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ON A SPECIAL FORM OF T-TENSOR AND $T2$-LIKE FINSLER SPACE

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A special form of T-tensor T_{hijk} of a Finsler space is proposed in this paper. It arises from the theory of $C2$-like Finsler space. A particular case of this special form of T-tensor gives a $T2$-like Finsler space. A three dimensional Finsler space having such special form of T_{hijk} have also been discussed.

1. INTRODUCTION

Let F^n be an n-dimensional Finsler space equipped with the metric function $L(x, y)$. If $C_{hij}(x, y)$ be the (h) hv-torsion tensor of F^n then its v-covariant differentiation denoted by $|_k$ is used in defining the T-tensor T_{hijk} in the form (Motsumoto 1972a).

$$T_{hijk} = LC_{hij}|_k + C_{ijk}l_h + C_{hjk}l_i + C_{hik}l_j + C_{hij}l_k, \qquad \ldots(1.1)$$

where l_i are covariant components of unit vector along the elements of support y^i.

In our previous paper (Singh *et al.* 1982), we have studied three special forms of T_{hijk}, given by

(A) $T_{hijk} = \rho(h_{hi}h_{jk} + h_{hj}h_{ik} + h_{hk}h_{ij})$

(B) $T_{hijk} = h_{hi}P_{jk} + h_{hj}P_{ik} + h_{hk}P_{ij} + h_{ik}P_{hj} + h_{ij}P_{hk} + h_{jk}P_{hi}$

and

(C) $T_{hijk} = a_h C_{ijk} + a_i C_{hjk} + a_j C_{hik} + a_k C_{hij} + h_{hi}Q_{jk} + h_{hj}Q_{ik}$
$\qquad + h_{hk}Q_{ij} + h_{ij}Q_{hk} + h_{ik}Q_{hj} + h_{jk}Q_{hi}$,

where ρ is certain scalar and h_{ij} is angular metric tensor defined by

$$h_{ij} = g_{ij} - l_i l_j,$$

P_{ij} and A_{ij} are components of certain tensor fields and a_h are components of a covariant vector. Examples of Finsler spaces having these forms of T_{hijk} are given in our previous paper.

In this paper we shall study the properties of non-Riemannian Finsler spaces in which T_{hijk} is of the form

$$T_{hijk} = \rho C_h C_i C_j C_k + a_h C_i C_j C_k + a_i C_h C_j C_k + a_j C_h C_i C_k + a_k C_h C_i C_j, \qquad \ldots(1.2)$$

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where ρ is a scalar function, positively homogeneous of degree three in y^i, a_h are components of covariant vector field which is positively homogeneous of degree two in y^i and $C_i = C^j_{ij}$ is the torsion vector.

The examples of Finsler spaces having the special form (1.2) will be given in the second article. The particular form of (1.2) in which a_h is null vector will be discussed in the last article.

2. Finsler Space with *T*-tensor of the Form (1.2)

First of all we shall discuss the two dimensional Finsler space F^2. With reference of Berwald's frame (l_i, m_i) the angular metric tensor and (h) hv-torsion tensor are given by (Ikeda 1979)

$$h_{ij} = m_i m_j, \; LC_{ijk} = I\, m_i m_j m_k \qquad \ldots(2.1)$$

where I is the principal scalar.

From (1.1) and (2.1) it follows that T-tensor of F^2 is given by

$$LT_{hijk} = I_{;2} m_h m_i m_j m_k \qquad \ldots(2.2)$$

where

$$I_{;2} = L \frac{\partial I}{\partial y^k} m^k.$$

Since $m_i = \dfrac{C_i}{C}$, where $C^2 = C_i C_j g^{ij}$, from (2.1), it follows that $I = LC$.

Thus, if we write

$$L\, a_h = a_1\, l_h + a_2\, m_h \qquad \ldots(2.3)$$

then from (1.2) and (2.2) it follows that

$$a_1 = 0, \; I_{;2} = L\, \rho\, C^4 + 4\, a_2 C^3. \qquad \ldots(2.4)$$

Hence

Theorem 2.1 – In a two dimensional Finsler space F^2 the T-tensor T_{hijk} can be expressed in the form (1.2) and the scalar components a_1 and a_2 satisfy the relations given in (2.4).

Motsumoto (1973) developed the theory of three dimensional Finsler space referring to the orthogonal frame $e^{\,i}_{(\alpha)}$, $\alpha = 1,2,3$(*) with reference to this frame the tensor C_{hij} is written as

$$LC_{hij} = C_{\alpha\beta\gamma}\, e_{(\alpha)h}\, e_{(\beta)i}\, e_{(\gamma)j} \qquad \ldots(2.5)$$

where the scalar components $C_{\alpha\beta\gamma}$ are such that

$$C_{1\beta\gamma} = 0, \; C_{222} = H, \; C_{233} = I, \; C_{333} = -\, C_{223} = J. \qquad \ldots(2.6)$$

The scalars H, I and J are called main scalars and satisfy the equation

$$H + I = LC. \qquad \ldots(2.7)$$

(*) In Motsumoto (1973) the variation of α is taken as 0,1,2 while we have taken it as 1,2,3.

The scalar components of LT_{hijk} are given by (Singh *et al.* 1982)

$$LT_{hijk} = [C_{\alpha\beta\gamma;\delta} + C_{\beta\gamma\delta}\,\delta_{1\alpha} + C_{\alpha\gamma\delta}\,\delta_{1\beta} + C_{\alpha\beta\delta}\,\delta_{1\gamma}]\, e_{(\alpha)h}\, e_{(\beta)i}\, e_{(\gamma)j}\, e_{(\delta)k} \quad \ldots(2.8)$$

where the semicolan denotes the v-scalar derivative (Motsumoto 1973). We shall use the following relations which have been obtained by Motsumoto (1973).

$$\left.\begin{aligned} &C_{1\beta\gamma;\delta} = -C_{\beta\gamma\delta},\ C_{222;\delta} = H_{;\delta} + 3Jv_\delta \\ &C_{223;\delta} = -J_{;\delta} + (H-2I)\,v_\delta \\ &C_{233;\delta} = I_{;\delta} - 3Jv_\delta,\ C_{333;\delta} = J_{;\delta} + 3Iv_\delta \end{aligned}\right\} \quad \ldots(2.9)$$

where v_δ are the scalar components of v-connection vector (Motsumoto 1973).

Let a_α be the scalar components of La_i

$$La_i = a_\alpha\, e_{(\alpha)i}. \quad \ldots(2.10)$$

We assume that the T-tensor of the space F^3 have the form (1.2) then in view of the relation (2.10), we have

$$\begin{aligned} LT_{hijk} = C^3\,[CL\,\rho\,\delta_{2\alpha}\,\delta_{2\beta}\,\delta_{2\gamma}\,\delta_{2\delta} &+ a_\alpha\,\delta_{2\beta}\,\delta_{2\gamma}\,\delta_{2\delta} \\ &+ a_\beta\,\delta_{2\alpha}\,\delta_{2\gamma}\,\delta_{2\delta} + a_\gamma\,\delta_{2\alpha}\,\delta_{2\beta}\,\delta_{2\delta} \\ &+ a_\delta\,\delta_{2\alpha}\,\delta_{2\beta}\delta_{2\gamma}]\, e_{(\alpha)h}\, e_{(\beta)i}\, e_{(\gamma)j}\, e_{(\delta)k}. \end{aligned} \quad \ldots(2.11)$$

Comparing (2.8) with (2.11), we get

$$\begin{aligned} &C_{\alpha\beta\gamma;\delta} + C_{\beta\gamma\delta}\delta_{1\alpha} + C_{\alpha\gamma\delta}\,\delta_{1\beta} + C_{\alpha\beta\delta}\,\delta_{1\gamma} \\ &\quad = C^4\,L\rho\,\delta_{2\alpha}\,\delta_{2\beta}\,\delta_{2\gamma}\,\delta_{2\delta} + C^3\,(a_\alpha\,\delta_{2\beta}\,\delta_{2\gamma}\,\delta_{2\delta} \\ &\quad\quad + a_\beta\delta_{2\alpha}\delta_{2\gamma}\delta_{2\delta} + a_\gamma\delta_{2\alpha}\delta_{2\beta}\delta_{2\delta} + a_\delta\delta_{2\alpha}\delta_{2\beta}\delta_{2\gamma}). \end{aligned} \quad \ldots(2.12)$$

Since T_{hijk} is an indicatory tensor, from (1.2) it follows that $a_i y^i = 0$, which in view of (2.10) gives $a_1 = 0$. Thus equations (2.6), (2.9) and (2.12) yield

$$\left.\begin{aligned} &\text{(a)}\quad H_{;\delta} + 3\,Jv_\delta = C^3\,(L\rho C + 3\,a_2)\,\delta_{2\delta} + C^3\,a_\delta \\ &\text{(b)}\quad -J_{;\delta} + (H-2I)v_\delta = C^3\,a_3\,\delta_{2\delta} \\ &\text{(c)}\quad I_{;\delta} - 3\,Jv_\delta = 0 \\ &\text{(d)}\quad J_{;\delta} + 3\,Iv_\delta = 0. \end{aligned}\right\} \quad \ldots(2.13)$$

These equations give

$$\left.\begin{aligned} &a_1 = 0,\ a_2 = \frac{1}{4}\left(\frac{L}{C^3}\,C_{;2} - LC\rho\right), \\ &a_3 = \frac{L}{C^2}\,v_2 \text{ and } v_3 = 0. \end{aligned}\right\} \quad \ldots(2.14)$$

Hence we have the following:

Theorem 2.2—In a three dimensional Finsler space if the T-tensor is of the form (1.2) then the scalar component v_3 of v-connection vector vanishes and the scalar components a_α of La_i are given by (2.14).

Since in any three dimensional Finsler space $v_1 = 0$ (Motsumoto 1973), Theorem 2.2 gives the following:

Theorem 2.3—In a three dimensional Finsler space with the T-tensor of the form (1.2), the ν-connection vector vanishes if the scalar component a_3 of $L\ a_i$ vanishes.

Now we shall give examples of n-dimensional Finsler spaces ($n > 2$) whose T-tensor is of the form (1.2).

A $C2$-like Finsler space is defined as an n ($n > 2$)-dimensional Finsler space in which (h) $h\nu$-torsion tensor is of the form (Motsumoto and Numata 1980)

$$C_{hij} = \frac{1}{C^2} C_h C_i C_j. \qquad \ldots(2.15)$$

Since $C_{hij} \mid_k - C_{kij} \mid_h = 0$, from (2.15), we have

$$C_h C_j \Gamma_{ik} + C_h C_i \Gamma_{jk} - C_k C_j \Gamma_{ih} - C_k C_i \Gamma_{jh} = 0 \qquad \ldots(2.16)$$

where

$$\Gamma_{ik} = C_i \mid_k - \frac{1}{2C^4} C^2 \mid_k C_i. \qquad \ldots(2.17)$$

Since $2\ C_i \mid_k C^i = C^2 \mid_k$, from (2.17), we have

$$\Gamma_{ik} C^i = 0 \text{ and } \Gamma_{ik} C^k = \frac{1}{2}\left(C^2 \mid_i - \frac{1}{C^2} C^2 \mid_k C^k C_i\right). \qquad \ldots(2.18)$$

Contracting (2.16) with g^{jh} and using (2.18) we get

$$C^2 \Gamma_{ik} = \alpha C^2 C_i C_k + \tfrac{1}{2} C_k C^2 \mid_i \qquad \ldots(2.19)$$

where

$$\alpha\ C^2 = \Gamma_{ik}\ g^{ik} - \frac{1}{2C^2} C^2 \mid_j C^j.$$

By virtue of eqns. (2.17) and (2.19), we have the following:

Lemma 2.1—In a $C2$-like Finsler space there exists a scalar α such that

$$C_i \mid_j = \alpha C_i C_j + \frac{1}{C} (C \mid_j C_i + C \mid_i C_j). \qquad \ldots(2.20)$$

Now we are in position to prove the following

Theorem 2.4—The T-tensor of a $C2$-like Finsler space is of the form (1.2).

PROOF : The ν-covariant differentiation of (2.15) gives

$$C_{hij} \mid_k = \frac{1}{C^2}(C_h \mid_k C_i C_j + C_i \mid_k C_h C_j + C_j \mid_k C_h C_i) - \frac{1}{C^4} C^2 \mid_k C_h C_i C_j$$

which in view of Lemma 2.1, gives

$$C_{hij} \mid_k = \frac{3\alpha}{C^2} C_h C_i C_j C_k + \frac{1}{C^3} (C \mid_h C_i C_j C_k + C \mid_i C_h C_j C_k + C \mid_j C_h C_i C_k + C \mid_k C_h C_i C_j). \qquad \ldots(2.21)$$

Equations (1.1), (2.15) and (2.21) give the form (1.2), where

$$\rho = \frac{3\alpha L}{C^2},\ a_i = \left(\frac{L\ C \mid_i}{C^3} + \frac{l_i}{C^2}\right).$$

There are other examples of a Finsler space whose *T*-tensor is of the form (2.1). If L is a metric function of a two dimensional Finsler space F^2 and $^*L$ is the metric function of $(n-2)$ dimensional Riemannian space R^{n-2} then the Finsler space $F^2 \times R^{n-2}$ with metric $\sqrt{L+{}^*L}$ is $C2$-like Finsler space (Motsumoto & Numata 1980). Hence by virtue of Theorem 2.4 the *T*-tensor of Finsler space $F^2 \times R^{n-2}$ is of the form (1.2).

3. *T*2-LIKE FINSLER SPACE

In this section we shall deal the particular form of (1.2) in which a_h is a null vector. Firstly we shall prove the following

Theorem 3.1—If the *T*-tensor T_{hijk} is written in decomposed form

$T_{hijk} = C_h T_{ijk}$

with $C_h \neq 0$ then T_{hijk} is written in the form

$$T_{hijk} = \rho\, C_h C_i C_j C_k. \qquad \ldots(3.1)$$

PROOF : Since T_{hijk} is symmetric in all its four indices, T_{ijk} is a symmetric tensor in all its indices. Since C_h is non-vanishing vector, we may suppose $C_1 \neq 0$, then $T_{1ijk} = T_{i1jk}$ implies that $C_1 T_{ijk} = C_i T_{1jk}$, which gives

$$T_{ijk} = C_i T_{jk} \qquad \ldots(3.2)$$

where

$$T_{jk} = T_{1jk}/C_1.$$

In view of relation (3.2), the identity $T_{1jk} = T_{j1k}$ gives $C_1 T_{jk} = C_j T_{1k}$, which gives $T_{jk} = C_j T_k$, where $T_k = T_{1k}/C_1$.

Since T_{jk} is symmetric tensor, we have

$$T_{1k} - T_{k1} = C_1 T_k - C_k T_1 = 0.$$

Thus

$$T_k = \rho\, C_k$$

where $\rho = T_1/C_1$.

Hence we get eqn. (3.1).

By virtue of eqn. (2.2), we have the following

Theorem 3.2—In any non-Riemannian Finsler space F^2 the *T*-tensor is of the form (3.1) where $\rho = I_{;2}\, C^{-4}\, L^{-1}$.

In view of this theorem we give the following definition:

Definition—A non-Riemannian Finsler space F^n $(n \geqslant 2)$ is called *T*2-like Finsler space, if the *T*-tensor T_{hijk} is written in the form (3.1).

In order to discuss the properties of three dimensional *T*2-like Finsler space, we shall use the equation (2.13). Since a_h is a null vector its scalar components a_α vanishes and the equation (2.13) yield

$$\left.\begin{aligned}&(a)\quad H;_\delta + 3Jv_\delta = L\rho C^4\delta_{2\delta}\\&(b)\quad -J;_\delta + (H-2I)v_\delta = 0\\&(c)\quad I;_\delta - 3Jv_\delta = 0\\&(d)\quad J;_\delta + 3Iv_\delta = 0.\end{aligned}\right\}\qquad ...(3\ 3)$$

These equations give

$$v_\delta = 0,\ I;_\delta = 0,\ J;_\delta = 0$$

and

$$H;_\delta = LC^4\rho\ \delta_{2\delta}. \qquad ...(3.4)$$

Theorem 3.3—In a three dimensional $T2$-like Finsler space the v-connection vector vanishes identically and the main scalars I, J are functions of position (co-ordinate) only. The main scalar H satisfies the equation

$$H;_\delta = L\rho C^4\ \delta_{2\delta}$$

where

$$\delta_{2\delta} = 0 \text{ when } \delta \neq 2$$
$$= 1 \text{ when } \delta = 2.$$

If $\rho = 0$, then from (3.1), we get $T_{hijk} = 0$ i.e. the Finsler space satisfies T-condition (Motsumoto 1975). Thus by virtue of Theorem 3.3.

Corollary 3.1—The T-condition, for a 3-dimensional non-Riemannian $T2$-like Finsler space, is equivalent to the fact that the v-connection vector v_j vanishes identically and the main scalars H, I and J are functions of position only.

Now, we shall discuss the properties of n-dimensional ($n \geqslant 3$) $T2$-like Finsler space.

Contracting (3.1) with g^{hk} and using (1.1), we get

$$L\ C_i \mid _j = \rho C^2 C_i C_j - l_i C_j - l_j C_i. \qquad ...(3.5)$$

Again contracting (3.5) with C^i and using the relation $2C_i \mid _j C^i = C^2 \mid _j$, we get

$$C \mid _j = \lambda\ C_j + \mu l_j$$

where

$$\lambda = \rho \frac{C^3}{L} \text{ and } \mu = -\frac{C}{L}.$$

Hence we have the following

Theorem 3.4—In a $T2$-like Finsler space relation (3.5) holds and $C^2 \mid _j$ is a linear combination of C_j and l_j.

For the $T2$-likeness of $C2$-like Finsler space, we have the following:

Theorem 3.5—A $C2$-like Finsler space is $T2$-like iff condition (3.5) holds.

PROOF : The necessary part of the theorem follows from the theorem (3.4).

Conversely, if (3.5) holds and F^n is $C2$-like then v-covariant differentiation of (2.15) gives

$$C_{hij} \mid_k = \frac{1}{C^2} (C_h \mid_k C_i C_j + C_i \mid_k C_h C_j + C_j \mid_k C_h C_i) - \frac{1}{C^4} C^2 \mid_k C_h C_i C_j. \quad ...(3.6)$$

Contracting (3.5) with C^i and using the relation $2C_i \mid_j C^i = C^2 \mid_j$, we get

$$L\, C^2 \mid_j = 2\, C^2 (\rho\, C^2 C_j - l_j). \quad ...(3.7)$$

Substituting (2.15), (2.19), (3.6) and (3.7) in (1.1), we get

$$T_{hijk} = \rho\, C_h C_i C_j C_k.$$

Hence F^n is $T2$-like Finsler space.

Theorem 3.6—A $C2$-like Finsler space is $T2$-like iff $C \mid_k$ is a linear combination of l_k and C_k.

PROOF : The necessary part follows from Theorem 3.4.

Conversely, if $C \mid_j$ is linear combination of C_j and l_j, then $C \mid_j = \lambda C_j + \mu l_j$, for some scalars λ and μ.

Since C is positively homogeneous of degree -1 in y^i, contracting the above equation with y^i, we get $\mu = - C/L$. Thus we have

$$C \mid_j = \lambda C_j - \frac{C}{L}\, l_j. \quad ...(3.8)$$

Substituting (2.15), (2.20), (3.6) and (3.8) in (1.1), we get

$$T_{hijk} = \rho C_h C_i C_j C_k$$

where

$$\rho = \frac{3L\alpha}{C^2} + \frac{4\lambda L}{C^3}.$$

Theorem 3.7—If a $T2$-like Finsler space is c-reducible then it satisfies T-condition.

PROOF : A C-reducible Finsler space F^n is a non-Riemannian Finsler space in which (*h*) *hv*-torsion tensor is of the form (Motsumoto 1972 b)

$$C_{hij} = \frac{1}{(n+1)} (C_h h_{ij} + C_i h_{hj} + C_j h_{hi}).$$

In a C-reducible Finsler space there exists scalar M such that (Motsumoto 1974)

$$T_{hijk} = M\, (h_{ij} h_{hk} + h_{hi} h_{jk} + h_{hj}\, h_{ik}). \quad ...(3.9)$$

By virtue of eqns. (3.1) and (3.9), we get

$$M\, (h_{ij} h_{hk} + h_{hi} h_{jk} + h_{hj} h_{ik}) = \rho C_h C_i C_j C_k$$

which after contraction with g^{hk}, gives

$$M\, (n+1)\, h_{ij} = \rho\, C^2 C_i C_j. \quad ...(3.10)$$

Since the rank of h_{ij} is $(n-1)$ and the rank of $C_i C_j$ is 1, therefore, for $n > 3$, eqn. (3.10) is valid only when $\rho = M = 0$.

This proves the theorem.

The v-curvature tensor S_{hijk} is given by

$$S_{hijk} = C_{hkr} C^r_{ij} - C_{hjr} C^r_{ik}.$$

The v-covariant differentiation of above equation and the application of equations (1.1) and (3.1), give

$$L\, S_{hijk} \mid_l = (C_{ij.}\, C_h C_k + C_{hk.}\, C_i C_j - C_{ik.}\, C_h C_j - C_{hj.}\, C_i C_k)\, C_l$$
$$- 2\, l_l S_{hijk} - l_h S_{lijk} - l_i S_{hljk} - l_j S_{hilk} - l_k S_{hijl} \qquad \ldots(3.11)$$

where the dot denotes the contraction with C^r. The indicatorised tensor T_{hijkl} of $S_{hijk} \mid_l$ is defined as

$$T_{hijkl} = L\, S_{hijk} \mid_l + 2 l_l S_{hijk} + l_h S_{lijk} + l_i S_{hljk} + l_j S_{hilk} + l_k S_{hijl}.$$

Equation (3.11) gives the following:

Theorem 3.8—The indicatorised tensor T_{hijkl} of $LS_{hijk} \mid_l$ for a $T2$-like Finsler space is of the form

$$T_{hijkl} = (C_{ij.}\, C_h C_k + C_{hk.}\, C_i C_j - C_{ik.}\, C_h C_j - C_{hj.}\, C_i C_k) C_l.$$

Contracting (3.11) with g^{hk}, we get

$$\text{(a)}\quad LS_{ij} \mid_l = \rho\, [C^2 (C_{ij.} + C_i C_j) - (C_{i\,..}\, C_j + C_{j\,..}\, C_i)\, C_l$$
$$- 2 l_l S_{ij} - l_i S_{lj} - l_j S_{il}, \qquad \ldots(3.12)$$

where

$$S_{ij} = S_{hij}\, g^{hk}.$$

Again contracting (a) with g^{ij}, we get

$$LS \mid_l = 2\rho\, (C^4 - C \ldots)\, C_l - 2 l_l S \qquad \ldots(3.13)$$

where

$$S = S_{ij}\, g^{ij}.$$

This gives the following

Theorem 3.9—In a $T2$-like Finsler space F^n if the v-scalar S vanishes identically and $C^4 \neq C \ldots$, then F^n satisfies T-condition.

An $S3$-like Finsler space is characterised by the relation (Fukui and Yamada 1979)

$$S_{hijk} = \frac{S}{(n-1)\,(n-2)} (h_{hk} h_{ij} - h_{hj} h_{ik}). \qquad \ldots(3.14)$$

For an $S3$-like Finsler space the function $L^2 S$ is a function of co-ordinates only (Fukui and Yamada 1979). Hence $(L^2 S) \mid_l = 0$. This result and (3.12) give

$$\rho (C^4 - C \ldots)\, C_l = 0.$$

Therefore, we get:

Theorem 3.10—If a $T2$-like Finsler space F^n is $S3$-like and $C^4 \neq C \ldots$, then F^n satisfies T-condition.

Since in every three dimensional Finsler space (3.13) holds, therefore, we get

Corollary 3.2—A $T2$-like 3-dimensional Finsler space with $C^4 \neq C \ldots$, is a Finsler space satisying T-condition.

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