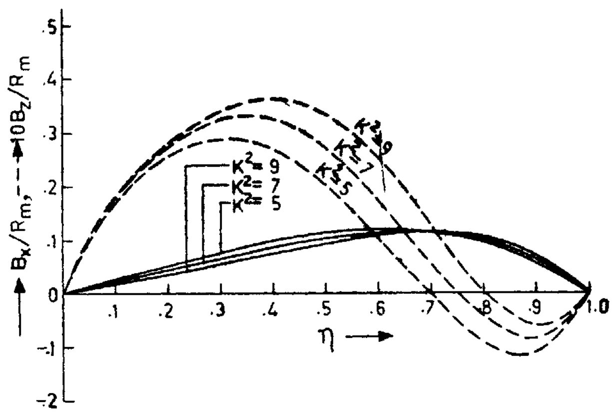

Fig. 4. Induced magnetic field profiles ($-B_x/R_m$, ——— 10 B_z/R_m) for $M^2=4.0$.

resultant shear stress τ_1 first increases, attains a maximum and then decreases with the increase in either K^2 or M^2 while it increases with the increase in K^2 for fixed $M^2 \geqslant 8$. In the case of Jana *et al.* (1977) the resultant shear stress at the moving plate increases with the increase in either K^2 or M^2. Fig. 6 shows that resultant shear stress τ_0 increases with the increase in M^2 for fixed K^2 while it decreases with increase in K^2 for fixed M^2.

The rates of heat transfer at both the plates and critical Eckert number E_c are presented in Tables I to III to observe the influence of the rotation and the magnetic parameters on the temperature field. In Tables I and II it is found that the rates of

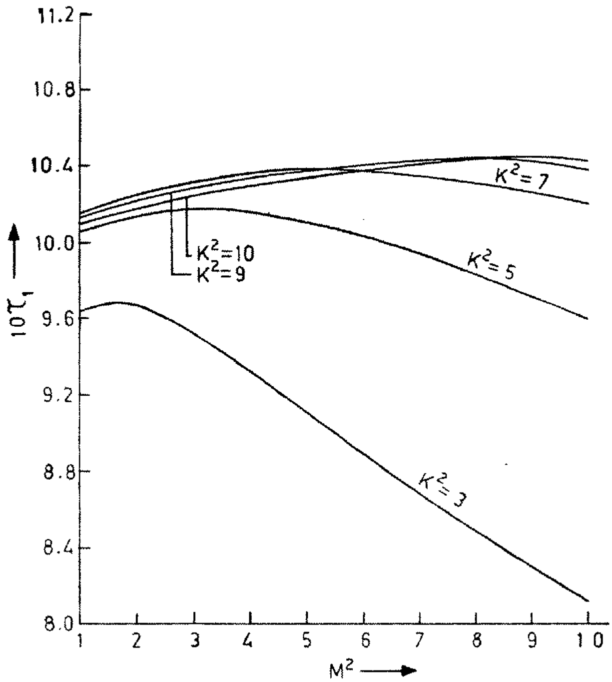

Fig. 5. Resultant shear stress τ_1 at $\eta=1$.

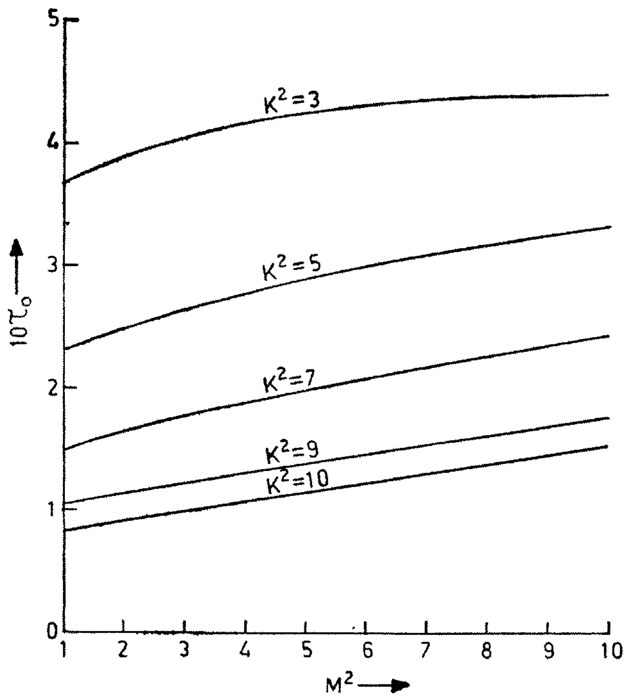

Fig. 6. Resultant shear stress τ_0 at $\eta=0$.

TABLE I

Rate of heat transfer $\left.\frac{d\theta}{d\eta}\right|_{\eta=0}$ *for* $P_r = 0.025$ *and* $E = 2.50$

K^2 \ M^2	1	3	5	7	9
6	1.032804	1.036091	1.039595	1.043163	1.046619
7	1.032597	1.035351	1.038209	1.041143	1.044076
8	1.032455	1.034870	1.037314	1.039798	1.042304
9	1.032350	1.034529	1.036698	1.038873	1.041056

TABLE II

Rate of heat transfer $\left.\frac{d\theta}{d\eta}\right|_{\eta=1}$ *for* $P_r = 0.025$ *and* $E = 2.50$

K^2 \ M^2	1	3	5	7	9
6	0.876999	0.875064	0.892012	0.945432	1.039385
7	0.862663	0.855408	0.857430	0.883175	0.947648
8	0.850046	0.839461	0.831367	0.836380	0.867709
9	0.838732	0.826143	0.811405	0.801613	0.807153

TABLE III

Critical Eckert number $10^{-2} \times E_c$ *for* $P_r = 0.025$

K^2 \ M^2	1	3	5	7	9
6	0.203250	0.200102	0.231507	0.451850	0.506216
7	0.182034	0.172900	0.175352	0.213995	0.477542
8	0.166718	0.155725	0.148251	0.152793	0.188977
9	0.155021	0.143796	0.132559	0.126016	0.129637

heat transfer at both the plates decrease with the increase in K^2 for fixed M^2 while, for fixed K^2, at the stationary plate it increases with the increase in M^2, and at the moving plate it first decreases, attains a minimum and then increases with the increase in M^2 which implies that the magnetic field helps in cooling the system (Rohsenow and Hartnett 1973) but Jana *et al.* (1977) have shown that the rates of heat transfer at both the plates decrease with the increase in M^2. Table III shows that the critical Eckert number, at the moving plate, decreases with the increase in K^2 for fixed M^2 while it first decreases. attains a minimum and then increases with increase in M^2 for fixed K^2.

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APPENDIX

$$a_1 = \sinh\alpha\cos\beta, \quad a_2 = (1 - \cosh\alpha\cos\beta),$$

$$b_1 = \cosh\alpha\sin\beta, \quad b_2 = \sinh\alpha\sin\beta,$$

$$a_3 = (a_1 a_2 - b_1 b_2)/(a_1^2 + b_1^2), \quad a_4 = 2(M^2 a_3 + K^2 \beta),$$

$$b_3 = (a_2 b_1 + b_2 a_1)/(a_1^2 + b_1^2), \quad b_4 = 2(M^2 b_3 + K^2 \alpha),$$

$$a_5 = \{M^2(a_3 a_4 + b_3 b_4) + 2K^2(b_3 a_4 - a_3 b_4)\}/(a_4^2 + b_4^2), \; a_6 = (a_3 a_5 - b_3 b_5),$$

$$b_5 = \{M^2(b_3 a_4 - a_3 b_4) - 2K^2(a_3 a_4 + b_3 b_4)\}/(a_4^2 + b_4^2), \; b_6 = (a_3 b_5 + b_3 a_5),$$

$$a_7 = (M^2 a_1 + 2K^2 b_1)/(a_1^2 + b_1^2), \; a_8 = (a_7 - M^2 a_6), \; a_9 = M^2(a_8 a_5 + b_8 b_5),$$

$$b_7 = (M^2 b_1 - 2K^2 a_1)/(a_1^2 + b_1^2), \; b_8 = (b_7 - M^2 b_6), \; b_9 = M^2(b_8 a_5 - a_8 b_5),$$

$$a_{10} = 2K^2 b_5, \; a_{11} = -a_8, \; b_{10} = -2K^2 a_5, \; b_{11} = -b_8,$$

$$a_{12} = M^2 (a_{11} a_5 + b_{11} b_5),\ a_{13} = M^2 (a_{10} a_5 + b_{10} b_5),\ a_{14} = M^2 (a_{10} a_{11} + b_{10} b_{11}),$$

$$b_{12} = M^2 (b_{11} a_5 - a_{11} b_5),\ b_{13} = M^2 (b_{10} a_5 - a_{10} b_5),\ b_{14} = M^2 (b_{10} a_{11} - a_{10} b_{11}),$$

$$c_{1,2} = \frac{1}{2} \left\{ (a_8^2 + b_8^2) \pm M^4 (a_5^2 + b_5^2) \right\},\ c_3 = (a_{10}^2 + b_{10}^2),$$

$$c_{4,5} = \frac{1}{2} \left\{ \pm (a_{11}^2 + b_{11}^2) + M^4 (a_5^2 + b_5^2) \right\},\ c_6 = 1/(M^4 + 4 K^4),$$

$$d_1 = M^2/(\alpha^2 + \beta^2),\ d_2 = (c_1 + c_4 d_1)/4 \alpha^2,\ d_3 = (a_{12} d_1 - a_9)/4 \alpha^2,$$

$$d_4 = (c_2 + c_5 d_1)/4 \beta^2,\ d_5 = (b_{12} d_1 + b_9)/4 \beta^2,\ d_6 = c_6 (M^2 a_{13} + 2 K^2 b_{13}),$$

$$d_7 = c_6 (M^2 b_{13} - 2 K^2 a_{13}),\ d_8 = c_6 (M^2 a_{14} + 2 K^2 b_{14}),\ d_9 = c_6 (M^2 b_{14} - 2 K^2 a_{14}),$$

$$d_{10} = c_3 d_1/2,\ d_{11} = (d_2 - d_4 + 2 d_1 d_6),$$

$$d_{12} = d_2 \cosh 2\alpha + d_3 \sinh 2\alpha - d_4 \cos 2\beta - d_5 \sin 2\beta + 2 d_1 (d_6 \cosh\alpha \cos\beta - d_7 \sinh\alpha \sin\beta + d_8 \sinh\alpha \cos\beta - d_9 \cosh\alpha \sin\beta) + d_{10} - d_{11},$$

$$A = \frac{1}{(\alpha^2+\beta^2)} \Big[(\alpha b_8 + \beta a_8) \cosh\alpha \cos\beta - (\alpha a_8 - \beta b_8) \sinh\alpha \sin\beta - M^2 \left\{ (\alpha b_5 + \beta a_5) \sinh\alpha \cos\beta - (\alpha a_5 - \beta b_5) \cosh\alpha \sin\beta \right\} \Big],$$

$$B = \frac{1}{(\alpha^2+\beta^2)} \Big[+ (\alpha a_8 - \beta b_8) \cosh\alpha \cos\beta + (\alpha b_8 + \beta a_8) \sinh\alpha \sin\beta - M^2 \left\{ (\alpha a_5 - \beta b_5) \sinh\alpha \cos\beta + (\alpha b_5 + \beta a_5) \cosh\alpha \sin\beta \right\} \Big].$$