STABILITY OF PLASMA IN HELICAL FIELD

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In this paper we have studied the problem of instability in a helical field. Generally the magnetic field in the containing devices is of helical type. But it is very difficult to solve the resulting equations in the presence of general axial and azimuthal fields. After establishing equations for general helical field, we have considered the field which is constant in the axial direction and the azimuthal component is produced by a small line current as a particular case. The plasma shell of finite thickness and of infinite conductivity is taken in the presence of such a field. The normal mode technique is applied to study the axisymmetric and azimuthal disturbances. A sixth degree equation in frequency for the various values of parameters is obtained. In the azimuthal disturbances a neutral stability curve plotted indicates a very strong dependence of stability on the thickness of the plasma shell.

1. INTRODUCTION

Many investigators have studied the problem of stability of cylindrical plasma in the presence of axial magnetic field which may be uniform or non-uniform due to its application in fusion devices. However, most of the experimental fusion devices such as toroidal pinch and stellarators, etc., have the azimuthal component of the magnetic field in addition to axial component. Auluck and Kothari (1957) have studied the problem of stability of an infinitely long gravitating cylinder of incompressible inviscid and infinitely conducting fluid in the presence of axial and azimuthal components separately. Auluck and Nayyar (1960) extended this problem to take account of both azimuthal and axial components together. Newcomb and Kaufman (1961) have also investigated the stability of a tubular pinch which has both axial and azimuthal components. However, in all the above investigations, the energy method is used to discuss the stability which does not give the quantitative results. Kruskal and Schwarzchild (1954), however, have applied the normal mode method to study the stability of a plasma in which the magnetic field is caused by an electric current within the plasma and found the system to be unstable

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against lateral distortions. Chakraborty and Bhatnagar (1960) have studied the stability of an ideally conducting infinite liquid column carrying a uniform volume current and uniform surface charge density and have established various criteria for axisymmetric and azimuthal disturbances. Bhat (1968) studied the problem of stability of a self-gravitating cylindrical plasma in the presence of a non-uniform axial magnetic field and the azimuthal field produced by a constant volume current by the method of normal mode technique.

In the present investigation, we have considered the cylindrical plasma in the presence of both the axial and azimuthal components of magnetic field given in section 2 which are non-uniform and depend on the radial coordinate only. For simplicity, the plasma is taken to be infinitely conducting, inviscid, incompressible and the displacement current is neglected. We have applied the normal mode technique and find that the resulting perturbation equations, which are recorded in sections 3 and 4 for the axisymmetric and azimuthal disturbances respectively, cannot be solved to get dispersion relations for a general type of helical field. In order to simplify the analysis, we have considered a particular case of the helical field which has a uniform axial component and the azimuthal component produced by a line current. Such a field has already been discussed by Bhatnagar and Bhat (1968) in context with the production of high magnetic fields. We have mentioned the field in section 2 and the resulting solutions of the perturbation equations, which are obtained by using Picard's approximate method of solution, are recorded in sections 3 and 4 for the two types of disturbances. The dispersion relations obtained after using the required boundary conditions for this particular helical field have been discussed in sections 5 and 6.

The fundamental aim of stability problems is to find out magnetic field configuration which will stabilize a plasma over time intervals necessary for the onset of thermonuclear reactions in laboratories. The magnetic field configurations studied have been used in some laboratory devices and, therefore, we have studied the stability of plasma in the presence of these fields.

2. Steady State

The magnetic field inside the plasma in the steady state is given by

$$\vec{B} = [0, B_\phi(r), B_z(r)]. \qquad (2.1)$$

We will see in the next two sections that it will be extremely difficult to solve the equations governing the perturbed quantities for the field (2.1). We, therefore, consider a particular case in which a uniform axial magnetic field is present due to the absence of azimuthal current density and an azimuthal magnetic field is present due to a constant line current. The magnetic field in the dimensionless form is given by

$$\vec{B} = \begin{bmatrix} \left(0, \dfrac{C}{r}, A\right) & m < r < 1 \\ \left(0, \dfrac{C}{r}, H_1\right) & r > 1 \end{bmatrix} \qquad \ldots \quad \ldots \quad \ldots \quad (2.2)$$

where C is related with the strength of the line current. A and H_1 are uniform fields inside and outside the plasma respectively. A_1 has been chosen as unity due to non-dimensionalization in our discussion. We also choose the inner boundary $r = m$ to be rigid, non-conducting and non-magnetic. In the steady state, we assume that the entire system is at rest. Hence, we can show that the electric field and current density are zero everywhere. Further, we will have an azimuthal surface current density given by

$$\vec{J}^* = (0, H_1 - A, 0) \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2.3)$$

and the plasma pressure will be constant and will be given by

$$p_0 = \tfrac{1}{2}(H_1^{\,2} - A^2). \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2.4)$$

3. Axisymmetric Disturbances

We perturb the steady state described in section 2 in an axisymmetric manner and assume that these disturbances are so small that we can linearize the governing equations. We assume that the perturbed quantities vary exponentially with time and axial coordinate. Thus the disturbance is of the type $X' = X \exp(i\omega t + ilz)$, where ω is the angular frequency and l is the axial wavenumber. Let us denote the amplitude of the perturbed quantities as $\vec{v}$, p, $\vec{b}$ and $\vec{e}$, the velocity, pressure, magnetic field and electric field respectively. Using the momentum equation and Maxwell's equation, we get the following equation for the radial component of the velocity for the field (2.1):

$$\left(B_z^{\,2} - \frac{\omega^2}{l^2}\right)\frac{d^2 v_r}{dr^2} + \left(-\frac{\omega^2}{l^2 r} + 2B_z \frac{dB_z}{dr} + \frac{B_z^{\,2}}{r}\right)\frac{dv_r}{dr} + \left[\omega^2 + \frac{\omega^2}{l^2 r^2} - \frac{4\omega^2 B_\phi^{\,2}}{r^2(\omega^2 - l^2 z^2)}\right.$$
$$\left. + \frac{2B_\phi^{\,2}}{r^2} - l^2 B_z^{\,2} + \frac{2B_z}{r}\frac{dB_z}{dr} - \frac{B_z^{\,2}}{r^2} + \frac{2B_\phi}{r}\frac{dB_\phi}{dr}\right] v_r = 0 \qquad \ldots \quad \ldots \quad (3.1)$$

and all other perturbed quantities are related to v_r by the following relations:

$$v_\phi = \frac{2l\, B_z B_\phi}{ir(\omega^2 - l^2 B_z^{\,2})} v_r \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3.2)$$

$$v_z = \frac{i}{lr}\frac{d}{dr}(r v_r) \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3.3)$$

$$b_r = \frac{lB_z}{\omega} v_r \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3.4)$$

$$b_\phi = \left[\frac{2\omega\, B_\phi}{ir(\omega^2 - l^2 B_z^2)} - \frac{1}{i\omega r}\,\frac{d}{dr}(r\,B_\phi)\right] v_r \qquad (3.5)$$

$$b_z = \frac{i}{\omega r}\,\frac{d}{dr}(r v_r B_z) \qquad (3.6)$$

$$p = \frac{B_z}{i\omega}\,\frac{dB_z}{dr}\, v_r + \frac{\omega}{il^2 r}\,\frac{d}{dr}(r v_r)$$

$$- \left[\frac{2B_\phi^2\omega}{ir(\omega^2 - l^2 B_z^2)} - \frac{B_\phi}{i\omega r}\,\frac{d}{dr}(r\,B_\phi)\right] v_r \qquad (3.7)$$

$$e_r = \frac{iB_\phi}{lr}\,\frac{d}{dr}(r v_r) - \frac{2l\,B_\phi B_z^2}{ir(\omega^2 - l^2 B_z^2)}\, v_r \qquad (3.8)$$

$$e_\phi = B_z v_r \qquad (3.9)$$

and

$$e_z = -B_\phi v_r. \qquad (3.10)$$

Equation (3.1) could be solved to obtain v_r if we specify the type of magnetic field. We, therefore, use the type of field given by eqn. (2.2) and thus eqn. (3.1) reduces to

$$\frac{d^2 v_r}{dr^2} + \frac{1}{r}\,\frac{dv_r}{dr} + \left[-l^2 - \frac{1}{r^2} + \frac{4C^2\omega^2 l^2}{r^4(l^2A^2 - \omega^2)^2}\right] v_r = 0. \qquad (3.11)$$

If we assume that the strength of the line current (C) is small, we can approximately solve the above equation by the method of variation of parameter. The resulting solution will be an integral equation in v_r as in the case of Picard's method. For simplicity, we will consider the solution up to first iteration. Thus we have

$$v_r = B_0\left[I_1(lr) + \alpha^2 \int^r \frac{I_1(l\xi)}{\xi^3}\{I_1(l\xi)K_1(lr) - I_1(lr)K_1(l\xi)\}\, d\xi\right]$$

$$+ D_0\left[K_1(lr) + \alpha^2 \int^r \frac{K_1(l\xi)}{\xi^3}\{I_1(l\xi)K_1(lr) - I_1(lr)K_1(l\xi)\}\, d\xi\right] \qquad (3.12)$$

where

$$\alpha^2 = \frac{4C^2\omega^2 l^2}{(l^2A^2 - \omega^2)^2}.$$

The other perturbed quantities in terms of v_r for the field of type (2.2) are

$$v_\phi = \frac{2l\,AC}{ir^2(\omega^2 - l^2A^2)}\, v_r \qquad (3.13)$$

$$v_z = \frac{i}{lr}\left(r\frac{dv_r}{dr} + v_r\right) \qquad (3.14)$$

$$b_r = \frac{lA}{\omega}\, v_r \qquad (3.15)$$

$$b_\phi = \frac{2C\omega}{ir^2(\omega^2 - l^2A^2)}\, v_r \qquad (3.16)$$

$$b_z = \frac{iA}{\omega r}\left(r\frac{dv_r}{dr}+v_r\right) \qquad \ldots (3.17)$$

$$p = \frac{\omega}{il^2}\frac{dv_r}{dr}+\left[\frac{\omega}{il^2 r}-\frac{2C^2\omega}{ir^3(\omega^2-l^2A^2)}\right]v_r \qquad \ldots (3.18)$$

$$e_r = \frac{iC}{lr}\frac{dv_r}{dr}-\left[\frac{C}{ilr^2}+\frac{2ClA^2}{ir^2(\omega^2-l^2A^2)}\right]v_r \qquad \ldots (3.19)$$

$$e_\phi = Av_r \qquad \ldots (3.20)$$

and

$$e_z = -\frac{C}{r}v_r. \qquad \ldots (3.21)$$

The solutions for the electromagnetic field in vacuum are

$$b_r = E\,K_1(lr), \qquad \ldots (3.22)$$

$$b_\phi = 0, \qquad \ldots (3.23)$$

$$b_z = -iE\,K_0(lr), \qquad \ldots (3.24)$$

$$e_r = M\,K_1(lr), \qquad \ldots (3.25)$$

$$e_\phi = \frac{\omega}{l}E\,K_1(lr) \qquad \ldots (3.26)$$

and

$$e_z = -iMK_0(lr) \qquad \ldots (3.27)$$

where B_0, D_0, E and M are constants of integration. I_0, K_0 and I_1, K_1 are the modified Bessel functions of zeroth and first order.

4. Azimuthal Disturbances

In this section, we consider the equations of the perturbed quantities due to azimuthal disturbances. Starting with the steady state, we apply the perturbation of the type exp $(i\omega t+in\phi)$, where n is the azimuthal wavenumber and ω is the frequency. Denoting by $\vec{v}$, p, $\vec{b}$ and $\vec{e}$, the perturbed velocity, pressure, magnetic field, electric field respectively, and using the momentum and electromagnetic field equations, we get the following equation for the radial component of velocity for the field (2.1):

$$\left(-\frac{\omega r^2}{n^2}+\frac{B_\phi^2}{\omega}\right)\frac{d^2v_r}{dr^2}+\left(-\frac{3r\omega}{n^2}+\frac{2B_\phi}{\omega}\frac{dB_\phi}{dr}+\frac{B_\phi^2}{\omega r}\right)\frac{dv_r}{dr}+\frac{(1-n^2)}{r^2}\left(-\frac{\omega r^2}{n^2}+\frac{B_\phi^2}{\omega}\right)v_r = 0. \qquad \ldots (4.1)$$

The equations governing other perturbed quantities are

$$v_\phi = \frac{i}{n}\frac{d}{dr}(rv_r) \qquad (4.2)$$

$$v_z = 0 \qquad (4.3)$$

$$b_r = \frac{nB_\phi}{\omega r}v_r \qquad (4.4)$$

$$b_\phi = \frac{i}{\omega}\frac{d}{dr}(B_\phi v_r) \qquad (4.5)$$

$$b_z = \frac{i}{\omega}\frac{dB_z}{dr}v_r \qquad (4.6)$$

$$p = \frac{\omega r}{in^2}\frac{d}{dr}(rv_r)+\left[\frac{B_\phi}{i\omega}\frac{dB_\phi}{dr}+\frac{B_\phi{}^2}{i\omega r}+\frac{B_z}{i\omega}\frac{dB_z}{dr}\right]v_r \qquad (4.7)$$

$$e_r = \frac{B_z}{in}\frac{d}{dr}(rv_r) \qquad (4.8)$$

$$e_\phi = B_z v_r \qquad (4.9)$$

and

$$e_z = -B_\phi v_r. \qquad (4.10)$$

As in section 3, we note that the solution of (4.1) could be obtained if we postulate the type of magnetic field. Thus we make use of the particular field given by (2.2) to get the following equation for v_r:

$$\frac{d^2v_r}{dr^2}+\frac{1}{r}\left[-1-\frac{4\omega}{\left(-\omega+\dfrac{C^2n^2}{\omega r^4}\right)}\right]\frac{dv_r}{dr}+\frac{(1-n^2)}{r^2}v_r = 0. \qquad (4.11)$$

In this case also, if we assume that the strength of the line current is small and expand the coefficient of the second highest derivative, we get Cauchy-Euler type of equation in zeroth order. If we assume that C is so small that C^4 and higher powers can be neglected, we can build up an approximate solution using the method of variation of parameter. Further, the resulting solution will be an integral equation in v_r as in the Picard's method. We can build up the set of solutions in various order of approximations which will be convergent. We thus consider the solution to be at first iteration for our convenience,

$$v_r = A_0\left[r^{n-1}+\frac{n^2(n-1)r^{n-5}C^2}{\omega^2(2n-4)}\right]+B_0\left[\frac{1}{r^{n+1}}+\frac{n^2(n+1)C^2}{\omega^2(2n+4)r^{n+5}}\right]. \qquad (4.12)$$

The solutions of other perturbed quantities in terms of v_r are

$$v_\phi = \frac{i}{n}\frac{d}{dr}(rv_r) \quad \text{.. (4.13)}$$

$$v_z = 0 \quad \text{.. (4.14)}$$

$$b_r = \frac{nC}{\omega r^2} v_r \quad \text{.. (4.15)}$$

$$b_\phi = \frac{iC}{\omega}\frac{d}{dr}\left(\frac{v_r}{r}\right) \quad \text{.. (4.16)}$$

$$b_z = 0 \quad \text{.. (4.17)}$$

$$p = \frac{\omega r}{in^2}\frac{d}{dr}(rv_r) \quad \text{.. (4.18)}$$

$$e_r = \frac{A}{in}\frac{d}{dr}(rv_r) \quad \text{.. (4.19)}$$

$$e_\phi = Av_r \quad \text{.. (4.20)}$$

and

$$e_z = -\frac{C}{r}v_r. \quad \text{.. (4.21)}$$

We also write the solutions in vacuum for this type of disturbance using Maxwell's equation

$$b_r = \frac{F}{r^{n+1}} \quad \text{.. (4.22)}$$

$$b_\phi = \frac{F}{i\, r^{n+1}} \quad \text{.. (4.23)}$$

$$b_z = 0 \quad \text{.. (4.24)}$$

$$e_r = \frac{N}{r^{n+1}} \quad \text{.. (4.25)}$$

$$e_\phi = \frac{N}{i\, r^{n+1}} \quad \text{.. (4.26)}$$

and

$$e_z = -\frac{\omega}{n}\frac{F}{r^n} \quad \text{.. (4.27)}$$

where A_0, B_0, F and N are constants of integration.

5. Dispersion Relation for Axisymmetric Disturbance and its Discussion

We apply the boundary conditions at the perturbed surface $r = 1+\delta r \exp(i\omega t+ilz)$. The inner boundary $r = m$ is taken to be rigid, non-magnetic and non-conducting.

The dispersion relation in this case will be

$$\omega^6[I_0(l)K_1(ml)+K_0(l)I_1(ml)]+\omega^4l^2\Big[-2A^2\{I_0(l)K_1(ml)+K_0(l)I_1(ml)\}$$

$$+4C^2\{-X(1)K_0(l)K_1(ml)+Y(1)[I_1(ml)K_0(l)-I_0(l)K_1(ml)]+Z(1)I_1(ml)I_0(l)\}$$

$$+\Big\{(H_1{}^2-A^2)I_1(ml)K_0(l)-A^2I_0(l)K_1(ml)-\frac{(H_1{}^2K_1(ml)K_0(l)I_1(l)}{K_1(l)}\Big\}\Big]$$

$$+\omega^2l^4\Big[A^4\{I_0(l)K_1(ml)+K_0(l)I_1(ml)\}-2A^2\Big\{(H_1{}^2-A^2)I_1(ml)K_0(l)$$

$$-A^2I_0(l)K_1(ml)-\frac{H_1{}^2K_1(ml)K_0(l)I_1(l)}{K_1(l)}\Big\}+4C^2\Big\{X(1)K_0(l)K_1(ml)(A^2-H_1{}^2)$$

$$+Y(1)\Big[A^2I_0(l)K_1(ml)+\frac{H_1{}^2K_0(l)K_1(ml)I_1(l)}{K_1(l)}+(H_1{}^2-A^2)K_0(l)I_1(lm)\Big]$$

$$-Z(1)\Big[A^2I_1(ml)I_0(l)+\frac{H_1{}^2K_0(l)I_1(ml)I_1(l)}{K_1(l)}\Big]\Big\}\Big]$$

$$+l^6A^4\Big[(H_1{}^2-A^2)K_0(l)I_1(ml)-A^2I_0(l)K_1(ml)-\frac{H_1{}^2K_1(ml)K_0(l)I_1(l)}{K_1(l)}\Big]=0 \quad (5.1)$$

where

$$X(r)=\int_m^r\frac{[I_1(l\xi)]^2}{\xi^3}d\xi \quad .. \quad .. \quad .. \quad .. \quad (5.2)$$

$$Y(r)=\int_m^r\frac{I_1(l\xi)K_1(l\xi)}{\xi^3}d\xi \quad .. \quad .. \quad .. \quad (5.3)$$

and

$$Z(r)=\int_m^r\frac{[K_1(l\xi)]^2}{\xi^3}d\xi. \quad .. \quad .. \quad .. \quad (5.4)$$

The lower limit to these integrals have been taken to be m, the ratio of outer to inner radius. If we put $\omega'=\omega^2$, we reduce (5.1) to a cubic

$$a\omega'^3+3b\omega'^2+3c\omega'+d=0 \quad .. \quad .. \quad .. \quad (5.5)$$

where a, $3b$, $3c$ and d are the coefficients of ω^6, ω^4, ω^2 and ω^0 of eqn. (5.1). The above cubic could be reduced to

$$(a\omega'+b)^3+3H(a\omega'+b)+G=0 \quad .. \quad .. \quad .. \quad (5.6)$$

where $H=ac-b^2$ and $G=a^2d-3abc+2c^3$. We have following cases:

(i) when G^2+4H^3 is negative, the roots of the cubic are all real. For this H should necessarily be negative. Further, if ω' is +ve real, the system will be stable as ω will also be real. Nevertheless, the system will be unstable if ω' is negative implying that ω is imaginary.

(ii) when G^2+4H^3 is positive, the cubic has a pair of complex conjugate roots. The system will be unstable in this case.

(iii) If $G^2+4H^3 = 0$, the cubic has two equal roots. Further, if the equal roots are positive, the system is stable and unstable if the equal roots are negative.

(iv) If $G = 0$, $H = 0$, the cubic has its three roots equal. In this case also, the system will be stable if roots are positive and unstable if the roots are negative.

Tables I to IV give the roots of the cubic for some specified values of parameters. We note the following points from the numerical results.

TABLE I

$H_1 = 1{\cdot}0$, $C = 0{\cdot}4$, $m = 0{\cdot}9$

l	1st root of cubic	Real part of 2nd root of cubic	Imaginary part of 2nd root of cubic	Real part of 3rd root of cubic	Imaginary part of 3rd root of cubic
1	$0{\cdot}109639\times 10$	$0{\cdot}101958\times 10$	0	$0{\cdot}953831$	0
2	$0{\cdot}461945\times 10$	$0{\cdot}404784\times 10$	0	$0{\cdot}394564$	0
3	$0{\cdot}111617\times 10^2$	$0{\cdot}906166\times 10$	0	$0{\cdot}893579\times 10$	0
4	$0{\cdot}211820\times 10^2$	$0{\cdot}160711\times 10^2$	0	$0{\cdot}159276\times 10^2$	0
5	$0{\cdot}351092\times 10^2$	$0{\cdot}250802\times 10^2$	0	$0{\cdot}249191\times 10^2$	0
6	$0{\cdot}536931\times 10^2$	$0{\cdot}360852\times 10^2$	0	$0{\cdot}359143\times 10^2$	0
7	$0{\cdot}891598\times 10^2$	$0{\cdot}491024\times 10^2$	0	$0{\cdot}488975\times 10^2$	0
8	$0{\cdot}253444\times 10^3$	$0{\cdot}641225\times 10^2$	0	$0{\cdot}638776\times 10^2$	0
9	$0{\cdot}147697\times 10$	$0{\cdot}809998\times 10^2$	$0{\cdot}168523$	$0{\cdot}809998\times 10^2$	$-0{\cdot}168523$
10	$0{\cdot}474879\times 10^2$	$0{\cdot}999987\times 10^2$	$0{\cdot}229347$	$0{\cdot}999987\times 10^2$	$-0{\cdot}229347$

TABLE II

$H = 1{\cdot}0$, $m = 0{\cdot}7$, $l = 10$

C	1st root	Real part of 2nd root	Imaginary part of 2nd root	Real part of 3rd root	Imaginary part of 3rd root
$0{\cdot}4$	$0{\cdot}408802\times 10^2$	$0{\cdot}999943\times 10^2$	$0{\cdot}880738$	$0{\cdot}999943\times 10^2$	$-0{\cdot}880738$
$0{\cdot}8$	$0{\cdot}408847\times 10^2$	$0{\cdot}999771\times 10^2$	$1{\cdot}762697$	$0{\cdot}999771\times 10^2$	$-1{\cdot}762697$

(i) For the fixed value of magnetic field outside vacuum, strength of the current and the plasma thickness, we note that the system becomes unstable for large values of wavenumber. Table I represents that for $l = 9$ and 10, we have four complex roots for the dispersion relation (5.1). Moreover, we note that for asymptotically large values of l, the dispersion relation (5.1) degenerates.

TABLE III

$C = 0{\cdot}4,\ m = 0{\cdot}5,\ l = 10$

H_1	1st root of cubic	Real part of 2nd root of cubic	Imaginary part of 2nd root of cubic	Real part of 3rd root of cubic	Imaginary part of 3rd root of cubic
1·0	$0{\cdot}119210\times10^3$	$0{\cdot}620670\times10^2$	0	$0{\cdot}610902\times10^2$	0
3·0	$0{\cdot}272901\times10^3$	$-0{\cdot}245284\times10^3$	0	$-0{\cdot}246316\times10^3$	0
5·0	$0{\cdot}580279\times10^3$	$-0{\cdot}860007\times10^3$	0	$-0{\cdot}861103\times10^3$	0
7·0	$0{\cdot}104135\times10^4$	$-0{\cdot}178174\times10^4$	0	$-0{\cdot}178364\times10^4$	0
9·0	$0{\cdot}165100\times10^4$	$-0{\cdot}301106\times10^4$	0	$-0{\cdot}301334\times10^4$	0

TABLE IV

$l = 9,\ H_1 = 1{\cdot}0,\ C = 0{\cdot}4$

m	1st root	Real part of 2nd root	Imaginary part of 2nd root	Real part of 3rd root	Imaginary part of 3rd root
0·5	$0{\cdot}113102\times10^3$	$0{\cdot}177505\times10^2$	0	$0{\cdot}158778\times10^2$	0
0·7	$0{\cdot}112721\times10^3$	$0{\cdot}182225\times10^2$	0	$0{\cdot}169104\times10^2$	0
0·9	$0{\cdot}147697\times10$	$0{\cdot}809998\times10^2$	0·168523	$0{\cdot}809998\times10^2$	$-0{\cdot}168523$

(ii) For fixed value of H_1, m and l, the system becomes more unstable by increasing the strength of the current. This is exhibited in Table II.

(iii) The instability of the system increases due to increase in the external magnetic field (outside the plasma in vacuum). The value of ω^2 is negative for the field greater than equal to 3 and thus the system is unstable.

(iv) Table IV shows the dependence of the plasma thickness on the instability of the system. We conclude that the system becomes unstable if the plasma thickness is appreciably small.

6. DISPERSION RELATION FOR AZIMUTHAL DISTURBANCE AND ITS DISCUSSION

We apply the boundary conditions at the perturbed surface $r = 1+\delta r \exp(i\omega t+in\phi)$ and the inner boundary $r = m$ to be the same as in the previous section. The dispersion relation in this type of disturbance is

$$\omega^2\left(\frac{1}{m^{n+1}}+m^{n-1}\right)+nc^2\left[\frac{n(n+1)}{2(n+2)m^{n+5}}+\frac{n(n-1)m^{n-5}}{2(n-2)}\right.$$
$$\left.-\frac{(n-1)(3n-4)}{2(n-2)m^{n+1}}+\frac{(n^2+n-4)m^{n-1}}{2(n+2)}\right] = 0 \quad \text{.. .. (6.1)}$$

or

$$\omega^2 = \Delta. \quad \text{.. .. .. .. (6.2)}$$

If $\Delta < 0$, the system is unstable and if $\Delta > 0$, the system is stable. The increase in the strength of the line current increases the growth rate of instability or adds to the stability of the system according as

$$\Phi = m^{2n+4}(n^2+n-4)(n-2)+n(n-1)(n+2)m^{2n} \\ -(n+2)(n-1)(3n-4)m^4+n(n+1)(n-2)$$

is greater than zero or less than zero respectively. The neutral stability curve has been plotted in Fig. 1.

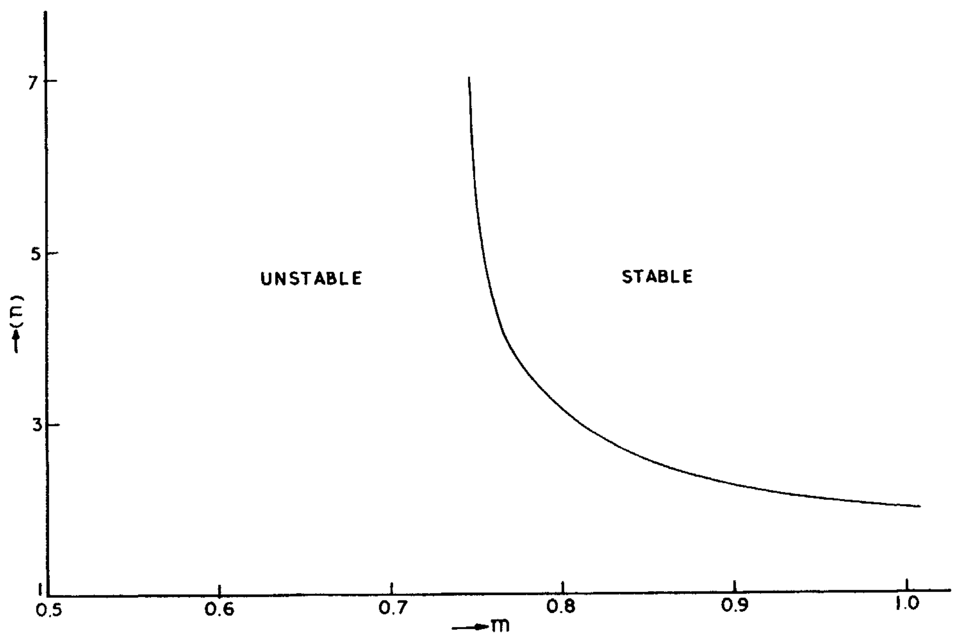

FIG. 1. Neutral stability curve.

We note that the stability of the system for this type of disturbance very strongly depends on the thickness of the plasma shell. The region to the right of the curve is stable and to the left is unstable.

REFERENCES

Auluck, F. C., and Kothari, D. S. (1957). The influence of a magnetic field on the longitudinal stability of a gravitating cylinder. *Z. Astrophys.*, **42**, 101.

Auluck, F. C., and Nayyar, N. K. (1960). The stability of a gravitating cylinder in the presence of magnetic fields. *Z. Astrophys.*, **50**, 7.

Bhat, P. K. (1968). Some Problems on Hydromagnetics. Ph.D. Thesis, Indian Institute of Science, Bangalore.

Bhatnagar, P. L., and Bhat, P. K. (1968). Stability of a collapsing or exploding cylindrical shell of conducting fluid in the presence of a magnetic field. *Proc. natn. Inst. Sci. India*, **34 A**, 115.

Chakraborty, B. B., and Bhatnagar, P. L. (1960). The stability of an infinitely conducting liquid column with uniform volume current and surface change. *Proc. natn. Inst. Sci. India*, **26** A (Suppl. II), 76.

Kruskal, M., and Schwarzchild, M. (1954). Some instabilities of a completely ionized plasma. *Proc. R. Soc.*, **223** A, 348.

Newcomb, W. A., and Kaufman, A. N. (1961). Hydromagnetic stability of tubular pinch. *Phys. Fluids*, **4**, 314.