IMPROVEMENT OF SASTRY'S NUMERICAL SOLUTION OF AN INTEGRAL EQUATION BY RICHARDSON EXTRAPOLATION

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Sastry's (1973) numerical solution of an integral equation is improved by applying Richardson's extrapolation to the values obtained for two different step sizes.

Sastry (1973) described a numerical procedure for solving integral equations of the form

$$f(s) = F(s) + \int_0^1 f(t)\,[P(s, t) \log | s - t | + Q(s, t)]\,dt \qquad ...(1)$$

where F, P, and Q are continuous, based on the generalized trapezoidal rule. Since the method has an error of order h^2, where h is the step size in the numerical integration, an improved solution can be obtained by solving for two different values of h and using Richardson's extrapolation to eliminate the leading error term. An explanation of the method and tables of coefficients are given by Salvadori and Baron (1971).

For the example treated by Sastry which he solved for 10 and 20 divisions, the improved solution is given by

$$v_{extr} = (4v_{20} - v_{10})/3. \qquad ...(2)$$

Table I shows that this reproduces the exact result to the four place accuracy of the calculation. It can be seen that it is thus possible to improve the accuracy significantly with very little additional work.

TABLE I

s (in degrees)	v_{10}	v_{20}	v_{extr}	Exact $v = 1{\cdot}5 \sin s$
18	0·4572	0·4619	0·4635	0·4635
36	0·8812	0·8816	0·8817	0·8817
54	1·2158	1·2141	1·2135	1·2135
72	1·4303	1·4275	1·4266	1·4266
90	1·5042	1·5011	1·5001	1·5000

References

Salvadori, M. G., and Baron, M. L. (1971). Numerical Methods in Engineering. Prentice-Hall, Inc., Englewood Cliffs, N.J., p. 86.

Sastry, S. S. (1973). A numerical solution of an integral equation of the second kind occurring in aerodynamics. *Indian J. pure appl. Math.*, **4**, 838–43.