VARIOUS TYPES OF DISLOCATION OF THE ELASTIC MATERIAL

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Dislocations of translation and rotation in plane strain and axial dislocation of translation in axial strain of an elastic material with different cross-sections can be obtained with stress free boundaries of the material by choosing suitable complex potentials. These potentials are found to be in the form of infinite series, while the axial dislocation requires annulling Saint-Venant torsion solution.

INTRODUCTION

Various authors like Weingarten (1901), Volterra (1907), etc. have developed the mathematical theory of elastic dislocation. Ghosh (1926) studied some problems of dislocation for an eccentric circular cross-section. Stevenson (1945) discussed the problems of dislocations in plane strain by using complex variables. Shivakumar (1960, 1962, 1963) obtained the solutions of dislocations for elliptic cross-section, while Chakravorty (1965) has obtained the solutions of dislocation of an elastic cylinder bounded by two squares with rounded corners. We use Stevenson's method to obtain some new results.

STRESSES AND DISPLACEMENTS

The stresses and the displacement of an isotropic elastic body in plane strain in terms of the complex potentials $\Omega(z)$ and $\omega(z)$ are given by (Ghosh 1926)

$$\tau_{xx} + \tau_{yy} = \tfrac{1}{2}[\Omega'(z) + \bar{\Omega}'(\bar{z})] \qquad \ldots(1)$$

$$\tau_{xx} - \tau_{yy} + 2i\tau_{xy} = -\tfrac{1}{2}[z\bar{\Omega}''(\bar{z}) + \bar{\omega}''(\bar{z})] \qquad \ldots(2)$$

$$8\mu D = (3 - 4\sigma)\,\Omega(z) - z\bar{\Omega}(\bar{z}) - \bar{\omega}'(\bar{z}) \qquad \ldots(3)$$

where

$$D = u + iv,\ u = u(x, y),\ v = v(x, y),\ z = x + iy,$$

$\bar{z} = x - iy$ and μ, σ are shear modulus and Poisson's ratio respectively.

For stress free boundaries it is obvious that

$$C_y D = i\beta z, \qquad \ldots(4)$$

$$2F = \Omega(z) + z\,\bar{\Omega}'(\bar{z}) + \bar{\omega}'(\bar{z}) \qquad \ldots(5)$$

$$= \text{constant on the boundaries,}$$

where β is a constant and $C_y D$ the change in D if we start from a point $P(z, \bar{z})$ in the material and also return to the same point *via* a path in the material.

The stress resultant gives

$$x + iy = -\tfrac{1}{2}\, i C_y F = 0 \qquad \ldots(6)$$

and the resultant couple N is given by

$$N = \text{Re.}\ \tfrac{1}{4}\, C_y \{\omega(z) - z\,\omega'(z)\} = 0 \qquad \ldots(7)$$

with no body force on the system.

We use the conformal transformation

$$z = C\left(\zeta + \frac{K}{p\zeta^p}\right) \qquad \ldots(8)$$

(p is an odd integer and C, K are constants).

The region considered here is bounded by hypotrochoids in the z-plane. It is conformally transformed into the region bounded by two concentric circles $r = a, b$ $(b > a)$ in the ζ-plane.

Using (8), (3) and (5) become

$$8\mu D = \frac{1}{\bar{z}'(\bar{\zeta})}\{(3 - 4\sigma)\,\Omega(\zeta)\,\bar{z}'(\bar{\zeta}) - z\bar{\Omega}'(\bar{\zeta}) - \bar{\omega}'(\zeta)\} \qquad \ldots(9)$$

and

$$2\bar{F} = \frac{1}{z'(\zeta)}\{z'(\zeta)\,\bar{\Omega}(\bar{\zeta}) + \bar{z}\Omega'(\zeta) + \omega'(\zeta)\} \qquad \ldots(10)$$

$$= \text{constant on } \zeta\bar{\zeta} = r^2\ (r = a, b),$$

where

$$\zeta = re^{i\theta},\ \bar{\zeta} = re^{-i\theta},\ z'(\zeta) = \frac{dz}{d\zeta} = C\left(1 - \frac{K}{\zeta^{p+1}}\right)$$

and

$$\Omega'(z) = \frac{1}{z'(\zeta)} \cdot \Omega'(\zeta).$$

Case I: Dislocation of Translation

We consider the cyclic complex potentials

$$\left.\begin{aligned} \Omega(z) &= \Omega(\zeta) = -Qia \log \zeta \\ \text{and} \quad & \\ \omega(z) &= \omega(\zeta) = Qi\bar{a}\, z \log \zeta \end{aligned}\right\} \quad \ldots(2.1)$$

where

$Q = \mu/\pi(1-\sigma)$ and a is a constant.

Then

$$C_y\, 8\mu D = (3-4\sigma)(-Qia \,.\, 2\pi i) - Qia \,.\, 2\pi i = 8\pi\, Qa\,(1-\sigma),$$

whence,

$$C_y\, D = a,$$

and from (7), we have

$$N = \text{Re.}\, \tfrac{1}{4}(2\pi i \,.\, Q\bar{a}z - z \,.\, 2\pi\, iQ\bar{a}) = 0.$$

Substituting (2.1) in (10) on the boundaries $\zeta\zeta = r^2$, we have

$$2\bar{F} = Qi\bar{a} \log r^2 - \frac{QiaC}{z'(\zeta)}\left(\frac{r^2}{\zeta^2} + \frac{k}{p} \cdot \frac{\zeta^{p-1}}{r^2 p}\right) + \frac{Qi\bar{a}C}{z'(\zeta)}\left(1 + \frac{K}{p\zeta^{p+1}}\right). \quad \ldots(2.2)$$

In order to make the boundaries stress free, we assume the non-cyclic complex potentials

$$\left.\begin{aligned} \Omega(\zeta) &= \sum_{-\infty}^{\infty} A_n \zeta^n \\ \text{and} \quad & \\ \omega(\zeta) &= C \sum_{-\infty}^{\infty} B_n \zeta^{n+1} - Qi\bar{a}C\left(\zeta - \frac{K}{p^2 \zeta^p}\right) \end{aligned}\right\} \quad \ldots(2.3)$$

where A_n and B_n are complex constants expressed by

$$A_n = a_n + ia_n'; \quad B_n = b_n + ib_n',$$

where a_n, a_n', b_n, b_n' are real constants and n is only even. Taking $2\bar{F} = M_r =$ constant on the boundaries $\zeta\zeta = r^2\,(r = a, b)$, we have to satisfy on the boundaries

$$2\bar{F} = \frac{M_r \cdot C}{z'(\zeta)}\left(1 - \frac{K}{\zeta^{p+1}}\right). \quad \ldots(2.4)$$

Now combining (2.1) with (2.3) and using $\zeta = re^{i\theta}$, $\zeta = re^{-i\theta}$ with the condition (2.4), we have on the boundaries $\zeta\bar{\zeta} = r^2\,(r = a, b)$,

$$- QiaC\left(\left(e^{-2i\theta} + \frac{K}{p} \cdot \frac{e^{i(p-1)\theta}}{r^{p+1}}\right) + \sum_{-\infty}^{\infty} \psi_n(r)\, e^{in\theta}\right.$$

$$= M_r\left(1 - \frac{K}{r^{p+1}} \cdot e^{-i(p+1)\theta}\right). \quad \ldots(2.5)$$

Putting, for convenience, $iQa = T$ in (5), we have

$$\sum_{-\infty}^{\infty} \psi_n(r)\, e^{in\theta} = M_r\left\{1 - \frac{K}{r^{p+1}} \cdot e^{-i(p+1)\theta}\right\}$$

$$+ T\left\{e^{-2i\theta} + \frac{K}{p} \cdot \frac{e^{i(p-1)\theta}}{r^{p+1}}\right\} \quad \ldots(2.6)$$

where

$$\psi_n(r) = \bar{A}_{-n}\, r^{-n} - K\bar{A}_{-n-p-i}\, r^{-n-2p-2} + (n+2)\, A_{n+2} \cdot r^{n+2}$$

$$+ \frac{k}{p}(n-p+1)\, A_{n-p+i}\, r^{n-2p} + (n+1)\, B_n\, r^n. \quad \ldots(2.7)$$

Putting $n = 0, \ -2;\ 2, \ -4; \ldots\ldots\ldots; \ (p-1), -(p+1)$ successively in (2.7), we have on $r = a$ and $r = b$

$$\left.\begin{aligned}
&\bar{A}_0 - K\bar{A}_{-(p+1)} \cdot r^{-2(p+1)} + 2A_2 r^2 - \frac{K(p-1)}{p} \\
&\qquad \times A_{-(p-1)} \cdot r^{-2p} + B_0 = M_r \\
&\bar{A}_2 r^2 - K\bar{A}_{-(p-1)} \cdot r^{-2p} - \frac{K(p+1)}{p} \\
&\qquad \times A_{-(p+1)} \cdot r^{-2(p+1)} - B_{-2}\, r^{-2} = T \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
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&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&\bar{A}_{-(p-1)} \cdot r^{-(p-1)} - K\bar{A}_{-2p} \cdot r^{-(3p+1)} + (p+1)\, A_{p+1} \\
&\qquad \times r^{p+1} + pB_{p-1} \cdot r^{p-1} = T \cdot \frac{K}{pr^{p+1}} \\
&\bar{A}_{p+1} \cdot r^{p+1} - K\bar{A}_0 \cdot r^{-(p+1)} - (p-1)\, A_{-(p+1)} \\
&\qquad \times r^{-(p-1)} - 2KA_{-2p} \cdot r^{-(3p+1)} - pB_{-(p+1)} \\
&\qquad \times r^{-(p+1)} = -\frac{K}{r^{p+1}} \cdot M_r.
\end{aligned}\right\} \quad \ldots(2.8)$$

Eliminating M_r from the first and the last equations of (2.8) we have a single equation to be satisfied on $r = a, b$. Thus finally (2.8) give $2p$ - equations.

Now taking A_0, B_0, B_{-2}, $B_{-4}, \ldots\ldots\ldots, B_{-(p+1)}$ to be zeros, we see that from eqns. (2.8) we can find (p is odd) unknowns namely

$$A_2,\ A_4, \ldots\ldots, A_{p+1};\ A_{-2},\ A_{-4}, \ldots\ldots, A_{-2p};\ B_2,\ B_4, \ldots\ldots\ldots, B_{p-1}.$$

Putting $n = p + m$ and $n = -(p + m + 2)$ in (2.7), we have

$$\left.\begin{aligned} &\bar{A}_{-(p+m)} \cdot r^{-(p+m)} - K\bar{A}_{-(2p+m+1)} \cdot r^{-(3p+m+2)} + (p + m + 2) \\ &\quad \times A_{p+m+2} \cdot r^{p+m+2} + \frac{k}{p} \cdot (m + 1) A_{m+1} \cdot r^{-(p-m)} \\ &\quad + (p + m + 1) B_{p+m} \cdot r^{p+m} = 0 \\ \text{and} \\ &\bar{A}_{p+m+2} \cdot r^{p+m+2} - K\bar{A}_{m+1} \cdot r^{-(p-m)} - (p + m) A_{-(p+m)} \cdot r^{-(p+m)} \\ &\quad - \frac{K}{p} (2p + m + 1) A_{-(2p+m+1)} \cdot r^{-(3p+m+2)} - (p + m + 1) \\ &\quad \times B_{-(p+m+2)} \cdot r^{-(p+m+2)} = 0. \end{aligned}\right\} \quad \ldots(2.9)$$

Equations (2.9) show that A_{p+m+2}, $A_{-(2p+m+1)}$, B_{p+m}, $B_{-(p+m+2)}$ can be found when $A_{-(p+m)}$, A_{m+1} are known. But A_2 and $A_{-(p+1)}$ are known from (2.8). Hence A_{p+3}, $A_{-(2p+2)}$, B_{p+1} and $B_{-(p+3)}$ can be found from (2.9). Proceeding in this manner we can find all A_n's and B_n's.

In the series for $\Omega(\zeta) = \sum_{-\infty}^{\infty} A_n \zeta^n$, A_n vanishes when $n = 0$ and in the series for

$$\omega(\zeta) = \sum_{-\infty}^{\infty} B_n \zeta^{n+1},$$

$$B_0 = B_{-2} = B_{-4} = \ldots\ldots\ldots = B_{-(p+1)} = 0.$$

Hence the complex potentials

$$\left.\begin{aligned} &\Omega(\zeta) = -Qia \log \zeta + \sum_{n=1}^{\infty} A_{2n} \zeta^{2n} + \sum_{n=1}^{\infty} A_{-2n} \zeta^{-2n} \\ \text{and} \\ &\omega(\zeta) = Qi\bar{a}\, z \log \zeta - Qi\bar{a}C\left(\zeta - \frac{K}{p^2 \zeta^p}\right) \\ &\quad + \sum_{n=1}^{\infty} B_{2n} \zeta^{2n+1} + \sum_{n=\frac{1}{2}(p+3)}^{\infty} B_{-2n} \zeta^{-2n+1} \end{aligned}\right\} \quad \ldots(2.10)$$

give the dislocation of translation.

CaseII: *Dislocation of Rotation*

Here we have to find suitable cyclic and non-cyclic complex potentials which satisfy (4), (7) and (10).

In this case we at first assume

$$\left.\begin{aligned} \Omega(\zeta) &= Q\beta z \log \zeta \\ \text{and} \qquad \omega(\zeta) &= 0 \end{aligned}\right\} \qquad \ldots(3.1)$$

where β is a real constant and $\theta = \mu/\pi\,(1-\sigma)$.

The above potentials give

$$C_y\, 8\mu D = 8\pi i\,(1-\sigma)\, Qbz. \qquad \ldots(3.2)$$

From (3.2), we get

$$C_y D = i\beta z$$

which gives the dislocation of rotation. Clearly (3.1) satisfies (4) and gives on substitution in (10),

$$2\bar{F} = \frac{Q\beta C}{z'(\zeta)} \left\{ \bar{z}\left(1 + \frac{K}{p\,\zeta^{p+1}}\right)\right\} + Q\beta\bar{z}\log r^2. \qquad \ldots(3.3)$$

To nullify the stresses on the boundaries, we take the potentials

$$\left.\begin{aligned} \Omega(\zeta) &= \sum_{-\infty}^{\infty} A_n\,\zeta^n - Q\beta C\left(\zeta - \frac{K}{p^2\zeta^p}\right) \\ \text{and} \qquad \omega(\zeta) &= C\sum_{-\infty}^{\infty} \frac{B_n\,\zeta^{n+1}}{n+1} \end{aligned}\right\} \qquad \ldots(3.4)$$

where A_n, B_n are real constants and n is odd.

Assuming $A_0 = 0$, $B_{-1} = 0$ and combining (3.1) with (3.4), we have on the boundaries $\zeta\bar{\zeta} = r^2$ $(r = a, b)$,

$$2\bar{F} = \frac{C}{z'(\zeta)}\left[\sum_{-\infty}^{\infty}\left\{\frac{A_{-n}}{r^{2n}} - \frac{KA_{-n-p-1}}{r^{2(n+p+1)}} + (n+2)\,A_{n+2}.r^2 + \frac{K}{p}\,.\,(n-p+1)\,\frac{A_{n-p+1}}{r^{2p}} + B_n\right\}\zeta^n + M(r)\,\zeta^p + \frac{L(r) - KM(r)}{\zeta} - \frac{KL(r)}{\zeta^{p+2}}\right] \qquad \ldots(3.5)$$

where

$$L(r) = Q\beta C r^2 (\log r^2 - 1)$$

and

$$M(r) = \frac{Q\beta CK}{pr^{2p}}\left(\log r^2 + \frac{1}{p}\right).$$

Taking n and p odd only, we have from (3.5)

$$2\bar{F} = 0, \text{ on } r = a, b.$$

Hence putting $n = -1,\ p, -(p+2),\ (2p+1),\ -(2p+3)$ successively in (3.5), we have

$$2A_1 r^2 - 2K\frac{A_{-p}}{r^{2p}} + L(r) - KM(r) = 0 \qquad \ldots(3.6)$$

$$\frac{A_{-p}}{r^{2p}} - \frac{KA_{-(2p+1)}}{r^{2(2p+1)}} + (p+2) A_{p+2} \,.\, r^2 + \frac{K}{p}\frac{A_1}{r^{2p}} + B_p + M(r) = 0 \qquad \ldots(3.7)$$

$$A_{p+2} \,.\, r^{2(p+2)} - KA_1 r^2 - pA_{-p} \,.\, r^2 - \frac{K(2p+1)}{pr^{2p}} A_{-(2p+1)} + B_{-(p+2)} - KL(r) = 0 \qquad \ldots(3.8)$$

$$\frac{A_{-(2p+1)}}{r^{2(2p+1)}} - \frac{KA_{-(3p+2)}}{r^{2(4p+3)}} + (2p+3) A_{2p+3} \,.\, r^2 + \frac{K}{p} \,.\, \frac{(p+2)}{r^{2p}} \,.\, A_{p+2} + B_{2p+1} = 0 \qquad \ldots(3.9)$$

$$A_{2p+3} \,.\, r^{2(2p+3)} - KA_{p+2} \,.\, r^{2(p+2)} - (2p+1) A_{-(2p+1)} \,.\, r^2 - \frac{K(3p+2)}{pr^{2p}} \,.\, A_{-(3p+2)} + B_{-(2p+3)} = 0. \qquad \ldots(3.10)$$

The above results are true on $r = a$ and $r = b$. From (3.6), we obtain A_1 and A_{-p}. Substituting these values in (3.7) and (3.8), we get A_{p+2}, $A_{-(2p+1)}$, B_p, $B_{-(p+2)}$ in terms of A_1 and A_{-p}. Again substituting these values in (3.9) and (3.10), we get A_{2p+3}, $A_{-(3p+2)}$, B_{2p+1}, $B_{-(2p+3)}$. Putting $n = -1 + m(p+1)$ and $n = -\{1 + m(p+1)\}$ in (3.5), we have

$$A_{1-m(p+1)} \,.\, r^{2\{1-m(p+1)\}} - \frac{K}{r^2} \,.\, A_{1-(m+1)(p+1)} \,.\, r^{2\{1-(m+1)(p+1)\}} + \{-p + (m+1)(p+1)\} A_{-p+(m+1)(p+1)} \,.\, r^2 + \frac{K}{p} \,.\, \frac{\{-p + m(p+1)\}}{r^{2p}} \,.\, A_{-p+m(p+1)} + B_{-1+m(p+1)} = 0,$$

$$\text{on } r = a \text{ and } r = b, \qquad \ldots(3.11)$$

and

$$A_{-p+(m+1)(p+1)} \cdot r^{2\{-p+(m+1)(p+1)\}} - KA_{-p+m(p+1)} \cdot r^{2\{-p+m(p+1)\}}$$
$$+ \{1 - m(p+2)\} A_{1-m(p+1)} \cdot r^2 + \frac{K}{p} \cdot \frac{\{1-(m+1)(p+1)\}}{r^{2p}}$$
$$\times A_{1-(m+1)(p+1)} + B_{-\{1+m(p+1)\}} = 0, \text{ on } r = a \text{ and } r = b. \quad \text{...(3.12)}$$

Equations (3.11) and (3.12) show that $A_{-p+(m+1)(p+1)}$, $A_{1-(m+1)(p+1)}$, $B_{-1+m(p+1)}$, $B_{-\{1+m(p+1)\}}$ can be found when $A_{1-m(p+1)}$, $A_{-p+m(p+1)}$ are known. But for $m = 1$ and 2, A's and B's are known from (3.6)–(3.10). Thus for $m = 3$, A's and B's may be found from (3.11) and (3.12) and then for $m = 4, 5, 6\ldots$ etc., the constants A's and B's can be calculated from (3.11) and (3.12).

In the series for $\Omega(\zeta) = \sum_{-\infty}^{\infty} A_n \zeta^n$, A_n vanishes unless $n = 1 + m(p+1)$ and in the series for

$$\omega(\zeta) = \sum_{-\infty}^{\infty} \frac{B_n \zeta^{n+1}}{n+1},$$

B_n vanishes unless $n = -1 + m(p+1)$, where m is zero or any + ve or − ve integer.

Hence the complex potentials

$$\left.\begin{aligned} \Omega(z) &= Q\beta z \log\zeta - Q\beta C\left(\zeta - \frac{K}{p^2\zeta^p}\right) + \sum_{m=-\infty}^{\infty} A_{1+m(p+1)} \cdot \zeta^{m(p+1)+1} \\ &\text{and} \\ \omega(z) &= C\sum_{m=-\infty}^{\infty} \frac{B_{-1+m(p+1)}}{m(p+1)} \cdot \zeta^{m(p+1)} \end{aligned}\right\} \quad \text{...(3.13)}$$

give the dislocation of rotation.

Putting $p = 0$ in (3.13), we get the solution for the dislocation or rotation for the elastic body of circular cross-section as obtained by Ghosh (1926).

Putting $p = 1$ in (3.13), we get the solution for elliptic cross-section as obtained by Shivakumar (1960, 1962, 1963).

Putting $p = 2$ in (3.13), we get the solution of dislocation for the elastic body whose cross-section is bounded by two squares with rounded corners as obtained by Chakravorty (1965).

Case III: *Axial Dislocation*

We define the axial strain by $u = 0$, $v = 0$, $\omega = \omega(z, \bar{z})$ by taking z-axis as the axis of the cylinder.

The stress-displacement relation is

$$\psi = \tau_{xz} + i\tau_{yz} = 2\mu \frac{\partial \omega}{\partial \bar{z}}, \qquad ...(4.1)$$

and the only non-vanishing body stress equation is

$$\frac{\partial \psi}{\partial z} + \frac{\partial \bar{\psi}}{\partial \bar{z}} = 0,$$

which gives

$$\frac{\partial^2 \omega}{\partial z \partial \bar{z}} = 0, \text{ i.e., } \omega = \omega_1'(z) + \bar{\omega}_1'(\bar{z}). \qquad ...(4.2)$$

To make the boundary of the cylinder free from applied traction, the boundary conditions for a dislocation is

$$\psi \, d\bar{z} - \bar{\psi} dz = 0. \qquad ...(4.3)$$

For the dislocation, we have

$$C_s \, \omega = \gamma.$$

Now,

$$\omega_1'(z) = -\frac{i\gamma}{2\pi} \log \zeta \qquad ...(4.4)$$

gives the required cyclicity. Obviously, $\psi = 2\mu \, \bar{\omega}_1''(\bar{z})$ with (4.4) is satisfied by (4.3) on $|\zeta| = r =$ constant.

The vanishing of the only end resultant requires the annulling of the twisting moment given by

$$N = \text{Re.} \int_s iz \, \bar{\psi} \, ds$$

$$= \text{Re.} \int_s 2iz\mu \, \omega_1''(z) \, ds$$

$$= \text{Re.} \, 2i\mu \int_c \frac{\partial}{\partial z} [z\bar{z} \, \omega_1''(z)] \, ds.$$

Using complex Stokes' theorem, we have

$$N = \text{Re.} \, \mu \int_c z\bar{z} \frac{d}{d\zeta} \omega_1 (\zeta) \, d\zeta$$

$$= -\mu C^2 \cdot \frac{i\gamma}{2\pi} \int \left(\zeta + \frac{K}{p\zeta^p}\right) \left(\bar{\zeta} + \frac{K}{p\bar{\zeta}^p}\right) \cdot \frac{1}{\zeta} \, d\zeta$$

$$= \mu C^2\gamma \left[r^2 + \frac{K^2}{p^2 r^{2p}} \right], \text{ on } \zeta\bar{\zeta} = r^2,\ r = a,\ b\ (b > a)$$

$$= \frac{\mu C^2 \gamma}{p^2 a^{2p} b^{2p}} \left[p^2 a^{2p}\, b^{2p}\, (b^2 - a^2) - K\,(b^{2p} - a^{2p}) \right]. \qquad \ldots(4.5)$$

Using a complementary Saint-Venant torsion solution expressed in a manner appropriate to the ring space by Stevenson (1945), the twisting moment N can be annulled.

If we take

$$\left. \begin{aligned} D &= i\tau z\,(x + iy) = i\tau z_s,\ \omega = \tfrac{1}{2}\,\tau\,\{\omega_3(\zeta) + \bar{\omega}_3(\bar{\zeta})\} \\ \omega_3(\zeta) &= \frac{iC^2K^2}{b^4 + a^4}\cdot \zeta^{p+1} + iC^2K^2 \cdot \frac{a^2b^2}{b^2 + a^2} \cdot \frac{1}{\zeta^{p+1}} \end{aligned} \right\} \qquad \ldots(4.6)$$

where z is the third spatial co-ordinate, the torsion moment is given by

$$N_1 = \mu\,\tau\,(I - J)$$

where

$$I = -\tfrac{1}{4}\, i \int_c z\bar{z}^2 z'(\zeta)\, d\zeta$$

and

$$J = -\tfrac{1}{2} \int_c z\bar{z}\, \omega_3'(\zeta)\, d\zeta. \qquad \ldots(4.7)$$

Using (8) and (4.6), we have

$$I = \frac{1}{2}\pi C^4 \left[(b^4 - a^4) - \frac{2K^2}{p^2}(p - 1)\,(b^{-2p+2} - a^{-2p+2}) - \frac{K^4}{p^3}\,(b^{-4p} - a^{-4p}) \right]$$

and

$$J = \pi C^4 K^3\,(1 + p^{-1}) \left[\frac{(b^2 - a^2)}{(b^4 + a^4)} - \frac{a^2 b^2}{b^2 + a^2}\,(b^{-2p} - a^{-2p}) \right]. \qquad \ldots(4.8)$$

Taking $N = -N_1$, we have

$$\tau = -\frac{N}{\mu\,(I - J)} \qquad \ldots(4.9)$$

where N and I, J are given by (4.5) and (4.8). Hence the complete solution for the problem of axial dislocation is given by

$$\left. \begin{aligned} \omega &= -\frac{i\gamma}{2\pi}\log\zeta + \frac{i\gamma}{2\pi}\log\bar{\zeta} + \frac{1}{2}\,\tau\,\{\omega_3(\zeta) + \bar{\omega}_3(\bar{\zeta})\} \\ D &= u + iv = i\tau z_s. \end{aligned} \right\} \qquad \ldots(4.10)$$

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