EFFECTS OF EXTERNAL CIRCUIT ON MHD COUETTE FLOW BETWEEN CONDUCTING WALLS WITH HEAT TRANSFER

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An analysis of MHD Couette flow has been presented under the following conditions: (i) the walls are electrically conducting, (ii) fully developed flow, (iii) the presence of the magnetic field outside the channel and (iv) the walls are either perfectly or not perfectly thermally conducting. Closed form solutions are derived for flow and temperature fields. The effects of M (Hartmann number); ϕ_1, ϕ_2 (electrical conductance ratios for upper and lower walls); ψ (the thermal conductance ratio for upper and lower walls); and I_y, S (non-dimensional parameters) have been discussed.

1. INTRODUCTION

MHD Couette flow of an electrically conducting, incompressible viscous fluid has been discussed by several research workers. Lehnert (1952), Pai (1962), Hughes and Young (1966), Agarwal (1962) and Soundalgekar (1967) have discussed the flow between non-conducting plates whereas Yen and Chang (1964) and Soundalgekar and Dhavale (1970) discussed the effects of conducting walls on the plane and generalized MHD Couette flow.

The heat transfer aspect in plane and generalized MHD Couette flow was discussed by Soundalgekar (1969) and Soundalgekar and Dhavale (1970). The effect of conducting walls on the flow field has been discussed with the help of the boundary conditions on the magnetic field at the conducting walls, first derived by Shercliff (1956), which help us to define the electrical conductance ratio for two walls of different conductivities and thicknesses. The corresponding problem of flow between two stationary conducting plates has been discussed by Chang and Yen (1962).

But in a recent monograph by Gold (1966), Shercliff's boundary conditions are generalized by taking into account the presence of the magnetic field in free space outside the channel. The corresponding heat transfer aspect of the channel flow has been presented by Soundalgekar and Desai (1971).

But how such a magnetic field, present in free space, affects the flow field in a MHD Couette flow between conducting walls, has not been investigated as yet. Again in all the problems, it was assumed that the plates are perfectly thermally conducting, i.e. the fluid temperature equals the walls temperature. But this assumption is not true for all the solids. From the knowledge of the solid state physics (Kittel 1966), it is known that for metallic walls, by the Wiedermann-Franz law, the ratio of the thermal conductivity to the electrical conductivity is given by

$$\frac{K_w}{\sigma_w} = \frac{\pi^2}{3}\left(\frac{K}{e}\right)^2 T$$

where K_w is the thermal conductivity, σ_w the electrical conductivity, K the Boltzmann constant, e the electronic charge and T the absolute temperature.

Hence in case of plates with arbitrary thermal and electrical conductivities, a more general wall condition on the temperature should be taken into account. Such boundary conditions in case of channel flow between conducting vertical plates were first derived by Yu and Yang (1969) who defined a thermal conductance ratio similar to the electrical conductance ratio as defined by Yen and Chang (1964). The heat transfer aspect of the horizontal MHD channel flow between conducting plates with Yu and Yang's modified boundary conditions on temperature has recently been discussed by Soundalgekar (*unpublished data*).

It is the object of the present paper to discuss a MHD Couette flow with Gold's general boundary conditions on the magnetic field and Yu and Yang's boundary conditions on the temperature. In section 2, the problem is posed and closed form solutions for the velocity, magnetic field, current density, skin friction are derived in case of fully developed flow. Then the energy equation is integrated for (1) perfectly thermally conducting plates and (2) for the plates which are not perfectly thermally conducting (Yu and Yang's boundary conditions). The flow field and the temperature field are shown graphically whereas the numerical values of the rate of heat transfer, mean mixed temperature, skin friction are entered in tables.

During the course of discussion, in section 3, the effects of different parameters on the flow and temperature fields have been brought out.

2. Mathematical Analysis

Here z'-axis is chosen along the center line of the channel whose walls are in relative motion with a velocity u. The x'-axis is chosen perpendicular to the z'-axis and y'-axis is chosen normal to $x'z'$-plane. A uniform transverse magnetic field is assumed to be applied parallel to x'-axis. Then the fully developed plane MHD Couette flow is governed by the following equations in non-dimensional form:

$$\frac{d^2v}{dx^2}+M^2\frac{dH}{dx}=0 \quad \ldots \quad (1)$$

$$\frac{d^2H}{dx^2}+\frac{dv}{dx}=0. \quad \ldots \quad (2)$$

The boundary conditions are

$$v=\pm 1 \quad \text{at } x=\pm 1 \quad \ldots \quad (3)$$

$$\left.\begin{aligned} \frac{dH}{dx}+\frac{H}{\phi_1}&=\frac{H_1}{\phi_1} \quad \text{at } x=1 \\ \frac{dH}{dx}-\frac{H}{\phi_2}&=-\frac{H_2}{\phi_2} \quad \text{at } x=-1 \end{aligned}\right\} \quad \ldots \quad (4)$$

Here the non-dimensional quantities are defined as follows:

$$\left.\begin{aligned} v&=\frac{v'}{u}, & M&=\mu_e H_0 a(\sigma/\mu)^{\frac{1}{2}} \\ H&=\frac{H_z}{H_0R_m}, & R_m&=\sigma\mu_e Ua \\ x&=\frac{x'}{a}, & \phi_1&=\frac{\sigma_1 d_1}{\sigma a} \\ H_1&=\frac{H'}{H_0}, & \phi_2&=\frac{\sigma_2 d_2}{\sigma a} \end{aligned}\right\} \quad \ldots \quad (5)$$

where a is the half-width between the two plates; H_0 the applied magnetic field; σ_1, σ_2 the electrical conductivities of the upper and lower plates; σ the electrical conductivity of the fluid; M the Hartmann number; R_m the magnetic Reynolds number and v the fluid velocity. Also H_1 and H_2 in (4) are the constant magnitudes of the magnetic field in the upper and lower free space respectively. The parameters ϕ_1 and ϕ_2 are called the electrical conductance ratios for the upper and lower plates respectively.

Now, before proceeding for the solutions of the system of eqns. (1) and (2), we describe the situation outside the channel in free space. If a constant external electric field is applied by forcing an electric current through the side walls of the channel in the Y'-direction, the total current I'_y is given by $I'_y = H'_{z2}-H'_{z1}$. In the absence of the external circuit, $I'_y=0$, i.e. $H'_{z2}=H'_{z1}=0$. For $I'_y\neq 0$, $-H'_{z1}$ and H'_{z2} are the amounts of the net current which return to the channel through an upper and lower path respectively. If we now assume the conducting path in the lower region only then $H'_{z1}=0$ and $H'_{z2}=I'_y$. Hence in non-dimensional form

$$H_2=\frac{H'_{z2}}{H_0R_m}=\frac{I'_y}{H_0R_m}=I_y.$$

It now becomes convenient to bring out the effects of the parameters ϕ_1, ϕ_2, I_y on the flow field.

Under these assumptions, the solutions of the eqns. (1) and (2) subject to conditions (3) and (4) are given as follows:

Case I: $\phi_1 \lessgtr \phi_2$

$$v = A\left(1-\frac{\cosh Mx}{\cosh M}\right)+\frac{\sinh Mx}{\sinh M} \quad \ldots \quad (6)$$

$$H = \frac{A}{M}\frac{\sinh Mx}{\cosh M}-\frac{\cosh Mx}{M\sinh M}+\left(\phi_1+\frac{\coth M}{M}\right)(1-A) \quad \ldots \quad (7)$$

where

$$A = \frac{1}{(\phi_1-\phi_2)}\left\{\phi_1+\phi_2+\frac{2\coth M}{M}+I_y\right\}.$$

Knowing the magnetic field, we can now determine the current density from

$$J = -\frac{dH}{dx} \quad \ldots \quad (8)$$

where

$$J = \frac{aJ_y}{H_0R_m}.$$

Hence from (7) and (8), we get

$$J = \frac{\sinh Mx}{\sinh M}-\frac{A\cosh Mx}{\cosh M} \quad \ldots \quad (9)$$

The coefficient of the viscous drag at the lower plate is given by

$$RT = \left.\frac{dv}{dx}\right|_{x=-1} \quad \ldots \quad (10)$$

and hence from (10) and (6), we obtain

$$RT = M(\coth M+A\tanh M). \quad \ldots \quad (11)$$

We can similarly obtain the coefficient of magnetic drag.

Case II: $\phi_1 = \phi_2 = \phi$

In this case the solutions corresponding to (6), (7), (9) and (11) are

$$v = \frac{\sinh Mx}{\sinh M}-MA_1(\cosh M-\cosh Mx) \quad \ldots \quad (12)$$

$$H = \frac{I}{2}+\phi\frac{\coth M}{M}-\frac{\cosh Mx}{M\sinh M}-A_1\sinh Mx \quad \ldots \quad (13)$$

$$J = \frac{\sinh Mx}{\sinh M}MA_1\cosh Mx \quad \ldots \quad (14)$$

$$RT = M\coth M-M^2A_1\sinh M \quad \ldots \quad (15)$$

where

$$A_1 = \frac{I}{2(\phi M\cosh M+\sinh M)}.$$

v, H and J are shown for different M, ϕ_1, ϕ_2 and I on Figs. 1–9 whereas the numerical values of RT are entered in Table I.

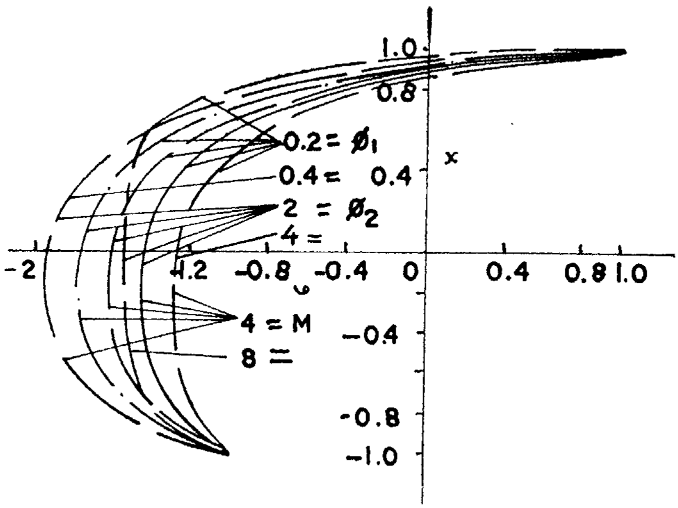

FIG. 1. Velocity profiles. $\phi_1 < \phi_2$; $I = 0$——, 0·3 – – –, 0·6 – · – · –

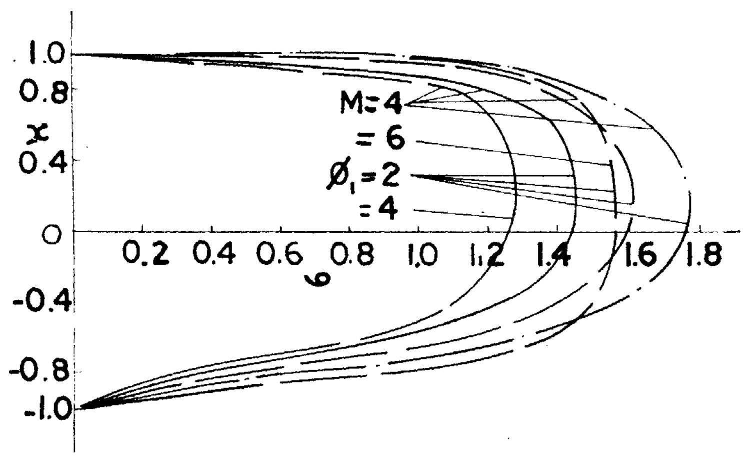

FIG. 2. Velocity profiles. $\phi_1 > \phi_2$; $\phi_2 = 0{\cdot}2$; $I = 0$ ——, 0·3 – – –, 0·6 – · – · –

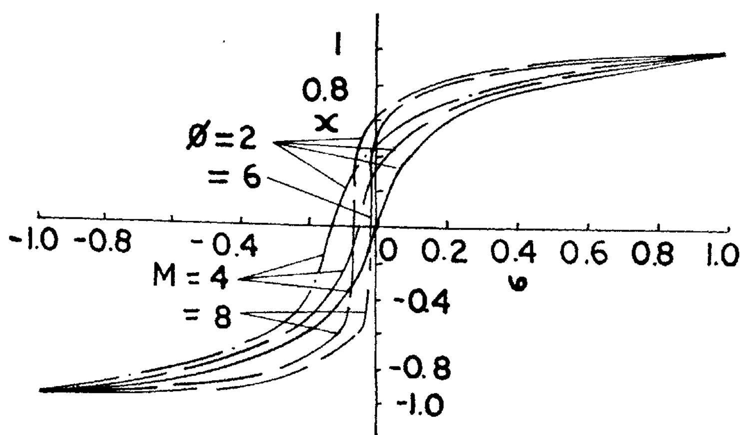

FIG. 3. Velocity profiles. $\phi_1 = \phi_2 = \phi$; $I = 0$ ——, 0·3 – – –, 0·6 – · – · –

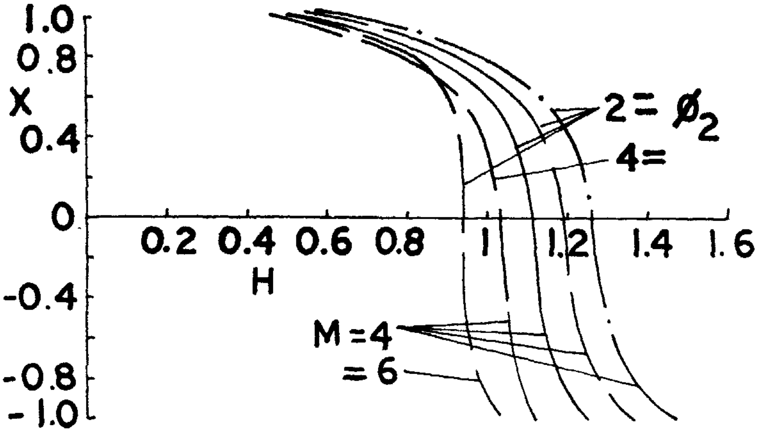

Fig. 4. Magnetic field. $\phi_1 < \phi_2$; $\phi_1 = 0{\cdot}2$; $I = 0$ ——, 0·3 – – –, 0·6 – · – · –

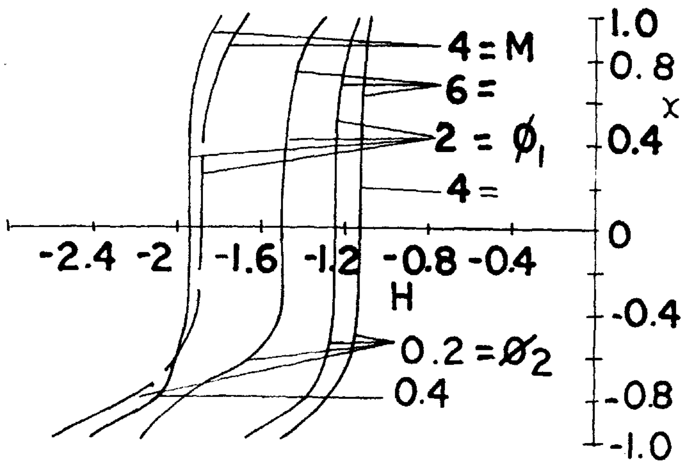

Fig. 5. Magnetic field. $\phi_1 > \phi_2$; $I = 0{\cdot}3$ ——, 0·6 – – – –

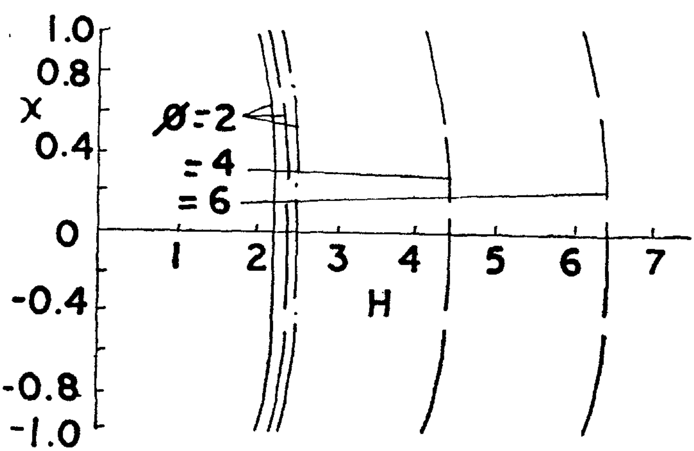

Fig. 6. Magnetic field. $\phi_1 = \phi_2 = \phi$; $M = 4$; $I = 0$ ——, 0·3 – – –, 0·6 – · – · –

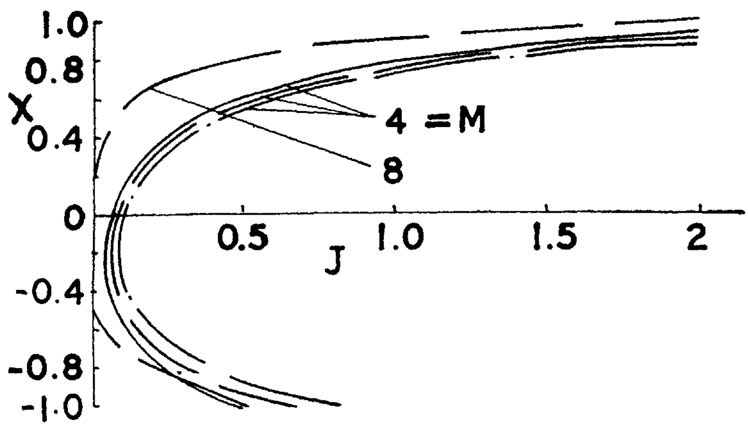

FIG. 7. Current density. $\phi_1 = 0{\cdot}2$; $\phi_2 = 2$; $I = 0$ ——, 0·3 – – –, 0·6 – · – ·

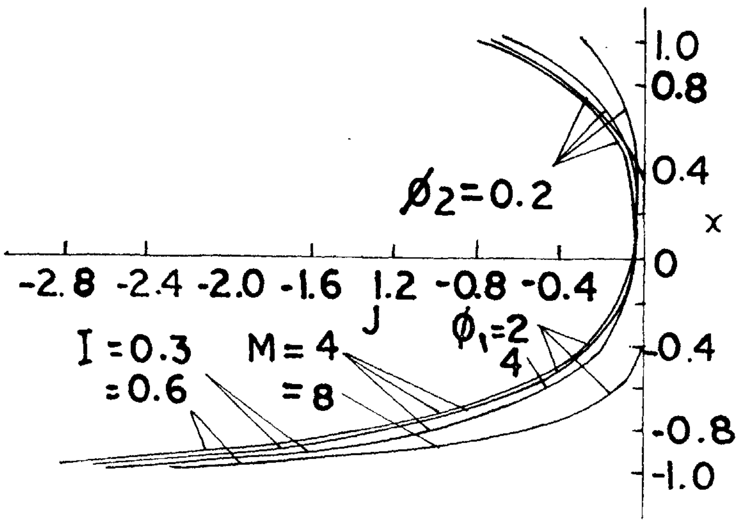

FIG. 8. Current density. $\phi_1 > \phi_2$

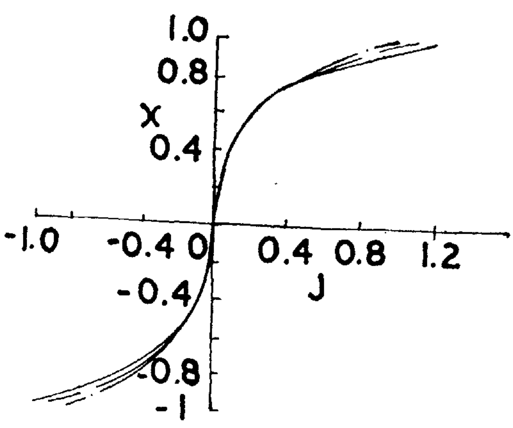

FIG. 9. Current density. $\phi_1 = \phi_2 = \phi = 2$;
$M = 4$; $I = 0$ ——, 0·3 – – –, 0·6 – · – ·

TABLE I

Values of the skin friction $RT|_{x=-1}$ [eqn. (11)]

$\phi_1 > \phi_2$

M	ϕ_1 \ ϕ_2	0·2	0·4	0·6	0·2	0·4	0·6
		$I = 0{\cdot}3$			$I = 0{\cdot}6$		
4	2	10·6656	11·9982	13·7114	11·3318	12·7477	14·5680
	4	9·2627	9·7769	10·3518	9·5782	10·1101	10·7045
	6	8·8272	9·1423	9·4807	9·0340	9·3565	9·7028
6	2	15·4444	17·3749	19·8571	16·4444	18·4999	21·1428
	4	13·6316	14·3889	15·2353	14·1052	14·8889	15·7647
	6	13·0690	13·5357	14·0370	13·3793	13·8571	14·3703

$\phi_1 < \phi_2$

M	ϕ_1 \ ϕ_2	2	4	6	2	4	6
		$I = 0{\cdot}3$			$I = 0{\cdot}6$		
4	0·2	−2·6603	−1·2573	−0·8219	−3·3265	−1·5729	−1·0286
	0·4	−3·9928	−1·7716	−1·1370	−4·7423	−2·1047	−1·3511
	0·6	−5·7060	−2·3464	−1·4754	−6·5626	−2·6991	−1·6974
6	0·2	−3·4443	−1·6314	−1·0668	−4·4443	−2·1051	−1·3791
	0·4	−5·3748	−2·3887	−1·5355	−6·4998	−2·8887	−1·8569
	0·6	−7·8569	−3·2351	−2·0369	−9·1426	−3·7645	−2·3702

The Energy Equation

Case I—If T_2 and T_1 are the temperatures of the lower and the upper walls respectively, then the energy equation for the fully developed flow is given in the following non-dimensional form:

$$\frac{d^2\theta}{dx^2}+P_rE\left[\left(\frac{dv}{dx}\right)^2+M^2\left(\frac{dH}{dx}\right)^2\right]=0 \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

where

$$\theta=\frac{T-T_1}{T_2-T_1}, \quad P_r=\frac{\mu C_p}{\lambda}, \quad E=\frac{v_0^2}{C_p(T_2-T_1)}.$$

Here P_r and E are respectively the Prandtl and the Eckert number. The boundary conditions are

$$\theta=0 \text{ at } x=1 \quad \text{and} \quad \theta=1 \text{ at } x=-1. \quad \ldots \quad \ldots \quad (17)$$

The solution of (16) satisfying (17), is

$$\theta = \tfrac{1}{2}(1-x)+\frac{P_rE}{4}\left(\frac{A^2}{\cosh^2 M}+\frac{1}{\sinh^2 M}\right)(\cosh 2M-\cosh 2Mx)$$
$$+AP_rE\left(\frac{\sinh 2Mx}{\sinh 2M}-x\right). \qquad (18)$$

θ is plotted in Figs. 10 and 11. The rate of heat transfer at the lower wall, expressed in terms of the Nusselt number, is given by

$$Nu = \frac{a}{T_2-T_1}\left(\frac{dT}{dx'}\right)_{x'=-a} \qquad (19)$$

which in non-dimensional form is given by

$$Nu = -\left(\frac{d\theta}{dx}\right)_{x'=-a}. \qquad (20)$$

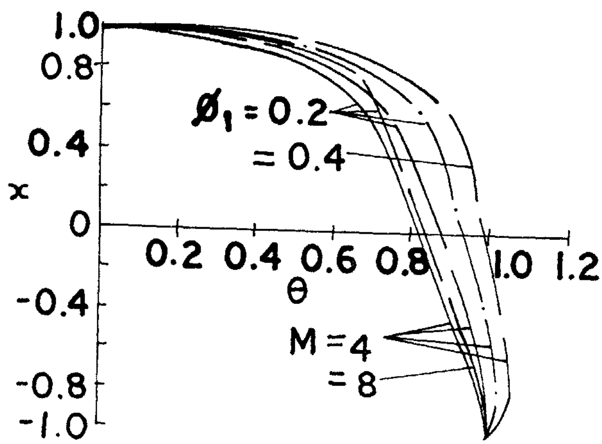

FIG. 10. Temperature profiles. $\phi_2 = 2\ (\phi_1 < \phi_2)$; $I = 0$ ——, 0·3 – – –, 0·6 – · – · –

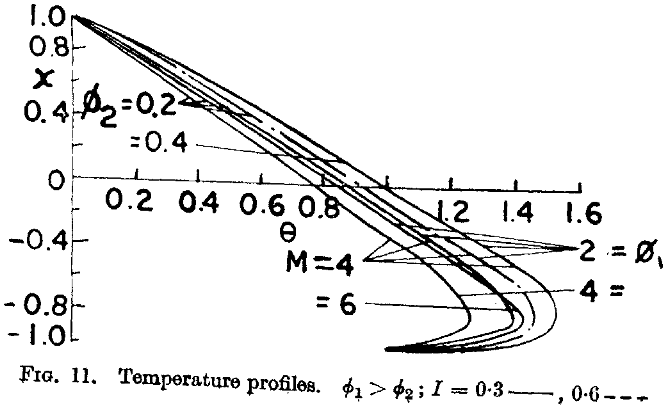

FIG. 11. Temperature profiles. $\phi_1 > \phi_2$; $I = 0{\cdot}3$ ——, 0·6 – – –

From (18) and (20), we get

$$Nu = \tfrac{1}{2} + P_r E\left[A - \left(\frac{A^2}{\cosh^2 M} + \frac{1}{\sinh^2 M}\right)\frac{M \sinh 2M}{2} + 2AM \coth 2M\right]. \quad (21)$$

The numerical values of Nu are entered in Table II.

TABLE II

Values of the Nusselt number, Nu [eqn. (21)]

				$\phi_2 > \phi_2$			
M	$\phi_1 \backslash \phi_2$	0·2	0·4	0·6	0·2	0·4	0·6
		$I = 0·3$			$I = 0·6$		
4	2	−4·8553	−6·2993	−8·4170	−5·5551	−7·1896	−9·5859
	4	−3·5271	−3·9911	−4·5411	−3·8086	−4·3058	−4·8949
	6	−3·1548	−3·4223	−3·7206	−3·3292	−3·6097	−3·9226
6	2	−7·1362	−9·1838	−12·1816	−8·1658	−10·4916	−13·8959
	4	−5·4396	−6·1217	−6·9292	−5·8618	−6·5930	−7·4587
	6	−4·9576	−5·3559	−5·8000	−5·2209	−5·6388	−6·1045
				$\phi_1 < \phi_2$			
M	$\phi_1 \backslash \phi_2$	2	4	6	2	4	6
		$I = 0·3$			$I = 0·6$		
4	0·2	−0·1882	−0·1574	0·2246	−0·4212	0·0968	0·1950
	0·4	−0·6887	0·0536	0·1778	−1·0640	−0·0278	0·1404
	0·6	−1·6163	−0·0937	0·1166	−2·1852	−0·2005	0·0702
6	0·2	−0·2103	0·1569	0·2263	−0·5066	0·0821	0·1906
	0·4	−0·8421	0·0302	0·1702	−1·3249	−0·0745	0·1231
	0·6	−2·0197	−0·1567	0·0938	−2·7911	−0·2979	0·0337

The mean mixed temperature is given by

$$T_m = \frac{\int_{-1}^{1} v\theta \, dx}{\int_{-1}^{1} v \, dx} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (22)$$

Substituting for v and θ from (6) and (18) respectively in (22) and carrying out the integration, we obtain

$$T_m = D_2 + \frac{\left[\frac{2D_1}{M}\left(\coth M - \frac{1}{M}\right) + \frac{2AP_rE}{3M}(2\coth 2M - \coth M) + \frac{AP_rE}{4}\left(\frac{A^2}{\cosh^2 M} + \frac{1}{\sinh^2 M}\right)\left\{\frac{2}{3M}(2\sinh 2M - \cosh 2M \,.\, \tanh M) - 1\right\}\right]}{2A\left(1 - \frac{\tanh M}{M}\right)} \quad \ldots \quad (23)$$

where

$$D_1 = -\tfrac{1}{2}(1 + 2AP_rE)$$

$$D_2 = \tfrac{1}{2}\left[1 + \frac{P_rE}{2}\left(\frac{A^2}{\cosh^2 M} + \frac{1}{\sinh^2 M}\right)\cosh 2M\right].$$

The numerical values of T_m are entered in Table III. The numerical values of RT, T_m and Nu when $\phi_2 = \phi_1 = \phi$ are entered in Table IV.

TABLE III

Values of mean mixed temperature T_m [eqn. (23)]

$\phi_1 > \phi_2$

M	ϕ_1 \ ϕ_2	0·2	0·4	0·6	0·2	0·4	0·6
		$I = 0·3$			$I = 0·6$		
4	2	0·6865	0·8722	1·1269	0·7785	0·9807	1·2642
	4	0·4904	0·5637	0·6434	0·5357	0·6100	0·6919
	6	0·4257	0·4729	0·5218	0·4568	0·5040	0·5533
6	2	0·5933	0·7663	0·9991	0·6826	0·8626	1·1280
	4	0·4261	0·4975	0·5745	0·4712	0·5433	0·6219
	6	0·3703	0·4168	0·4648	0·4015	0·4478	0·4958

$\phi_1 < \phi_2$

M	ϕ_1 \ ϕ_2	2	4	6	2	4	6
		$I = 0·3$			$I = 0·6$		
4	0·2	1·2858	1·2495	1·2433	1·3233	1·2518	1·2504
	0·4	1·3716	1·2552	1·2497	1·4373	1·2638	1·2498
	0·6	1·5382	1·2723	1·2507	1·6422	1·2877	1·2537
6	0·6	1·4321	1·2242	1·2113	1·5242	1·2364	1·2127

TABLE IV

Values of RT, T_m and Nu when $\phi_1 = \phi_2 = \phi$

M	ϕ	$RT\|_{x=-1}$		T_m		Nu	
		$I = 0{\cdot}3$	0·6	0·3	0·6	0·3	0·6
	2	3·7362	3·4697	2·4432	1·5064	−0·2107	−0·1281
4	4	3·8616	3·7205	4·1107	2·3391	−0·2521	−0·2057
	6	3·9067	3·8108	5·7784	3·1727	−0·2674	−0·2352
	2	5·5847	5·1693	1·7808	1·1804	−0·5535	−0·4184
6	4	5·7841	5·5681	2·8916	1·7346	−0·6224	−0·5478
	6	5·8541	5·7082	4·0027	2·2898	−0·6472	−0·5958
	2	7·4353	6·8706	1·4680	1·0268	−0·8962	−0·7084
8	4	7·7091	7·4182	2·3010	1·4420	−0·9930	−0·8903
	6	7·8041	7·6082	3·1343	1·8584	−1·0275	−0·9570

Case II—If the temperature of the walls is assumed to vary linearly along the walls, in the direction of the flow, then the temperature can be assumed in the form

$$T = a_1 z + G(x') \qquad \text{(24)}$$

where a_1 is the temperature gradient along the wall and G is the temperature of the fluid. In this case, the temperature field is governed by the following energy equation, viz.

$$\rho c_p v_z \frac{\partial T}{\partial z} = \lambda \frac{\partial^2 T}{\partial x'^2} + \frac{1}{\sigma}\left(\frac{dH_z}{dx'}\right)^2. \qquad \text{(25)}$$

Here the term representing the heat due to viscous dissipation is neglected as compared with the heat due to conduction. On substituting for T from (24), eqn. (25) reduces to the following non-dimensional form:

$$Sv = \frac{d^2\theta_1}{dx^2} + M^2 P_r E\left(\frac{dH}{dx}\right)^2 \qquad \text{(26)}$$

where

$$\theta_1 = \frac{G}{\theta_0}, \quad S_1 = \frac{a\, a_1}{\theta_0}, \quad R = \frac{a\rho u}{\mu} \text{ and } S = S_1 R P_r.$$

The modified boundary conditions due to Yu and Yang (1969) are given by

$$\frac{d\theta_1}{dx} \pm \frac{\theta_1}{\psi} = 0 \quad \text{at } x = \pm 1 \qquad \text{(27)}$$

where ψ is the thermal conductance ratio defined by

$$\psi = \frac{\lambda h}{K_w a}.$$

Here h is the thickness of the walls and K_w is the thermal conductivity of the plates. Substituting for v and H from (6) and (7) respectively in (26) and solving it under the boundary conditions (27), we obtain

$$\theta_1 = \left[\frac{SA}{2} - \frac{M^2P_rE}{4}\left(\frac{A^2}{\cosh^2 M} - \frac{1}{\sinh^2 M}\right)\right]x^2 + \frac{S}{M^2}\left(\frac{\sinh Mx}{\sinh M} - \frac{A\cosh Mx}{\cosh M}\right)$$
$$-\frac{P_rE}{8}\left(\frac{A}{\cosh^2 M}+\frac{1}{\sinh^2 M}\right)\cosh 2Mx + \frac{P_rEA}{2}\cdot\frac{\sinh 2Mx}{\sinh 2M} + D_5x + D_6 \quad \ldots \quad (28)$$

where

$$D_5 = -\frac{\psi\left(\frac{2S}{M}\coth M + 2P_rEAM\coth 2M\right)+\frac{2S}{M^2}+P_rEA}{2(1+\psi)}$$

$$D_6 = \frac{P_rE}{8}\left(\frac{A^2}{\cosh^2 M}+\frac{1}{\sinh^2 M}\right)\cosh 2M + \frac{SA}{2}\left(\frac{2}{M^2}-1\right)$$
$$+\frac{M^2P_rE}{4}\left(\frac{A^2}{\cosh^2 M}-\frac{1}{\sinh^2 M}\right) - \frac{\psi}{2}\left[2AS\left(1-\frac{\tanh M}{M}\right)\right.$$
$$\left.-M^2P_rE\left(\frac{A^2}{\cosh^2 M}-\frac{1}{\sinh^2 M}\right) - \frac{MP_rE}{2}\left(\frac{A^2}{\cosh^2 M}+\frac{1}{\sinh^2 M}\right).\sinh^2 2M\right].$$

θ_1 is shown under different conditions in Figs. 12–14.

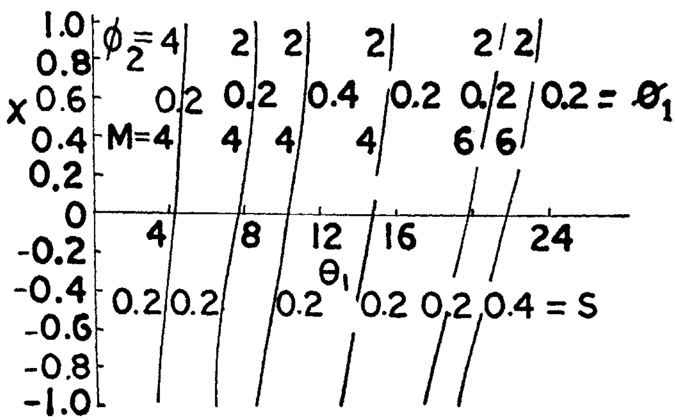

FIG. 12. Temperature profiles. $\phi_1 < \phi_2$; $I = 2$; $\psi = 2$ ——, 4 – – –

The rate of heat transfer is given by

$$q = -\lambda\left(\frac{dT}{dx'}\right)_{x'=-a}$$

which in non-dimensional form becomes

$$h = \frac{aq}{\lambda\theta} = -\left(\frac{d\theta_1}{dx}\right)_{x=-1}. \quad \ldots \quad \ldots \quad \ldots \quad (29)$$

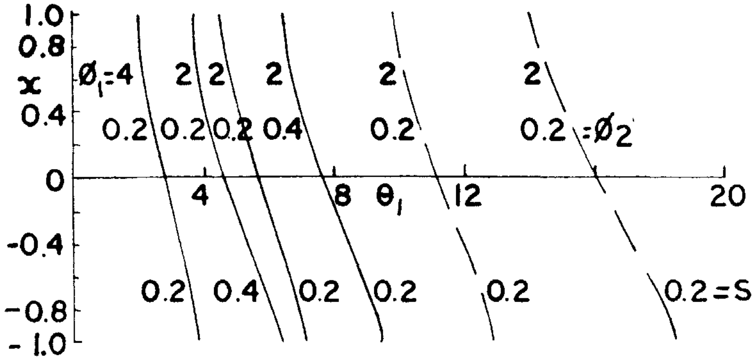

FIG. 13. Temperature profiles. $\phi_1 > \phi_2$; $I = 2$, $M = 4$; $\psi = 2$ ——, 4 - - -

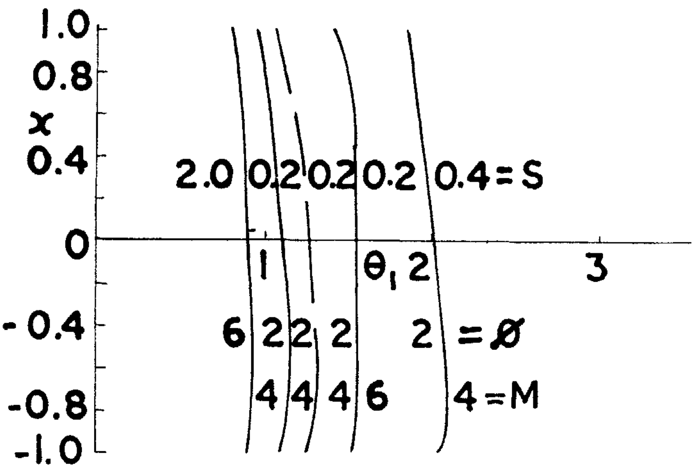

FIG. 14. Temperature profiles. $\phi_1 = \phi_2 = \phi$; $\psi = 2$ ——, 4 - - -; $I = 2$

From (28), we obtain

$$-h = \frac{M^2 P_r E}{2}\left(\frac{A^2}{\cosh^2 M} - \frac{1}{\sinh^2 M}\right) - SA + \frac{S \coth M}{M} + \frac{SA \tanh M}{M}$$

$$+\frac{MP_r E}{4}\left(\frac{A^2}{\cosh^2 M} + \frac{1}{\sinh^2 M}\right)\sinh 2M + P_r EAM \coth 2M + D_5. \quad (30)$$

The numerical values of $-h$ are entered in Table V.

3. Discussion

(1) From Figs. 1–3, we can conclude that the anti-symmetry of the velocity profiles in case of MHD Couette flow between walls of the same or zero conductivity is distorted due to the presence of the conducting walls of different conductivities. This has been explained by Yen and Chang as due to the Lorentzian forces acting on the fluid elements. However, if the magnetic field is present outside the channel whose effect is represented here by the parameter I, it is observed that an increase in I leads to an increase in the

TABLE V

Values of $-h$ (eqn. 30) *for* $I = 2$

$\phi_1 < \phi_2$

M	ψ	$\phi_1 \backslash \phi_2$	2	4	6	2	4	6
			$S = 0{\cdot}2$			$S = 0{\cdot}4$		
4	0	0·2	1·7398	0·7154	0·5150	2·1691	1·0175	0·7775
	2		2·9334	1·5132	1·1899	3·3377	1·7903	1·4275
	4		3·1722	1·6728	1·3249	3·5714	1·9448	1·5575
	0	0·4	2·5209	0·8579	0·5767	3·0179	4·3972	4·6731
	2		3·9252	1·7274	1·2934	1·1831	2·0275	2·1964
	4		4·2060	1·9013	1·4367	0·8527	1·5443	1·6827
	0	0·6	3·7658	1·0398	0·6502	4·3499	1·3906	0·9405
	2		5·4409	1·9893	1·4116	6·0000	2·3151	1·6769
	4		5·7759	2·1792	1·5639	6·3300	2·5000	1·8242
6	0	0·2	2·0833	0·7968	0·5534	2·5308	1·1111	0·8264
	2		3·9117	2·0391	1·6138	4·3408	2·3349	1·8683

$\phi_1 > \phi_2$

M	ψ	$\phi_1 \backslash \phi_2$	0·2	0·4	0·6	0·2	0·4	0·6
			$S = 0{\cdot}2$			$S = 0{\cdot}4$		
4	0	2	4·6121	5·8897	7·7730	4·2578	5·4677	7·2639
	2		3·3685	4·4354	6·0478	2·9892	3·9884	5·5138
	4		3·1198	4·1446	5·7028	2·7355	3·6926	5·1638
	0	4	2·6548	2·9663	3·3368	2·4278	2·7162	3·0610
	2		1·8069	2·0467	2·3372	1·5549	1·7717	2·0364
	4		1·6374	1·8629	2·1373	1·3803	1·5828	1·8315
6	0	2	6·7846	8·6406	11·3679	6·3926	8·1753	10·8084
	2		4·9191	6·4526	8·7653	4.5086	5·9688	8·1872

$\phi_1 = \phi_2 = \phi$

M	$\phi \backslash \psi$	0	2	4	0	2	4
		$S = 0{\cdot}2$			$S = 0{\cdot}4$		
4	2	0·4379	0·4109	0·4056	0·4748	0·4229	0·4125
	4	0·4368	0·4108	0·4056	0·4740	0·4230	0·4128
	6	0·4364	0·4107	0·4056	0·4738	0·4230	0·4129
6	2	0·6280	0·6092	0·6055	0·6558	0·6185	0·6109
	4	0·6278	0·6092	0·6054	0·6556	0·6184	0·6110
	6	0·6278	0·6092	0·6054	0·6555	0·6184	0·6110

velocity for $\phi_1 > \phi_2$, but there is a decrease in velocity with increasing I when $\phi_1 = \phi_2 = \phi$ and $\phi_1 < \phi_2$. Hence the presence of the magnetic field outside the channel enhances the Lorentzian force if $\phi_1 > \phi_2$, i.e. the electrical conductance ratio of the upper plate is greater than that of the lower plate. When the two plates are having the same electrical conductance ratios, say ϕ, we observe from Fig. 3 that an increase in ϕ leads to an increase in the velocity, i.e. Lorentzian force is enhanced.

In Fig. 9, the effects of I on the distribution of the current density is shown when $\phi_1 = \phi_2 = \phi$. It is interesting to note here that the current density is not affected by I in the central region of the channel, but J is effected by I only near the walls of the channel. The current density decreases with increasing I.

The temperature distribution is shown in Figs. 10 and 11. The temperature profiles are found to be deflected towards the wall of smaller electrical conductance ratio. However, for the same value of ϕ_1, ϕ_2 and M, an increase in I leads to an increase in the temperature of the fluid.

Figures 12–14 bring out the effects of the modified boundary conditions, viz. eqn. (27), on the temperature distribution. An increase in ψ, the thermal conductance ratio for the plates, leads to an increase in the temperature of the fluid when ϕ_1, ϕ_2, M and I are maintained at constant values.

We have the following observations from Tables I–V.

Table I

(1) An increase in I leads to an increase in the skin friction RT when M, ϕ_1, ϕ_2 are constant.

(2) When $\phi_1 > \phi_2$, for the same M and ϕ_2, an increase in ϕ_1 leads to a decrease in RT whereas for the same M and ϕ_1, an increase in ϕ_2 leads to an increase in RT. Exactly opposite is the case when $\phi_1 < \phi_2$.

(3) An increase in M leads to a decrease in RT when $\phi_1 < \phi_2$ whereas it leads to an increase in RT when $\phi_1 > \phi_2$.

Table II

(1) The Nusselt number Nu decreases with increasing I when M, ϕ_1 and ϕ_2 are constant.

(2) Nu decreases with increasing ϕ_2 when $\phi_1 > \phi_2$ whereas it increases with increasing ϕ_2 when $\phi_1 < \phi_2$ while M, ϕ_1 and I are constant.

(3) Nu increases with increasing ϕ_1 when $\phi_1 > \phi_2$ whereas it decreases with increasing ϕ_1 when $\phi_1 < \phi_2$ while I, ϕ_2 and M are constant.

Table III

(1) The mean mixed temperature T_m increases with increasing I.

(2) For the same value of M and ϕ_1, an increase in ϕ_2 leads to an increase in T_m when $\phi_1 > \phi_2$ whereas it leads to a decrease in T_m when $\phi_1 < \phi_2$.

(3) An increase in M leads to a decrease in T_m for the same value of ϕ_1, ϕ_2 and I.

Table IV ($\phi_1 = \phi_2 = \phi$)

(1) An increase in I leads to a decrease in RT when M and ϕ are constant.
(2) An increase in ϕ or M leads to an increase in RT when I is constant.
(3) T_m decreases with increasing I when M and ϕ are constant.
(4) T_m increases with increasing ϕ when M and I are constant.
(5) For the same ϕ and I, an increase in M leads to a decrease in T_m.
(6) An increase in I leads to an increase in Nu when M and ϕ are constant.
(7) An increase in ϕ leads to a decrease in Nu when M and I are constant.
(8) An increase in M leads to a decrease in Nu when ϕ and I are constant.

Table V

(1) An increase in S leads to an increase in the rate of heat transfer $(-h)$ when M, ψ, ϕ_1 and ϕ_2 are constant and $\phi_1 < \phi_2$ or $\phi_1 = \phi_2 = \phi$. However, $(-h)$ decreases as S increases when $\phi_1 > \phi_2$.

(2) For the same value of M, ψ, S and ϕ_1, an increase in ϕ_2 leads to a decrease in $(-h)$ when $\phi_1 < \phi_2$, whereas $(-h)$ increases with increasing ϕ_2 when $\phi_1 > \phi_2$.

(3) For the same value of M, ψ, S and ϕ_2, an increase in ϕ_1 leads to an increase in $(-h)$ when $\phi_1 < \phi_2$. But when $\phi_1 > \phi_2$, $(-h)$ decreases with increasing ϕ_1.

(4) When M, ϕ_1, ϕ_2, S are constant an increase in ψ, the thermal conductance ratio, leads to an increase in $(-h)$ when $\phi_1 < \phi_2$ whereas $(-h)$ decreases with increasing ψ when $\phi_1 > \phi_2$ and $\phi_1 = \phi_2$.

(5) When the values of ψ, ϕ_1, ϕ_2, S and I are constant, an increase in M leads to an increase in $(-h)$.

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