THE INTERACTION BETWEEN ISOLATED MOVING SCREW DISLOCATION AND A CIRCULAR CYLINDRICAL RIGID INCLUSION

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The paper discusses the interaction between an isolated moving screw dislocation and a cylindrical rigid inclusion contained within an infinite elastic body subject to an uniform applied shear stress at infinity. Smith (1968*a*, *b*) has discussed the same problem by omitting the inertial forces set up by the motions induced in the elements of the body by the moving dislocation. The problem is analysed by means of the assumption that the velocity of sound or shear waves in the medium plays the same role of a limiting velocity as does the velocity of light in the relativity theory.

1. INTRODUCTION

Because of the importance of the technological advantages to be gained from the development of reinforced materials and also the extent to which the fracture characteristic of such type of reinforced materials are affected, appreciable attention has been given in recent years to the understanding of the behaviour of isolated screw dislocations in reinforced medium. Particular emphasis has been given to the development of theories for stationary screw dislocations as the partial differential equations governing their behaviour are of a simpler form than those describing the corresponding edge dislocation. Barnett and Tetelman (1966) have recently discussed the antiplane strain problem of an infinite body free from shear forces at infinity and containing a circular cylindrical rigid inclusion (of radius a), the axis of which is parallel to the z-axis; and $\sigma_{yz} = p_1 - p_A$ for $a < x < a+L$, $(y = 0)$, where L is the length of the screw dislocation on the x-axis. Smith (1968*a*) has solved the same problem subject to the applied shear stress $\sigma_{yz} = p_A$ at infinity and in addition $\sigma_{yz} = p_1$ $(< p_A)$ for $a < x < a+L$ along $y = 0$. Moreover, Smith (1968*a*) has discussed the interaction between isolated screw dislocation and a cylindrical inhomogeneity contained within an infinite body subject to a uniform stress at infinity, for the case when the dislocation is external to the inhomogeneity which is assumed to have the form of a hole. However, these problems only discussed the case of a stationary dislocation and consequently leave aside the inertial stresses which set up a motion in the elements of the body by the moving dislocation.

The present paper's main aim is to consider a rigid reinforced solid model where the applied shear stress is uniform at infinity. The approach is based on the well-known analogy (Webester and Johnson 1965) between antiplane strain deformation and two-dimensional perfect fluid motion along with the use of complex variables and conformal transformation techniques, by applying an extension to the circle theorem in hydrodynamics (Milne-Thomson 1955). The stresses and displacement in the elastic body have been obtained when the dislocation is moving uniformly in the direction of the x-axis, and for the static dislocation as special case.

2. Theory

The model is illustrated in Fig. 1; a rigid cylinder of radius a and whose axis is normal to the plane of the figure, lies within an infinite body, of shear modulus μ, which subject to uniform applied shear stress $\sigma_{zx} = p_{zx}$ and $\sigma_{zy} = p_{zy}$ at infinity.

A right-hand (positive) screw dislocation of Burgers vector λ parallel to the cylinder axis intersecting the plane of Fig. 1 at the point $z^0 = x^0+iy^0$, $|z^0| > a$, moving with a uniform velocity $v = v_x$ in x-direction is considered.

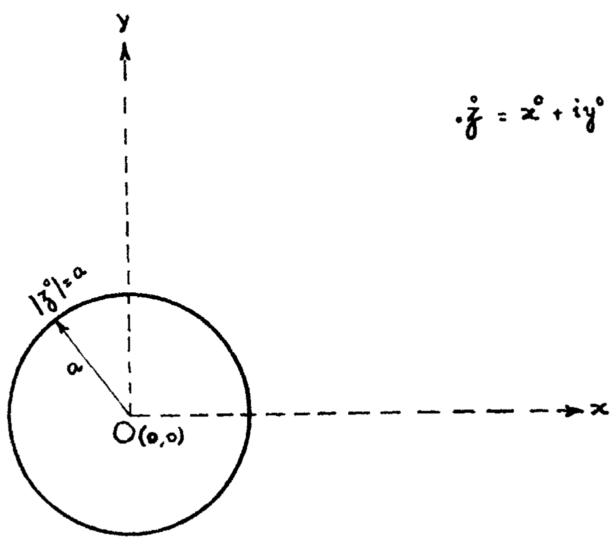

Fig. 1. External shear stresses at infinity ($\sigma_{zx} = p_{zx}$; $\sigma_{zy} = p_{zy}$).

In antiplane strain deformation the displacement is normal to the xy plane and equations of motion reduce to only one equation

$$\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}=\frac{1}{c^2}\frac{\partial^2 w}{\partial t^2} \qquad \text{(2.1)}$$

where t is the time limit and c is the speed of transverse sound or shear waves in the medium. This speed depends on the shear modulus μ and density ρ of the medium through the relation

$$c = (\mu/\rho)^{\frac{1}{2}}. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2.2)$$

The appropriate stress components outside the rigid inclusion are given by

$$\left.\begin{aligned} \sigma_{zx} &= \mu \frac{\partial w}{\partial x} \\ \sigma_{zy} &= \mu \frac{\partial w}{\partial y} \end{aligned}\right\}. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2.3)$$

The eqn. (2.1) is a typical wave equation and it is known (Hirth and Lothe 1968) that steady propagational solution can be obtained by making a Lorentz transformation of relativity, introducing the variables (x, y) with the relations

$$\left.\begin{aligned} x_1 &= \gamma(x-vt) \\ y_1 &= y \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2.4)$$

where

$$\gamma = (1-v^2/c^2)^{-\frac{1}{2}}.$$

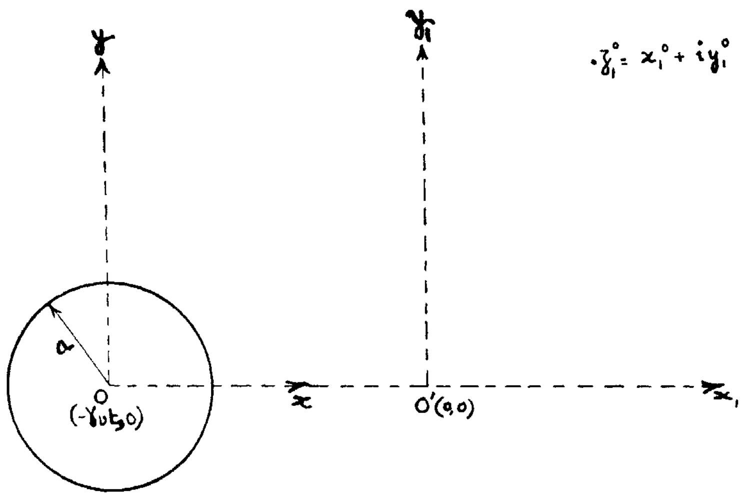

FIG. 2. External shear stresses at infinity ($\sigma_{x_1 z} = \gamma p_{xz}$; $\sigma_{y_1 z} = p_{yz}$).

When corresponding points transform as indicated above, for large value of (x, y) relations (2.4) simplifies to

$$x_1 = \gamma x; \; y_1 = y \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2.5)$$

and hence the boundary conditions for the problem in x_1y_1 plane are (Fig. 2)

$$\left.\begin{aligned} \sigma_{x_1 z} &= \gamma p_{zx} \\ \sigma_{y_1 z} &= p_{zy} \end{aligned}\right\} \text{at infinity.} \quad \ldots \quad \ldots \quad \ldots \quad (2.6)$$

Noting that in the case of a steady propagational solution

$$\frac{\partial^2}{\partial t^2} = v^2 \frac{\partial^2}{\partial x^2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2.7)$$

eqn. (2.1) becomes

$$\frac{\partial^2 w}{\partial x_1^2}+\frac{\partial^2 w}{\partial y_1^2}=0. \quad \ldots \quad (2.8)$$

Accordingly, there exists some potential function $F(z_1)$ where $z_1=x_1+y_1$, such that displacement outside the rigid inclusion can be given by

$$w=\mathrm{Rl}\,\frac{1}{\mu}.[F(z_1)] \quad \ldots \quad (2.9)$$

when the material is homogeneous (shear modulus μ), the appropriate form for $F(z_1)$ is analogous to the stationary dislocation (Smith 1968*b*) and can be given by

$$f(z_1)=(\gamma p_{zx}-ip_{zy})z_1+\frac{\mu\lambda}{2\pi i}\log(z_1-z_1^0). \quad \ldots \quad (2.10)$$

By extending the circle theorem of hydrodynamics (Milne-Thomson 1955), it can be shown that in the presence of the reinforced rigid cylindrical body of radius a, whose centre lies at $Z(-\gamma vt, 0)$ with respect to the new coordinates, the appropriate form of the complex function outside the rigid inclusion is found to be

$$F(z_1)=(\gamma p_{zx}-ip_{zy})z_1+\frac{\mu\lambda}{2\pi i}\log(z_1-z_1^0)$$
$$+(\gamma p_{zx}+ip_{zy})\frac{a^2}{z_1-Z}-\frac{\mu\lambda}{2\pi i}\log\left(\frac{a^2}{z_1-Z}-\bar{z}_1^0\right) \quad \ldots \quad (2.11)$$

from which the displacement w is given by

$$w=\frac{1}{\mu}(x_1\gamma p_{zx}+y_1p_{zy})+\frac{a^2}{\mu(x^2\gamma^2+y_1^2)}(\gamma^2xp_{zx}+y_1p_{zy})$$
$$+\frac{\lambda}{2\pi}\tan^{-1}\left(\frac{y-y^0}{x_1-x^0}\right)-\frac{\lambda}{2\pi}\tan^{-1}\left[\frac{y_1^0(x_1-Z)^2+y_1^0y_1^2-y_1a^2}{a^2(x_1-Z)-x_1^0(x_1-Z)^2-x_1^0y_1}\right]. \quad (2.12)$$

This on transforming in (x, y) system of coordinates reduces to

$$W=\frac{1}{\mu}\left[p_{zx}(x-vt)\gamma+yp_{zy}+\frac{a^2}{(x^2\gamma^2+y^2)}(x\gamma^2p_{zx}+yp_{zy})\right]+\frac{\lambda}{2\pi}\tan^{-1}\left(\frac{y-y^0}{x-x^0}\gamma\right)$$
$$-\frac{\lambda}{2\pi}\tan^{-1}\left[\frac{y^0(x^2\gamma^2+y^2)-ya^2}{a^2x\gamma-(x^0-vt)(x^2\gamma^2+y^2)}\right] \quad \ldots \quad (2.13)$$

and consequently the stresses are given by

$$\left.\begin{aligned}\sigma_{zx}&=\gamma p_{zx}+\frac{a^2\gamma p_{zx}}{(x^2\gamma^2+y^2)}-\frac{2a^2x\gamma^2}{(x^2\gamma^2+y^2)^2}(x\gamma^2p_{zx}+yp_{zy})-\frac{\mu\lambda}{2\pi}\frac{(y-y^0)\gamma}{[(x-x^0)^2+\gamma^2(y-y^0)^2]}\\&\quad-\frac{\mu\lambda}{2\pi}\frac{2a^2x^2y^0\gamma^3-a^2y^0\gamma(x^2\gamma^2+y^2)+ya^4\gamma-2xya^2\gamma^3(x^0-vt)}{[a^2x\gamma-\gamma(x^0-vt)(x^2\gamma^2+y^2)]^2+[y^0(x^2\gamma^2+y^2)-ya^2]^2}\\\sigma_{zy}&=p_{zy}+\frac{a^2p_{zy}}{(x^2\gamma^2+y^2)^2}-\frac{2a^2y}{(x^2\gamma^2y^2)^2}(x\gamma^2p_{zx}+yp_{zy})+\frac{\mu\lambda}{2\pi}\frac{\gamma(x-x^0)^2}{(x-x^0)^2+\gamma^2(y-y^0)^2}\\&\quad-\frac{\mu\lambda}{2\pi}.\frac{2a^2x.y.y^0\gamma-xa^4\gamma-2a^2y^2\gamma(x^0-vt)+a^2\gamma(x^0-vt)(x^2\gamma^2+y^2)}{[a^2x\gamma-\gamma(x^0-vt)(x^2\gamma^2+y^2)]^2+[y^0(x^2\gamma^2+y^2)-ya^2]^2}\end{aligned}\right\}.$$

$$\ldots \quad (2.14)$$

3. Special Cases

If $v \to 0$, i.e. when the dislocation is of stationary nature, $x_1 \to x$, $y_1 \to y$, $\gamma = 1$, and displacement and stresses reduce to

$$w = \frac{1}{\mu}(xp_{zx}+yp_{zy})+\frac{a^2}{\mu(x^2+y^2)}(xp_{zx}+yp_{zy}) + \frac{\lambda}{2\pi}\tan^{-1}\left(\frac{y-y^0}{x-x^0}\right)-\frac{\lambda}{2\pi}\tan^{-1}\left[\frac{y^0(x^2+y^2)-ya^2}{a^2x-x^0(x^2+y^2)}\right] \quad \ldots \quad (3.1)$$

and

$$\left.\begin{aligned} \sigma_{zx} &= p_{zx}+\frac{a^2p_{zx}}{x^2+y^2}-\frac{2a^2x}{(x^2+y^2)^2}(xp_{zx}+yp_{zx}) \\ &\quad -\frac{\mu\lambda}{2\pi}\cdot\frac{(y-y^0)}{[(x-x^0)^2+(y-y^0)^2]}-\frac{\mu\lambda}{2\pi}\cdot\frac{2a^2x^2y^0-a^2y^0(x^2+y^2)+ya^4-2xya^2x^0}{[a^2x-x^0(x^2+y^2)]^2+[y^0(x^2+y^2)-ya^2]^2} \\ \sigma_{zy} &= p_{zy}+\frac{a^2p_{zy}}{(x^2+y^2)}-\frac{2a^2y}{(x^2+y^2)^2}(xp_{zx}+yp_{zy})+\frac{\mu\lambda}{2\pi}\,\frac{(x-x^0)^2}{[(x-x^0)^2+(y-y^0)^2]} \\ &\quad -\frac{\mu\lambda}{2\pi}\,\frac{2a^2x\,.\,y\,.\,y^0-a^4x-2a^2y^2x^0+a^2x^0(x^2+y^2)}{[a^2x-x^0(x^2+y^2)]^2+[y^0(x^2+y^2)-ya^2]^2} \end{aligned}\right\}. \quad \ldots \quad (3.2)$$

4. Discussion

Moving dislocation criteria for rigid reinforced infinite body were determined for general models by application of the theory of antiplane strain deformation with the help of Lorentz transformation. As no real crystal or elastic body can support an infinite shear stress, so that the elastic solution must break down at a velocity somewhat lower than c (shear wave velocity). The above analysis may be extended to consider the interaction between a moving dislocation and inhomogeneity caused by fibre reinforced materials of different shear modulus for arbitrary cross-section.

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