EFFECT OF MAGNETIC FIELD OVER THE STRUCTURE OF TURBULENCE *

by H. C. Khare *and* R. S. L. Srivastava, *Department of Mathematics, University of Allahabad, Allahabad 2*

(Communicated by N. R. Dhar, F.N.A.)

(*Received 24 July 1970*)

Theodorson has shown that equation of fluid motion can be written in short and exact form with the vorticity $\bar{\omega}$ as the variable. Later on he discussed the structure of turbulence. In the present paper the authors have discussed the effect of magnetic field over the structure of turbulence.

Introduction

Theodorson (1961) had shown that the equation of fluid motion of viscous and incompressible fluid can be written in short and exact form with the variable $\bar{\omega}$ the vorticity, the rate of change of ω^2 is shown to be composed of two parts, one positive and the other negative. He also defined the structure of turbulence to be horse-shoe. In the present paper we have studied the effect of Alfvén waves over the structure of turbulence and motion of fluid for time short compared to the diffusion time of the field.

Calculations

Equation of motion for viscous incompressible fluid in the presence of a uniform magnetic field when gravitational field is also present is given (Huges and Young 1966) by

$$\rho\left[\frac{\partial \bar{V}}{\partial t}+\nabla\left(\frac{V^2}{2}\right)-\bar{V}\times(\nabla\times\bar{V})\right] = -\nabla p-\rho\nabla\psi+\rho\nu\nabla^2\bar{V}$$
$$+(\zeta+\tfrac{1}{3}\nu\rho)\nabla(\nabla\bar{V})+\mu(\nabla\times\bar{H})\times\bar{H}. \quad \ldots \quad (1)$$

Diffusion equation is given (Jackson 1962) by

$$\frac{\partial \bar{B}}{\partial t}=\nabla\times(\bar{V}\times\bar{B}). \quad \ldots \quad (2)$$

Maxwells equation is

$$\nabla\times\bar{H}=\bar{J} \quad \ldots \quad (3)$$

and other relations used are

$$\bar{B}=\mu\bar{H} \quad \text{and} \quad \bar{J}=\rho\bar{V} \quad \ldots \quad (4)$$

* This work has been financially assisted by the University Grants Commission of India, New Delhi.

where ρ, $\bar{V}$, p, ν, μ, ψ, $\bar{B}$, $\bar{H}$, $\bar{J}$, ζ and t represent density, velocity, pressure, kinematic viscosity, magnetic permeability, gravitational potential, magnetic induction, magnetic field (strength), current density, second coefficient of viscosity and time of the fluid motion respectively.

For incompressible fluids divergence of the velocity is zero, therefore, the fourth term on the right-hand side of eqn. (1) vanishes. Now writing the equation of motion (1) in the form

$$\rho\left[\frac{\partial \bar{V}}{\partial t}-\bar{V}\times(\nabla\times\bar{V})\right]=-\rho\nabla\left(\frac{p}{\rho}+\psi+\frac{1}{2}V^2\right)+\rho\nu\nabla^2\bar{V}+\mu(\nabla\times\bar{H})\times\bar{H}, \qquad (5)$$

taking curl and writing curl $\bar{V}=\bar{\omega}$ the vorticity, we have

$$\rho\left[\frac{\partial\bar{\omega}}{\partial t}-\text{curl }(\bar{V}\times\bar{\omega})\right]=\rho\nu\nabla^2\bar{\omega}+\mu\nabla\times[(\nabla\times\bar{H})\times\bar{H}]. \qquad (6)$$

Cancelling ρ and using eqns. (3) and (4), we get

$$\frac{\partial\bar{\omega}}{\partial t}-\text{curl }(\bar{V}\times\bar{\omega})=\nu\nabla^2\bar{\omega}+\nabla\times(\bar{V}\times\bar{B}) \qquad (7)$$

or

$$\frac{\partial\bar{\omega}}{\partial t}-\text{curl }(\bar{V}\times\bar{\omega})=\nu\nabla^2\bar{\omega}+\frac{\partial\bar{B}}{\partial t} \qquad (8)$$

where eqn. (2) has been used in writing eqn. (8). Making vector operations after multiplying the $\bar{\omega}$ we get the simple relation

$$\frac{D}{Dt}\left(\frac{\omega^2}{2}\right)=\omega^2\left(\frac{d}{ds}\right)V_s+\nu\bar{\omega}\,.\,\nabla^2\bar{\omega}+\bar{\omega}\,.\,\frac{\partial\bar{B}}{\partial t} \qquad (9)$$

where

$$\frac{D}{Dt}=\frac{\partial}{\partial t}+\bar{V}\cdot\nabla$$

and V_s represents the magnitude of $\bar{V}$ along s (which is taken along $\bar{\omega}$).

Discussion and Conclusion

The positive term $\omega^2\frac{d}{ds}V_s$ of eqn. (9) causes the turbulent flow. Since the term $\frac{d}{ds}V_s$ is always positive for a stretching vortex element, the structure of turbulence may be discussed on above ground. The behaviour of the fluid in the presence of magnetic field is governed to a large extent by the magnitude of conductivity quite different from the behaviour of the field. Equation (2) is equivalent to the statement that the magnetic flux through any loop moving with the local fluid velocity is constant in time. The magnetic Reynold number is used to distinguish between situations in which diffusion of the field lines relative to the fluid occurs and those in which the lines of force are frozen in. Transport of the lines of force with the fluid dominates over diffusion if $R_M \gg 1$ (Pai 1958). The tension in the lines of force tends to restore them to straight line form, thereby causing transverse oscillations. These magnetohydrodynamic waves are called Alfv́en waves. The lines of force are frozen in and move with the fluid. When field is not present neither a tangential nor an axial vortex will change the length of stretching vortex and thus only a radially pointing vector can be subjected to stretching and this leads to the formation of loops which take the horse-shoe shape.

Due to the presence of magnetic field we have a diffusion term in the equation and as we have seen that lines of force are frozen in fluid and moves with it. Therefore, the magnetic force acts along the axis of flow. Hence the horse-shoe structure of the turbulence changes its shape in the presence of magnetic field and takes the form of elliptical shell.

References

Huges, W. F., and Young, F. J. (1966). The Electromagnetodynamics of Fluids. John Wiley and Sons, Inc., New York.

Jackson, John David (1962). Classical Electrodynamics. John Wiley and Sons, Inc., New York.

Pai, S. I. (1958). Introduction to the Theory of Compressible Flow. D. Van Nostrand Company, Inc., Princeton.

Theodorson, Th. (1961). Fluid Dynamics and Applied Mathematics, ed. by J. B. Diaz and S. I. Pai. University of Maryland.