UNSTEADY MHD FREE CONVECTION FLOW PAST AN INFINITE VERTICAL FLAT PLATE WITH VARIABLE SUCTION

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An analysis of the effects of a variable suction and the horizontal magnetic field on the free convective flow past infinite, vertical, porous plate has been carried out. Approximate solutions for the velocity, fluctuating parts of the velocity, amplitude and phase of the skin friction are obtained. This is followed by a detail comparative discussion of different parameters on the free convective flow of mercury and ionized air.

INTRODUCTION

The unsteady free convection flow of an incompressible, viscous fluid past an infinite porous plate and a semi-infinite plate was studied by Nanda and Sharma (1962, 1963). In their first paper, they assumed suction velocity to be proportional to $t^{-1/2}$ whereas in the second paper the plate temperature was assumed to oscillate in time about a constant non-zero mean with isothermal free stream. Extending the work of Lighthill (1954) and Stuart (1955), Messiha (1966) studied the unsteady flow past an infinite porous plate with variable suction when the free stream oscillates about a non-zero constant mean. The hydromagnetic problem corresponding to Messiha's problem was recently presented independently by Soundalgekar (1969) and Pop (1967). The free convection flow past an infinite porous plate, when the plate temperature oscillates about a non-zero constant mean, was analysed by Messiha (1965) in case of constant suction velocity, and by Pop (1968) in case of variable suction velocity. The plate temperature was assumed by them to be oscillating with a frequency equal to that of the variable suction velocity. Recently, Lal (1969) has considered the same problem as Pop (1968). Lal first assumes the wall temperature to be an arbitrary function of time and then takes it as constant. The hydromagnetic case corresponding to that in Messiha (1965) has been presented in a recent note by Pop (1969).

It is the object of the present paper to study the effects of the horizontal magnetic field on the unsteady free convection flow, as studied by Pop (1968), past a porous vertical plate when a variable suction is assumed to be applied at the plate. The plate temperature is assumed to oscillate with the same frequency as that of the variable suction velocity. We also assume that the induced magnetic field is negligible which is true for very small magnetic Reynolds number (Shercliff 1965).

Expressions for velocity, fluctuating parts of the velocity, skin friction, amplitude of the skin friction, etc., are derived. Velocity profiles and the fluctuating parts of it are shown on graphs, whereas the numerical values of the amplitude, phase and the skin friction are entered in tables.

Mathematical Analysis

Here x'-axis is assumed to be along the vertical, porous, infinite plate, in the upward direction, and the y'-axis is taken perpendicular to it. The origin of the coordinate system is assumed to be at the lowest point of the plate. If B_0 is the strength of the magnetic induction applied parallel to y'-axis, then the unsteady free convective flow of an electrically conducting, viscous, incompressible fluid is governed by the following set of equations:

$$\frac{\partial v'}{\partial y'} = 0 \qquad (1)$$

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = f_x \beta (T' - T'_\infty) + \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho'} u' \qquad (2)$$

$$\frac{\partial v'}{\partial t'} = -\frac{1}{\rho'} \frac{\partial p'}{\partial y'} \qquad (3)$$

$$\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \frac{k}{\rho' C_p} \frac{\partial^2 T'}{\partial y'^2}. \qquad (4)$$

Here u', v' are the velocity components in the x', y'-directions, f_x the acceleration due to gravity, β the coefficient of the volume expansion, ν the kinematic viscosity, T' the temperature in the boundary layer, T'_∞ temperature of the free-stream, t' the time, and σ, k, ρ', C_p respectively the electrical conductivity, thermal conductivity, density and heat capacity of the fluid. With the usual assumptions of free convective flows, the heat due to viscous and ohmic dissipation is neglected in eqn. (4). As the suction velocity is assumed to be time-dependent, to first order in ϵ, where ϵ is a small positive constant, we have from eqn. (1)

$$v' = -v_0(1 + \epsilon A e^{i\omega' t'}) \qquad (5)$$

where v_0 is a non-zero constant suction velocity and A is a real positive constant such that $\epsilon A \leq 1$. The negative sign in (5) indicates that the suction velocity normal to the wall is directed towards the wall. Also ω' is the frequency of the suction velocity.

The boundary conditions are:

$$\left.\begin{array}{lll} \text{(i)} & t' < 0, & u' = v' = T' = 0 \quad \text{for } y' \geq 0 \\ \text{(ii)} & t' \geq 0, & u' = 0, \quad T' = T'_w(t) \text{ for } y' = 0 \\ & & u' \to 0, \quad T' \to T'_\infty \quad \text{as } y' \to \infty \end{array}\right\}. \qquad (6)$$

Substituting for v' from (5), eqns. (2) and (4) reduce to the following non-dimensional form:

$$\frac{\partial^2 u}{\partial y^2}+(1+\epsilon\, Ae^{i\omega t})\frac{\partial u}{\partial y}-\frac{1}{4}\frac{\partial u}{\partial t}-Mu=0 \quad \ldots \quad (7)$$

$$\frac{\partial^2 \theta}{\partial y^2}+P(1+\epsilon\, Ae^{i\omega t})\frac{\partial \theta}{\partial y}-\frac{P}{4}\frac{\partial \theta}{\partial t}=0 \quad \ldots \quad (8)$$

where

$$\left.\begin{aligned} y &= \frac{|v_0|\,y'}{\nu}, \quad t=\frac{{v_0}^2 t'}{4\nu}, \quad \omega=\frac{4\nu\omega'}{{v_0}^2} \\ P &= \frac{\mu C_p}{k}, \quad G=\frac{\nu f_x \beta(T'_w-T'_\infty)}{|{v_0}^3|} \\ u &= u'/|v_0|\,G, \quad \theta=(T'-T'_\infty)/(T'_w-T'_\infty) \\ M &= 4\nu\sigma {B_0}^2/\rho' {v_0}'^2 \end{aligned}\right\} \quad \ldots \quad (9)$$

Here P is the Prandtl number, G is the Grashof number and M the hydromagnetic parameter.

By virtue of (9), the boundary conditions (6) now become

$$\left.\begin{aligned} u=0, \quad \theta=T_w(t), \quad \text{for } y=0 \\ u=0, \quad \theta\to 0 \text{ as } y\to\infty \end{aligned}\right\} \quad \ldots \quad (10)$$

where

$$T_w(t)=1+\epsilon\, e^{i\omega t}.$$

To solve eqns. (7) and (8), we assume that

$$\theta(y,t)=1+\epsilon\, e^{i\omega t}-f_1(y)-\epsilon\, e^{i\omega t} f_2(y) \quad \ldots \quad (11)$$

and

$$u(y,t)=g_1(y)+\epsilon\, e^{i\omega t} g_2(y). \quad \ldots \quad (12)$$

Substituting (11) and (12) in (7) and (8), comparing harmonic terms and neglecting coefficients of ϵ^2, we get

$$f''_1+Pf'_1=0 \quad \ldots \quad (13)$$

$$f''_2+Pf'_2-\frac{i\omega P}{4}f_2=-\frac{i\omega P}{4}-APf_1^2 \quad \ldots \quad (14)$$

$$g''_1+g'_1-Mg_1=f_1-1 \quad \ldots \quad (15)$$

$$g''_2+g'_2-\left(\frac{i\omega}{4}+M\right)g_2=-Ag'_1+f_2-1. \quad \ldots \quad (16)$$

Here primes denote the differentiation with respect to y.

The corresponding boundary conditions on f's and g's are:

$$\left.\begin{aligned} f_1=f_2=0, \quad g_1=g_2=0 \text{ at } y=0 \\ f_1\to 1, \quad f_2\to 1, \quad g_1=g_2=0 \text{ as } y\to\infty \end{aligned}\right\}. \quad \ldots \quad (17)$$

Solving (13–15) with boundary conditions (17), substituting in (11) and (12), we have

$$\theta(y, t) = e^{-Py} - \epsilon\, e^{i\omega t}[(S-1)e^{-Py} - S\, e^{-Phy}] \qquad (18)$$

and

$$u(y, t) = \frac{e^{-my} - e^{-Py}}{P^2 - P - M} + \epsilon\, e^{i\omega t}\left[\frac{A}{P^2-P-M}\left\{\frac{m(e^{-my} - e^{ny})}{m^2 - m - M - \frac{i\omega}{4}} - \frac{P(e^{-Py} - e^{-ny})}{P^2 - P - M - \frac{i\omega}{4}}\right\}\right.$$

$$\left. + \frac{S-1}{P^2 - P - M - \frac{i\omega}{4}}(e^{-Py} - e^{-ny}) - \frac{S}{P^2h^2 - Ph - M - \frac{i\omega}{4}}(e^{-Phy} - e^{-ny})\right] \qquad (19)$$

where

$$S = 1 - \frac{4AiP}{\omega}, \qquad h = \tfrac{1}{2}\left\{1 + \left(1 + \frac{i\omega}{P}\right)^{\frac{1}{2}}\right\}$$

$$m = \frac{1 + \sqrt{1+4M}}{2}, \quad n = \tfrac{1}{2}\{1 + \sqrt{1 + 4M + i\omega}\}.$$

The non-dimensional form of the skin friction is given by

$$\tau = \frac{\tau'}{\rho' v_0'^2} = \left(\frac{\partial u}{\partial y}\right)_{y=0}. \qquad (20)$$

Hence from (19) and (20), we obtain

$$\tau = \frac{P-m}{P^2 - P - M} + \epsilon\, e^{i\omega t}\left[\frac{A}{P^2 - P - M}\left\{\frac{m(n-m)}{m^2 - m - M - \frac{i\omega}{4}} - \frac{P(n-P)}{P^2 - P - M - \frac{i\omega}{4}}\right\}\right.$$

$$\left. + \frac{(S-1)(n-P)}{P^2 - P - M - \frac{i\omega}{4}} - \frac{S(n-Ph)}{P^2h^2 - Ph - M - \frac{i\omega}{4}}\right]. \qquad (21)$$

To obtain the fluctuating parts of the velocity profile, we have from (19),

$$u(y, t) = \frac{e^{-my} - e^{-Py}}{P^2 - P - M} + \epsilon[M_r \cos \omega t - M_i \sin \omega t] \qquad (22)$$

where M_r, M_i are the fluctuating parts, given by

$$
\begin{aligned}
M_r = & \frac{Am}{A_1\left(A_2^2+\frac{\omega^2}{16}\right)}\left\{A_2 e^{-my}-e^{-n_r y}\left(A_2 \cos n_i y+\frac{\omega}{4}\sin n_i y\right)\right\} \\
& -\frac{AP}{A_1\left(A_1^2+\frac{\omega^2}{16}\right)}\left\{A_1 e^{-Py}-e^{-n_r y}\left(A_1 \cos n_i y+\frac{\omega}{4}\sin n_i y\right)\right\} \\
& -\frac{4AP}{A_1\omega}\left\{e^{-n_r y}\left(A_1 \sin n_i y-\frac{\omega}{4}\cos n_i y\right)+\frac{\omega}{4}e^{-Py}\right\} \\
& -\frac{\left[e^{-Ph_r y}\left\{\left(A_3-\frac{4AP}{\omega}A_4\right)\cos Ph_i y+\left(A_4+\frac{4AP}{\omega}A_3\right)\sin Ph_i y\right\} -e^{-n_r y}\left\{\left(A_3-\frac{4AP}{\omega}A_4\right)\cos n_i y+\left(A_4+\frac{4AP}{\omega}A_3\right)\sin n_i y\right\}\right]}{A_3^2+A_4^2}
\end{aligned} \tag{23}
$$

$$
\begin{aligned}
M_i = & \frac{Am}{A_1\left(A_2^2+\frac{\omega^2}{16}\right)}\left\{\frac{\omega}{4}e^{-my}-e^{-n_r y}\left(\frac{\omega}{4}\cos n_i y-A_2 \sin n_i y\right)\right\} \\
& -\frac{AP}{A_1\left(A_1^2+\frac{\omega^2}{16}\right)}\left\{\frac{\omega}{4}e^{-my}-e^{-n_r y}\left(\frac{\omega}{4}\cos n_i y-A_1 \sin n_i y\right)\right\} \\
& +\frac{4AP}{A_1\omega}\left\{A_1 e^{-Py}-e^{-n_r y}\left(A_1 \cos n_i y-\frac{\omega}{4}\sin n_i y\right)\right\} \\
& -\frac{\left[e^{-Ph_r y}\left\{\left(A_4+\frac{4AP}{\omega}A_3\right)\cos Ph_i y-\left(A_3-\frac{4AP}{\omega}A_4\right)\sin Ph_i y\right\} -e^{-n_r y}\left\{\left(A_4+\frac{4AP}{\omega}A_3\right)\cos n_i y-\left(A_3-\frac{4AP}{\omega}A_4\right)\sin n_i y\right\}\right]}{A_3^2+A_4^2}
\end{aligned} \quad .. \tag{24}
$$

where

$$h_r = \tfrac{1}{2}+\tfrac{1}{2}\left\{\tfrac{1}{2}\left(\left(1+\frac{\omega^2}{P^2}\right)^{\frac{1}{2}}+1\right)\right\}^{\frac{1}{2}}$$

$$h_i = \tfrac{1}{2}\left\{\tfrac{1}{2}\left(\left(1+\frac{\omega^2}{P^2}\right)^{\frac{1}{2}}-1\right)\right\}^{\frac{1}{2}}$$

$$A_1 = P^2-P-M, \quad A_2 = m^2-m-M$$

$$A_3 = P^2(h_r^2-h_i^2)-Ph_r-M$$

$$A_4 = 2P^2 h_r h_i - Ph_i - \frac{\omega}{4}.$$

Also, from (21), we have

$$\tau = \frac{P-m}{P^2-P-m} + \epsilon\,|B|\cos(\omega t+\alpha) \qquad (25)$$

where the amplitude of the skin friction, $|B|$, is given by

$$|B| = \frac{A}{P^2-P-M}\left\{\frac{m(n-m)}{m^2-m-M-\frac{i\omega}{4}} - \frac{P(n-P)}{P^2-P-M-\frac{i\omega}{4}}\right\}$$

$$+\frac{(S-1)(n-P)}{P^2-P-M-\frac{i\omega}{4}} - \frac{S(n-Ph)}{P^2h^2-Ph-M-\frac{i\omega}{4}} = B_r+iB_i \qquad (26)$$

$$\alpha = \tan^{-1}(B_i/B_r) \qquad (27)$$

$$B_r = \frac{A}{A_1}\left[\frac{m\left(A_2(n_r-m)-\frac{\omega n_i}{4}\right)}{A_2^2+\frac{\omega^2}{16}} - \frac{P\left(A_1(n_r-P)-\frac{\omega n_i}{4}\right)}{A_2^2+\frac{\omega^2}{16}}\right] - \frac{4AP}{\omega}\,\frac{A_1 n_i+\frac{\omega}{4}(n_r-P)}{A_1^2-\frac{\omega^2}{16}}$$

$$-\frac{A_3(n_r-Ph_r)-A_4(n_i-Ph_i)-\frac{4AP}{\omega}\{A_3(n_i-Ph_i)+A_4(n_r-Ph_r)\}}{A_3^2+A_4^2} \qquad (28)$$

$$B_i = \frac{A}{A_1}\left[\frac{m\left(\frac{\omega}{4}(n_r-m)+n_iA_2\right)}{A_2^2+\frac{\omega^2}{16}} - \frac{P\left(n_iA_1+\frac{\omega}{4}(n_r-P)\right)}{A_1^2+\frac{\omega^2}{16}}\right] + \frac{4AP}{\omega}\,\frac{A_1(n_r-P)-\frac{\omega n_i}{4}}{A_1^2+\frac{\omega^2}{16}}$$

$$-\frac{A_3(n_i-Ph_i)+A_4(n_r-Ph_r)+\frac{4AP}{\omega}\{A_3(n_r-Ph_r)-A_4(n_i-Ph_i)\}}{A_3^2+A_4^2} \qquad (29)$$

where

$$n_r = \tfrac{1}{2}+\tfrac{1}{2}\left\{\frac{\sqrt{(1+4M)^2+4\omega^2}+1+4M}{2}\right\}^{\frac{1}{2}}$$

$$n_i = \tfrac{1}{2}\left\{\frac{\sqrt{(1+4M)^2+4\omega^2}-(1+4M)}{2}\right\}^{\frac{1}{2}}.$$

Results and Discussion

For large values of ω, with constant M and A, we have

$$h \sim (i\omega/P)^{\frac{1}{2}}, \quad n \sim (i\omega)^{\frac{1}{2}}.$$

Hence from (19),

$$u(y,\,t) = \frac{e^{-my}-e^{-Py}}{P^2-P-M} + \epsilon\, e^{i\omega t}\left[\frac{Ai}{P^2-P-M}\left\{\frac{4m}{\omega}\left(e^{-my}-e^{-y\sqrt{i\omega}}\right)+\frac{P}{\omega}\left(e^{-Py}-e^{-y\sqrt{i\omega}}\right)\right\} + \frac{i(P-\frac{1}{4})(1-4AiP)}{\omega}\left(e^{-y\sqrt{i\omega P}}-e^{-y\sqrt{i\omega}}\right)\right] \quad \ldots \quad (30)$$

and from (26),

$$|B| = \frac{A}{P^2-P-M}\left\{\frac{4mi}{\omega}\left(\sqrt{i\omega}-m\right)-\frac{4Pi}{\omega}\left(\sqrt{i\omega}-P\right)\right\} + 16AP\sqrt{\frac{i}{\omega^3}} + \frac{i\sqrt{i}}{\sqrt{\omega}}(P-\tfrac{1}{4})(1-4AiP)(1-\sqrt{P}). \quad \ldots \quad (31)$$

To study the effects of different parameters on the flow field, calculations have been carried out for different values of P, ω, M and A. To be realistic, the fluids considered here are mercury ($P = 0{\cdot}025$) and ionized air ($P = 0{\cdot}72$). From eqn. (22), the expression for the velocity when $\omega t = \pi/2$ is obtained as

$$u(y) = \frac{e^{-my}-e^{-Py}}{P^2-P-M} - \epsilon M_i. \quad \ldots \quad (32)$$

The velocity profiles are shown on Figs. 1 and 2. The effects of the magnetic field on the velocity profiles, in case of mercury, can be seen from Fig. 1 and the curves I, II, III and IV from Fig. 2. Due to the presence of the magnetic field, the velocity is reduced appreciably when A and ω are constant. However, an increase in A leads to an increase in the velocity for all values of M and ω. In case of an ionized air, there is reduction in velocity due to presence of the magnetic field. However, it is not so appreciable. An increase in ω leads to a decrease in the velocity.

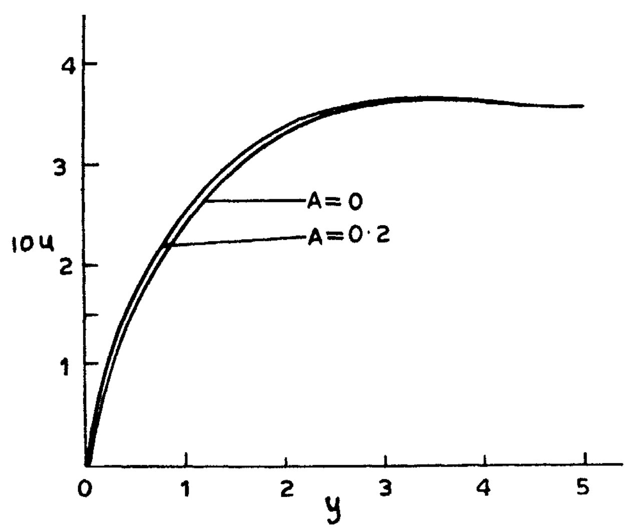

Fig. 1. Velocity profiles. $\omega = 10$, $P = 0{\cdot}025$, $\omega t = \pi/2$, $\epsilon = 0{\cdot}2$, $M = 0$.

But it is interesting to note that the velocity, in case of air, fades away very near the vertical plate than it is in case of mercury.

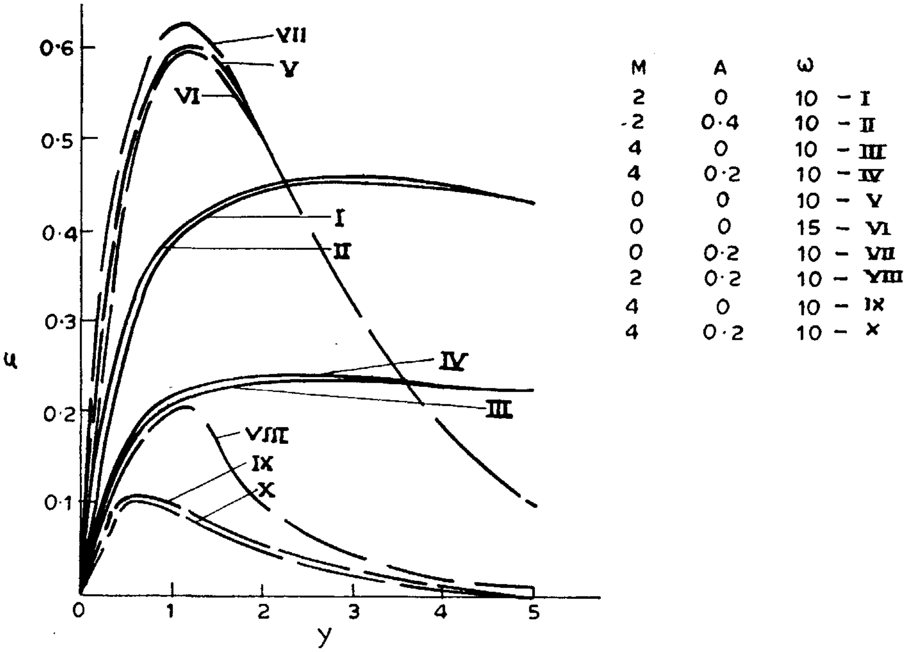

FIG. 2. Velocity profiles. $P = 0{\cdot}025$ — ; $0{\cdot}72$ —— ; $\epsilon = 0{\cdot}2$ - - - .

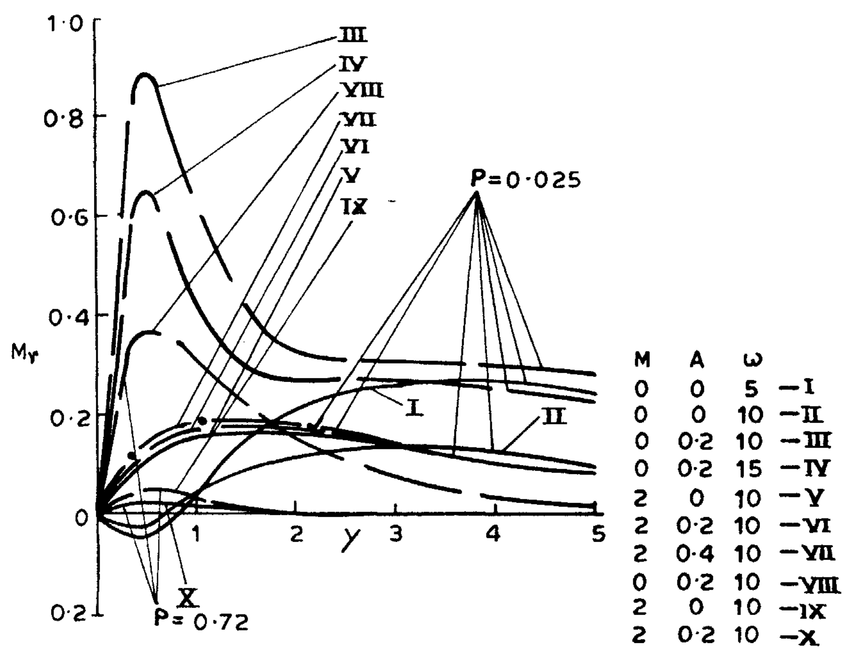

FIG. 3. Fluctuating part of the velocity profiles. $M_rA = 0$ —— ; $0{\cdot}2$ - - - ; $0{\cdot}4$ - · - · - · - .

The fluctuating parts of the velocity profiles are shown in Figs. 3 and 4. In case of mercury, M_r is negative for small values of ω and constant suction velocity ($A = 0$) near the plate. However, an increase in ω leads to a decrease in M_r when A or M is constant. But due to the application of the magnetic field, M_r is positive in case of mercury, which can be observed from curves V, VI, VII in Fig. 3. An increase in A leads to an increase in M_r for all values of M and ω.

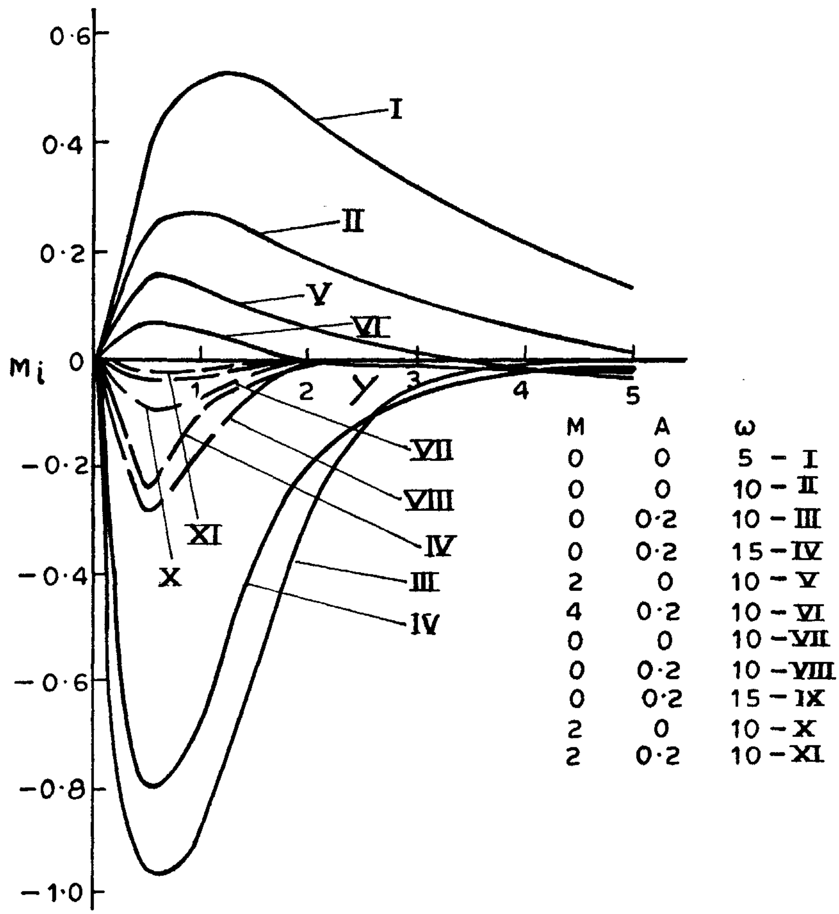

FIG. 4. Fluctuating part of the velocity profiles. $P = 0{\cdot}025$ —— ; $0{\cdot}72$ - - - .

But in case of an ionized air, M_r is positive and also decreases with increasing the strength of the magnetic field. The effect of A is the same as in the case of mercury.

M_i is shown on Fig. 4. In case of mercury ($P = 0{\cdot}025$), it has interesting features. For $M = 0$, $A = 0$, M_i is positive whereas for $M = 0$ and $A \neq 0$, M_i is negative for all ω. M_i decreases with increasing ω when M and A are constant. M_t also decreases in the presence of the magnetic field which can be seen from curves II, III, V and VI.

In case of an ionized air, M_t is completely negative for all M and A. M_t decreases with increasing A (curves VII and VIII, Fig. 4) when $M = 0$, but increases with increasing A in the presence of the magnetic field for the same ω (curves X, XI, Fig. 4). An increase in ω leads to an increase in M_t (curves VIII, IX, Fig. 4).

In Table I the numerical values of the amplitude and phase of the skin friction and the skin friction when $\omega t = \pi/2$ are entered. A close study of the table reveals that the amplitude $|B|$ of the skin friction, in case of mercury, is greater than that of the ionized air. An increase in ω leads to a decrease in $|B|$. An increase in M also leads to a decrease in $|B|$. But an increase in A leads to an increase in $|B|$.

TABLE I

P	M	A	ω	$\|B\|$	$\tan \alpha$	τ
0·025	0	0	5	1·2611	−2·3444	40·23
			10	0·8603	−1·7873	40·15
			15	0·6917	−1·5988	40·11
		0·2	5	6·9798	−0·4780	40·60
			10	5·2875	−0·6120	40·55
			15	4·4578	−0·6757	40·49
		0·4	5	14·9297	−0·5486	41·43
			10	11·3331	−0·6643	41·25
			15	9·5521	−0·7194	41·11
	2	0	5	0·8028	1·1066	0·8564
			10	3·7027	3·5161	0·8404
		0·2	5	0·8629	0·8712	0·8622
			10	0·7009	2·2120	0·8478
0·72	0	0	5	0·7616	−0·2472	1·4254
			10	0·5185	−0·5081	1·4358
			15	0·3984	−0·6356	1·4316
		0·2	5	0·9092	−0·3579	1·4501
			10	0·6742	−0·5582	1·4546
			15	0·5408	−0·6574	1·4483
		0·4	5	1·0634	−0·4421	1·4749
			10	0·8305	−0·5906	1·4733
			15	0·6833	−0·6704	1·4649
	2	0	5	0·3545	−0·3352	0·6039
			10	0·2865	−0·3636	0·6009
		0·2	5	0·4466	−0·4172	0·6157
			10	0·3487	−0·4516	0·6101
	4	0·2	5	0·4228	−0·3792	0·4682
			10	0·3410	−0·4668	0·4671

It is interesting to note that the phase of the skin friction is negative for all M, A and ω in case of an ionized air, whereas it is negative, in case of mercury only, in the absence of the magnetic field. It is observed that the phase of the skin friction is positive, in case of mercury, when the magnetic field is present. Only in case of mercury when $A = 0$ (constant suction), an increase in ω leads to an increase in tan α. Otherwise, in all cases, an increase in ω leads to a decrease in tan α. An increase in A also leads to a decrease in tan α. Tan α increases with increasing M but in the presence of the magnetic field, in case of mercury, an increase in ω leads to an increase in tan α.

The numerical values of the skin friction τ, as calculated from (21), are interesting. In case of mercury, the application of the magnetic field reduces the skin friction quite appreciably for all values of A and ω. For constant A and M, an increase in ω leads to a decrease in τ. But an increase in A leads to an increase in τ when M and ω are constant.

In conclusion, as the temperature field is not affected by the magnetic field, probably due to neglecting the induced magnetic field, the effects of A and ω on the temperature field have not been discussed here. It has already been discussed by Pop (1968).

In the next paper, to be presented soon, the problem will be presented without neglecting the induced magnetic field.

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