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MAGNETIC FIELD EFFECTS ON THE FREE CONVECTION AND MASS TRANSFER FLOW THROUGH POROUS MEDIUM WITH CONSTANT SUCTION AND CONSTANT HEAT FLUX

M. ACHARYA[†], G. C. DASH[‡] AND L. P. SINGH[*]

[†]*Department of Physics, College of Basic Science and Humanities, O.U.A.T. Bhubaneswar* 751 003

[‡]*Department of Mathematics, C.E.T., O.U.A.T. Bhubaneswar* 751 003

[*]*Department of Physics, Utkal University, Bhubaneswar* 751 004

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An analysis of steady two-dimensional free convection and mass transfer flow of a viscous incompressible electrically conducting fluid through a porous medium bounded by a vertical infinite surface with constant suction velocity and constant heat flux in the presence of a uniform magnetic field is presented. The effects of Grashof number for heat transfer ($Gr > 0$, corresponds to externally cooled plate and $Gr < 0$ specifies condition for externally heated plate), Grashof number for mass transfer, Schmidt number, Eckert number, permeability prameter and magnetic number on velocity and temperature profiles are discussed numeriacally and shown graphically. All numerical calculations are done with respect to air (Pr = 0.71 at 20 °C). The effects of the above mentioned parameters on skin friction coefficient at the surface are also presented.

Key Words : Conducting Fluid; Porous Medium; Suction, Temperature Distribution; Diffusing Species

1. INTRODUCTION

Porous media are widely used in high temperature heat exchangers, turbine blades, jet nozzles, etc. In practice cooling of porous structure is achieved by forcing the liquid or gas through capillaries of solid. Actually, they are used to insulate a heated body to maintain its temperature. Porous media are considered to be useful in diminishing the natural free convection which would otherwise occur intensely on a vertical heated surface. In order to make heat insulation of surface more effective it is necessary to study the free convection flow through a porous medium and to estimate its effect in heat and mass transfer. Study of origin of flow through porous media is heavily based on Darcy's experimental law. By using Darcy's law, Yamamoto and Yoshida[1] considered suction and injection flow with convective acceleration through a plane porous wall specifically for the flow outside a vortex layer. The generalisation of the above study was presented by Yamamoto and Iwamura[2]. Chawla and Singh[3] studied oscillatory flow past a porous bed. The steady two-dimensional flow of viscous fluid through a porous medium bounded by a porous surface subjected to a constant suction velocity by taking account of free convection currents (both velocity and temperature fields are constant along x-axis) was studied by Raptis[4] *et al.* The effect of variable permeability on combined free and forced convection in porous media was studied by Chandrasekhara and Namboodiri[5]. Later on mixed convection in porous media adjacent to a vertical uniform heat flux surface was studied by Joshi and Gebhart[6]. Heat and mass transfer in a porous medium was discussed by Bejan and Khair[7]. The above problem was studied in presence of buoyancy effect by Trevisan and Bejan[8]. Lai and Kulacki[9] studied the effect of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium. The free convection effect on the flow of an ordinary viscous fluid past an infinite vertical porous plate with constant suction and constant heat flux was investigated by Sharma[10]. The steady two dimensional flow through a porous medium bounded by a vertical infinite surface with constant suction velocity and constant heat flux in presence of free convection current (where both velocity and temperature fields are constant along x-axis) was studied by Sharma[11]. Convection in a porous medium with inclined temperature gradient was investigted by Nield[12]. The problem of mixed convection along an isothermal vertical plate in porous medium with injection and suction was studied by Hooper[13] *et al.*

The objective of the present study is to investigate the free convection and mass transfer flow of a viscous in compressible and electrically conducting fluid through a porous medium (assumed highly porous) bounded by vertical infinite surface with constant suction velocity and constant heat flux under the action of uniform magnetic field applied normal to the direction of flow.

2. Mathematical Formulation

We consider steady two dimensional motion of viscous incompressible electrically conducting fluid through a porous medium occupying semi-infinite region of space bounded by a vertical infinite surface under the action of uniform magnetic field applied normal to the direction of flow. The effect of induced magnetic field is neglected. The magnetic Reynolds number is assumed to be small. Further magnetic field is not strong enough to cause Joule heating[14] (electrical dissipation). Hence, the term due to electrical dissipation is neglected in energy equation (3). The x-axis is taken along the surface in the upward direction and y-axis is taken normal to it. The fluid properties are assumed constant except for the influence of density in the body force term. As the bounding surface is infinite in length, all the variables are functions of y only. Hence, by the usual boundary layer approximation the basic equations for steady flow through highly porous medium are :-

$$v_y = 0, \qquad \dots (1)$$

$$vu_y = \nu u_{yy} + g\beta(T - T_\infty) + g\beta^* (C - C_\infty) - (\sigma B_0^2/\rho)u - (\nu/k)u, \qquad \dots (2)$$

$$vT_y = (\lambda/\rho C_p)T_{yy} + (\nu/C_p)u_y^2 \qquad \dots (3)$$

and

$$vC_y = DC_{yy} \qquad \dots (4)$$

where u and v are the corresponding velocity components along and perpendicular to the surface, ν the kinematic viscosity, g the acceleration due to gravity, β the coefficient of volume expansion for the heat transfer and β^* is the volumetric coefficient of expansion with species concentration, T the fluid temperature, T_∞ the fluid temperature at ∞, λ the thermal conductivity, ρ the density of the fluid, C_p specific heat at constant pressure, C the species concentration, C_∞ the species concentration at ∞. D the chemical molecular diffusivity and k the permeability of porous medium.

3. Method of Solution

The equation of continuity (1) gives

$$v = \text{constant} = -v_0, \qquad \text{... (5)}$$

where $v_0 > 0$ corresponds to steady suction velocity (normal) at the surface. In view of eq. (5), eq. (2) (3) and (4) are reduced to

$$-v_0 u_y = \nu u_{yy} + g\beta (T - T_\infty) + g\beta^* (C - C_\infty) - (\sigma B_0^2/\rho)u - (\nu/k)u, \qquad \text{... (6)}$$

$$-v_0 T_y = (\lambda/\rho C_p)\, T_{yy} + (\nu/C_p) u_y^2 \qquad \text{... (7)}$$

and $$-v_0 C_y = D C_{yy}. \qquad \text{... (8)}$$

The relevant boundary conditions are

$$u = 0, \quad T + T_\infty, C = C_\infty \quad \text{for all} \quad y \; t \leq 0,$$

$$u = 0, \quad T_y = -q/\lambda, \quad C_y = \dot{m}/D, \quad y = 0 \; t > 0 \qquad \text{... (9)}$$

$$u = 0, \quad T = T_\infty, C = C_\infty \quad y \to \infty \quad t > 0.$$

Introducing following non-dimensional parameters into eqs. (6), (7) and (8) and dropping asterisk.

$f(\eta) = u/v_0$	(velocity)
$\eta = v_0 y/\nu$	(distance)
$Pr = \mu\, C_p/\lambda$	(Prandtl number)
$Sc = \nu/D$	(Schmidt number)
$\theta = (T - T_\infty) v_0\, \lambda/q\nu$	(Temperature)
$C^* = (C - C_\infty) v_0 D/\dot{m}\nu$	(species concentration)
$\alpha = v_0^2\, K/\nu^2$	(permeability parameter)
$Gr = g\beta q \nu^2/v_0^4 \lambda$	(Grashof number for heat transfer)
$Gm = g\beta^* \dot{m} \nu^2/v_0^4 D$	(Grashof number for mass transfer)

$M = \sigma B_0^2 \nu/\rho v_0^2$ (Magnetic number)

$E = \lambda v_0^3/q \nu c_p$ (Eckert number),

where q is the heat flux per unit area and $\dot{m}$ is the mass flux per unit area.

We get

$$f'' + f' - f(\alpha^{-1} + M) = -Gr\theta - GmC, \quad \text{... (10)}$$

$$-Pr\theta' = \theta'' + Pr(f')^2 E \quad \text{... (11)}$$

and

$$C'' + ScC' = 0, \quad \text{... (12)}$$

where prime denotes differentiation with respect to η.

The corresponding boundary conditions become

$$\eta = 0, \quad f = 0, \quad \theta' = -1, \quad C' = -1$$

$$\eta \to \infty, \quad f = 0, \quad \theta \to 0, \quad C \to 0. \quad \text{... (13)}$$

In order to obtain a solution of above coupled nonlinear system of eqs. (10), (11) & (12), we expand f, θ and C in powers of Eckert number E, assuming that it is very small. This is justified in low speed incompressible flow. Hence, we can write

$$f(\eta) = f_0(\eta) + Ef_1(\eta) + O(E^2)$$

$$\theta(\eta) = \theta_0(\eta) + E\theta_1(\eta) + O(E^2),$$

and

$$C(\eta) = C_0(\eta) + EC_1(\eta) + O(E^2). \quad \text{... (14)}$$

Substituting eq. (14) into eqs. (10), (11) & (12), equating coefficients of same power of E and neglecting higher order terms in E, we find

$$f_0'' + f_0' - f_0(\alpha^{-1} + M) = -Gr\theta_0 - GmC_0, \quad \text{... (15)}$$

$$f_1'' + f_1' - f_1(\alpha^{-1} + M) = -Gr\theta_1 - GmC_1, \quad \text{... (16)}$$

$$\theta_0'' + Pr\theta_0' = 0, \quad \text{... (17)}$$

$$\theta_1'' + Pr\theta_1' = -Pr(f_0')^2, \quad \text{... (18)}$$

$$C_0'' + ScC_0' = 0 \quad \text{... (19)}$$

and $$C_1'' + ScC_1' = 0 \qquad \ldots (20)$$

with corresponding boundary conditions

$$f_0 = 0,\ f_1 = 0,\ \theta_0' = -1,\ \theta_1' = 0,\ C_0' = -1,\ C_1' = 0 \quad \text{at} \quad \eta = 0$$

$$f_0 \to 0,\ f_1 \to 0, \theta_0 \to 0,\ \theta_1 \to 0,\ C_0 \to 0,\ C_1 \to 0 \quad \text{as} \quad \eta \to \infty.$$

Solving eq. (15)-(20) under boundary conditions (21), we obtain :

$$f_0 = K_2 \exp(K_3\eta) - (K_5/Pr) \exp(-Pr\eta) - (K_6/Sc) \exp(-Sc\eta); \qquad \ldots (22)$$

$$f_1 = -2K_5K_8 \exp(K_3\eta) - 2K_5K_9(Sc/Pr) \exp(K_3\eta) - K_5K_{11}((Pr+Sc)/Pr) \exp(K_3\eta)$$
$$+ K_9Gr \exp(K_3\eta)/(4Sc^2 - 2Sc - K_1) + K_{11}Gr \exp d(K_3\eta)/[(Pr+Sc)^2$$
$$-(Pr+Sc) - K_1] + GrK_8 \exp(K_3\eta)/(4Pr^2 - 2Pr - K_1)$$
$$+ 2K_5K_8 \exp(-Pr\eta) + 2K_9K_5(Sc/Pr) \exp(-Pr\eta)$$
$$+ K_{11}K_5((Pr+Sc)/Pr) \exp(-Pr\eta) - K_9Gr \exp(-2Sc\eta)/(4Sc^2 - 2Sc - K_1)$$
$$- GrK_{11} \exp(-(Pr+Sc)\eta)/[(Pr+Sc)^2 - (Pr+Sc) - K_1]$$
$$- GrK_8 \exp(-2Pr\eta)/[4Pr^2 - 2Pr - K_1];$$

$$\theta_0 = \exp(-Pr\eta)/Pr; \qquad \ldots (23)$$

$$\theta_1 = -2K_8 \exp(-Pr\eta) - 2K_9(Sc/Pr) \exp(-Pr\eta) - K_{11}((Pr+Sc)/Pr) \exp(-Pr\eta)$$
$$+ K_9 \exp(-2Sc\eta) + K_{11} \exp(-(Pr+Sc)\eta) + K_8 \exp(-2Pr\eta); \qquad \ldots (25)$$

and $$C_0 = \exp(-Sc\eta)/Sc, \qquad \ldots (26)$$

where

$$K_1 = \alpha^{-1} + M,\ K_2 = K_5/Pr - K_6/Sc,$$

$$K_3 = (-1 - (1 + 4K_1)^{1/2})/2,\ K_4 = K_2K_3,$$

$$K_5 = Gr/(Pr^2 - Pr - K_1),\ K_6 = Gm/(Sc^2 - Sc - K_1),$$

$$K_8 = -K_5^2/2Pr,\ K_9 = -PrK_6^2/(4Sc^2 - 2ScPr)$$

and $$K_{11} = -2PrK_5K_6/(Sc(Pr + Sc)).$$

4. Skin Friction

The skin friction coefficient at the surface is given by

$$\tau = [\tau_{xy'}/\rho V_0^2]_{y'=0} \qquad \dots (27)$$

$$\tau = K_4 + K_5 + K_6 - 2EK_3K_5K_8 - 2EK_3K_5K_9(Sc/Pr)$$

$$- EK_3K_5K_{11}(Pr + Sc)/Pr + EGrK_3K_9/(4Sc^2 - 2Sc - K_1)$$

$$+ EGrK_3K_{11}/[(Pr + Sc)^2 - (Pr + Sc) - K_1]$$

$$+ EGrK_3K_8/[4Pr^2 - 2Pr - K_1]$$

$$- 2K_5EPrK_8 - 2EK_5K_9Sc - K_5K_{11}E(Pr + Sc)$$

$$+ 2ScK_9EGr/(4Sc^2 - 2Sc - K_1) + (Pr + Sc)GrEK_{11}/[(Pr + Sc)^2 - (Pr + Sc) - K_1]$$

$$+ 2PrGrK_8E/(4Pr^2 - 2Pr - K_1).$$

5. Results and Discussion

Using eqs. (14), (22), (23), (24) (25) & (26) we find required equations for velocity, temperature and concentration. During the course of numerical Calculations of velocity, temperature and species concentration functions the value of Prandtl number Pr is chosen to be 0.71 which represent air at 20 °C. The values of Schmidt number Sc are chosen in such a way that they represent the diffusing chemical spceies of most common interest in air. (For example, the values of Schmidt number for H_2, H_2O, NH_3 and propyl benzene in air is 0.22, 0.60, 0.78 and 2.62 respectively, Perry[15]). Here Grashof number for heat transfer $Gr < 0$ corresponds to an externally heated plate as the free convection currents are carried towards the plate. $Gr > 0$ corresponds to an externally cooled plate. As the species concentration is assumed to be very low, thus only positive values are chosen.

The fluid velocity and temperature variations in case of externally cooled surface are shown in Fig. 1. for various values of Schmidt number Sc. It is observed that for heavier diffusing foreign species i.e., increasing Schmidt number reduction in velocity level both in magnitude and extent and thining of thermal boundary layer occurs. Substantial increase in velocity distribution profiles is observed near the plate with decreasing Schmidt number (lighter diffusing particle) : Comparison of solid curves (Gr = 10.0) and broken curves (*Gr* = 5.0) for a particular foreign species indicate that greater cooling results in an increase in velocity and thermal boundary layer thickness. Comparison of velocity distribution curves shows that curves fall gradually after attaining maximum value near the plate or on the plate, where as temperature distribution profile falls from maximum value on the plate.

Fig. 2 exhibits variation of velocity and temperature profiles of the fluid for different values of Schmidt number in case of externally heated plate ($Gr < 0$). The velocity of fluid layer decreases in magnitude for thicker diffusing species and substantial decrease is observed near the plate. Here also thinning effect in thermal boundary layer is observed for smaller values of *Sc*. For a particular value of *Sc* greater heating causes decrease in fluid velocity (comparing solid and broken curves) and increase in thermal boundary layer thickness. Comparing the curves of Fig. 1 and Fig. 2, it is concluded that cooling results increase in velocity and temperature while heating results decrease in velocity and temperature. The velocity and temperature profiles for various values of Grashof number for mass transfer (*Gm*) is plotted both for externally heated and externally cooled plate (Fig. 3). Increasing Gm results in increase in magnitude and extent of velocity and temperature distribution in case of externally cooled plate (solid lines), where as velocity increases and thining effect is observed for thermal boundary layer in case of externally heated plate (broken lines). Again substantial increase or decrease occur in velocity distribution near the plate. As discussed earlier cooling increases both velocity and temperature while heating decreases velocity and temperature both in magnitude and extent.

In Fig. 4, we have studied the effect of Eckert number on velocity and temperature distribution profiles for externally cooled plate. Increasing *E* from –0.04 to 0.04, velocity of fluid layer and thickness of thermal boundary layer increases both in magnitude and extent. A readily visible inflection point occurs in the curve of $E = -0.01$, while the curves for $E = +\ 0.01$, $E = +\ 0.02$, $E = +\ 0.03$ and $E = +\ 0.04$ displays "hill" in the respective curves of velocity distribution. Fig. 5 exhibit the same effect for an externally heated plate. Reduction in velocity and increase in thermal distribution layer is observed both in magnitude and extent with increasing Eckert number. It is interesting to note that in this case the velocity distribution fluctuates which is mainly due to large heating and magnetic field. All the velocity curves fall from same point and substantial decrease occur near the plate.

Fig. 6 depicts the effect of permeability parameter α on velocity and temperature distribution. Velocity and temperature increases both in magnitude and extent both for externally heated plate (solid curves) and externally cooled plate (broken curves) with increasing permeability parameter. However, substantial increase (broken curves) and decrease (solid curves) occur near the plate. Effect of magnetic field is exhibited in Fig. 7. As magnetic number increases velocity and temperature at any point decreases both in magnitude and extent. Concentration profiles for various values of Schmidt number are shown in Fig. 8. Which shows that for thicker diffusing species concentration layer thickness decreases both in magnitude and extent.

It is inferred from Table I that skin friction coefficient increases both for externally cooled plate and externally heated plate with increasing Sc but the magnitude being negative for $Gr < 0$. Similarly, increasing Grashof number for mass transfer skin friction coefficient decreases for both the cases and magnitude being negative for $Gr < 0$ (Table II). It is also noted that skin friction

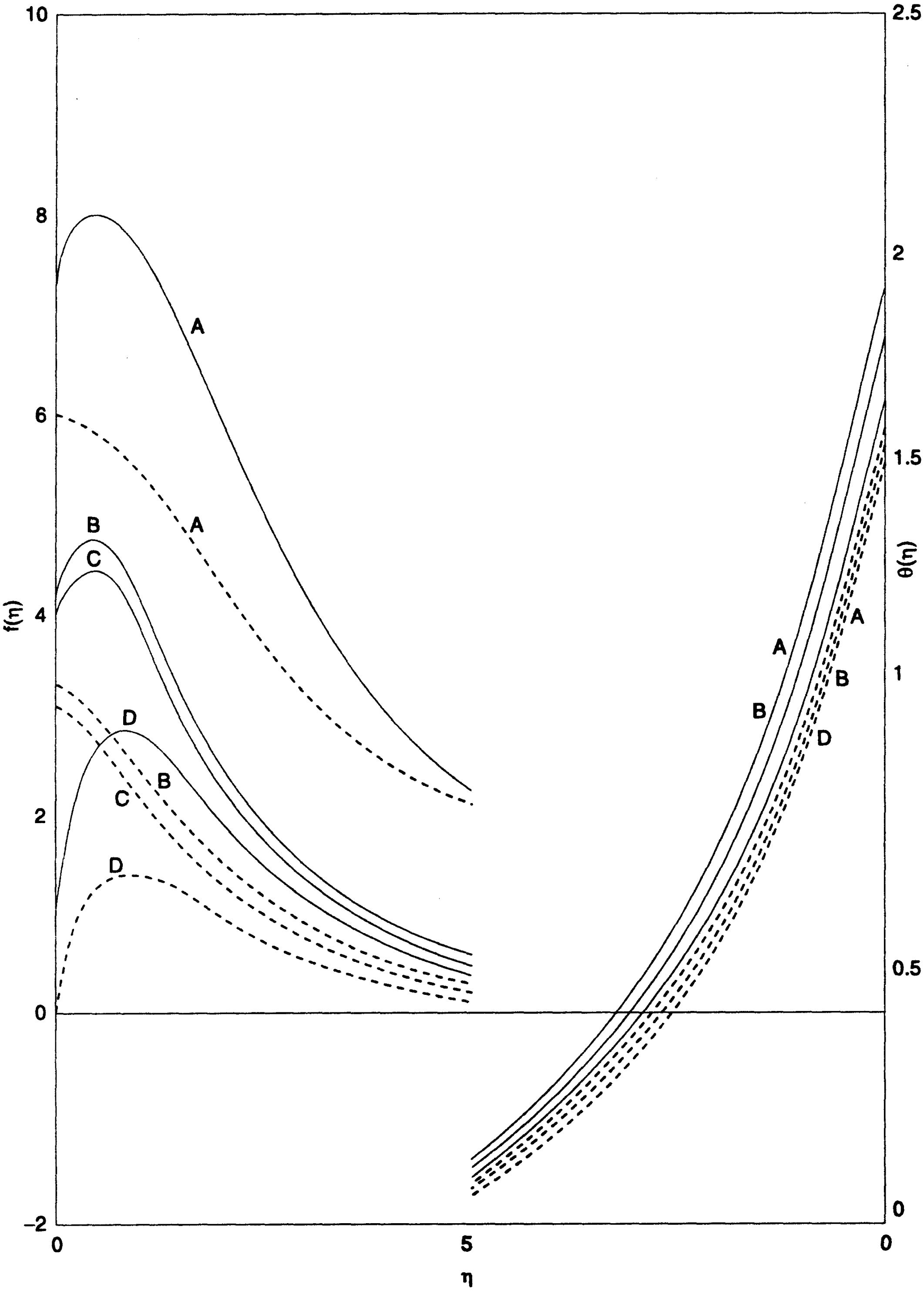

FIG. 1. Velocity and temperature profiles against η for various values of Sc. $Pr = 0.71$, $Gm = 2.0$, $\alpha = 1.0$, $M = 1.0$, $E = 0.02$ $Gr > 0$ (externally cooled plate) --- $Gr = 5.0$, — $Gr = 10.0$ values of Sc for curve $A = 0.22$, $B = 0.60$, $C = 0.78$, $D = 2.62$

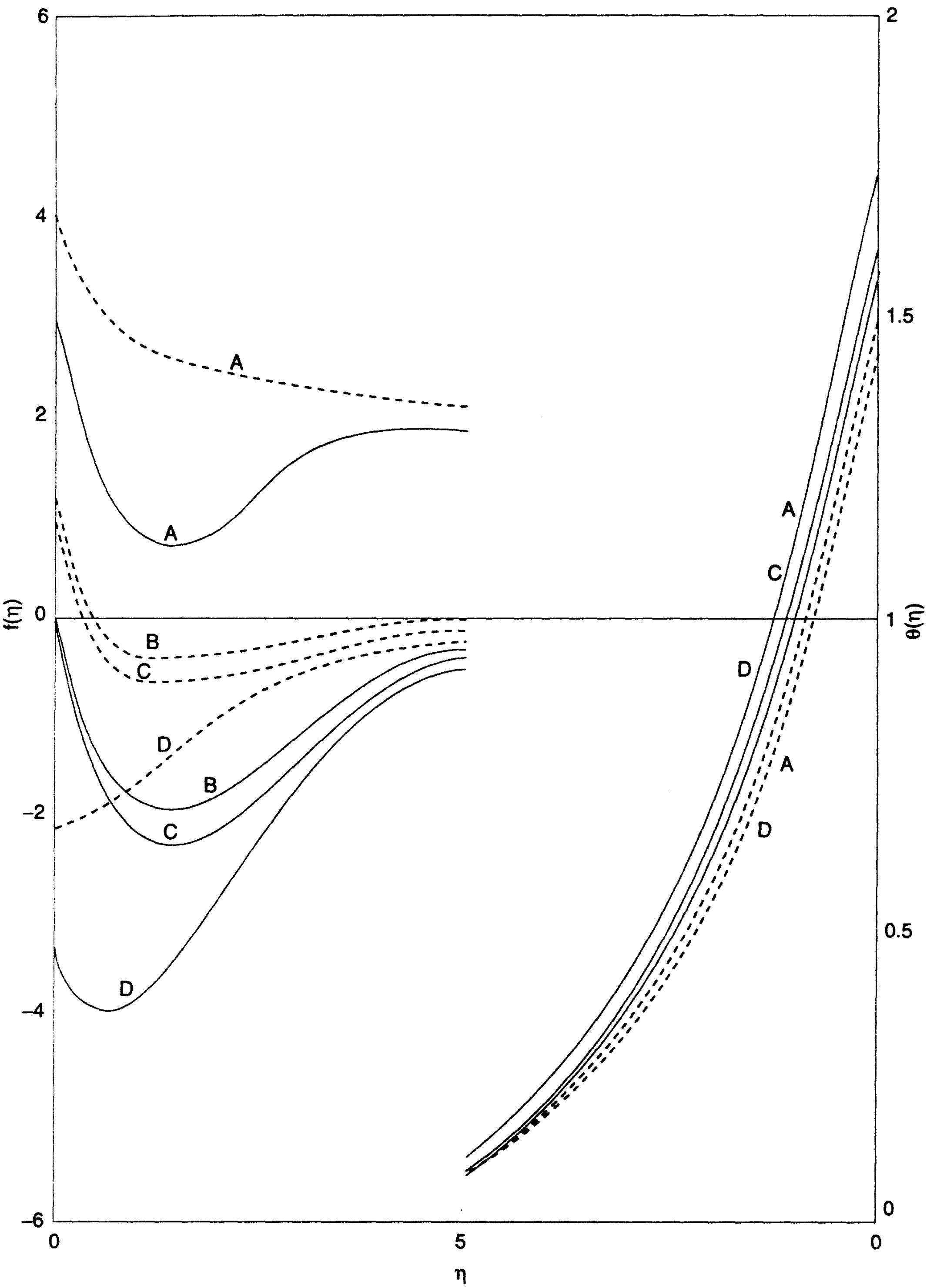

FIG. 2. Velocity and temperature profiles against η for various values of Sc. $Pr = 0.71$, $Gm = 2.0$, $\alpha = 1.0$, $M = 1.0$, $E = 0.02$ $Gr < 0$ (externally cooled plate) --- $Gr = -5.0$, ---- $Gr = -10.0$ values of Sc for curve $A = 0.22$, $B = 0.60$, $C = 0.76$, $D = 2.62$

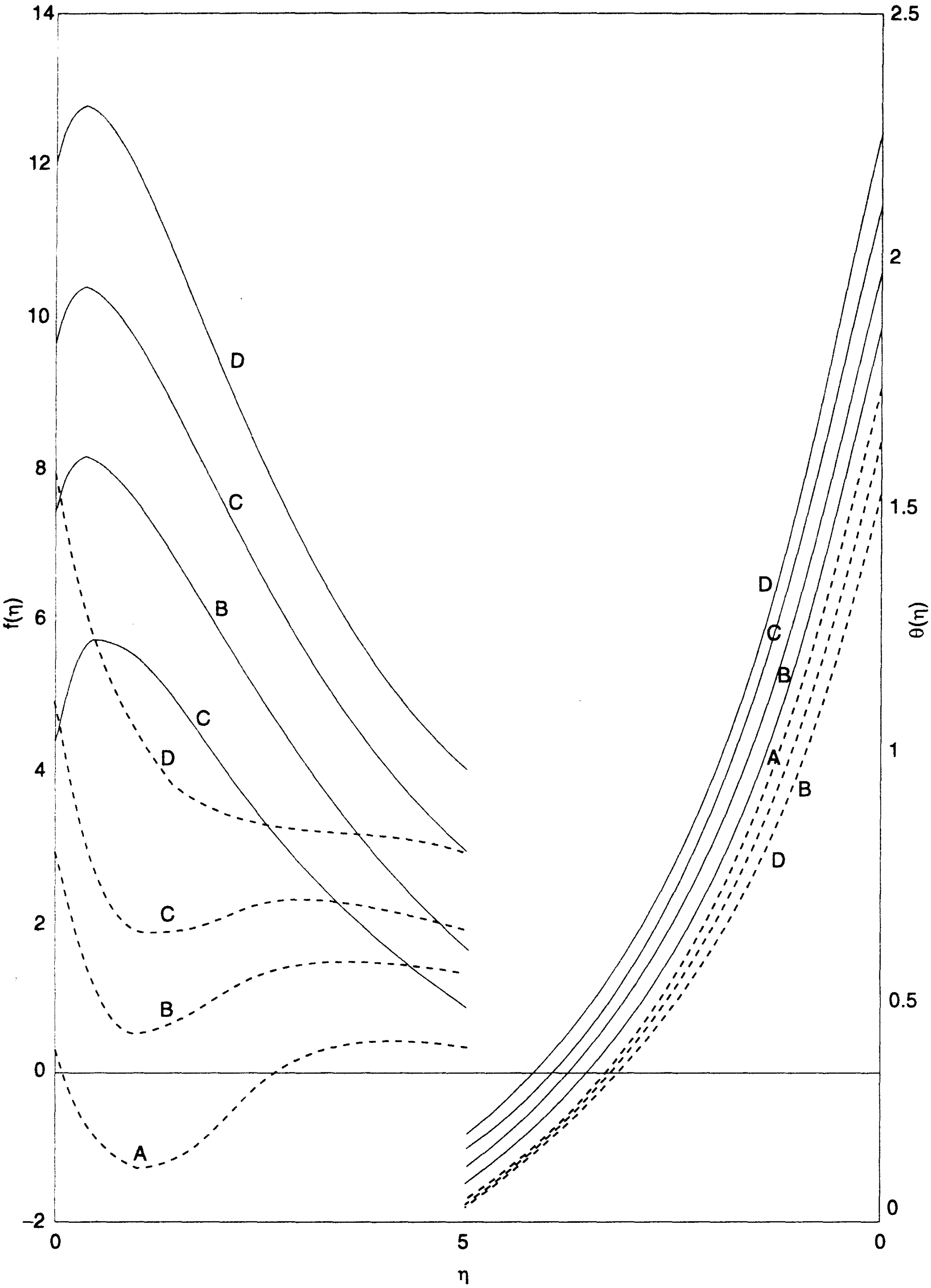

FIG. 3. Velocity and temperature profiles against η for various *Gm*. (— Gr = 10.0, externally cooled plate), (--- $Gr = -1.0$, externally heated plate) $Pr = 0.71$, $Sc = 0.6$, $\alpha = 1.0$, $M = 1.0$, $E = 0.2$, values of *Gm* for curve $A = 1.0$, $B = 2.0$, $C = 3.0$, $D = 4.0$

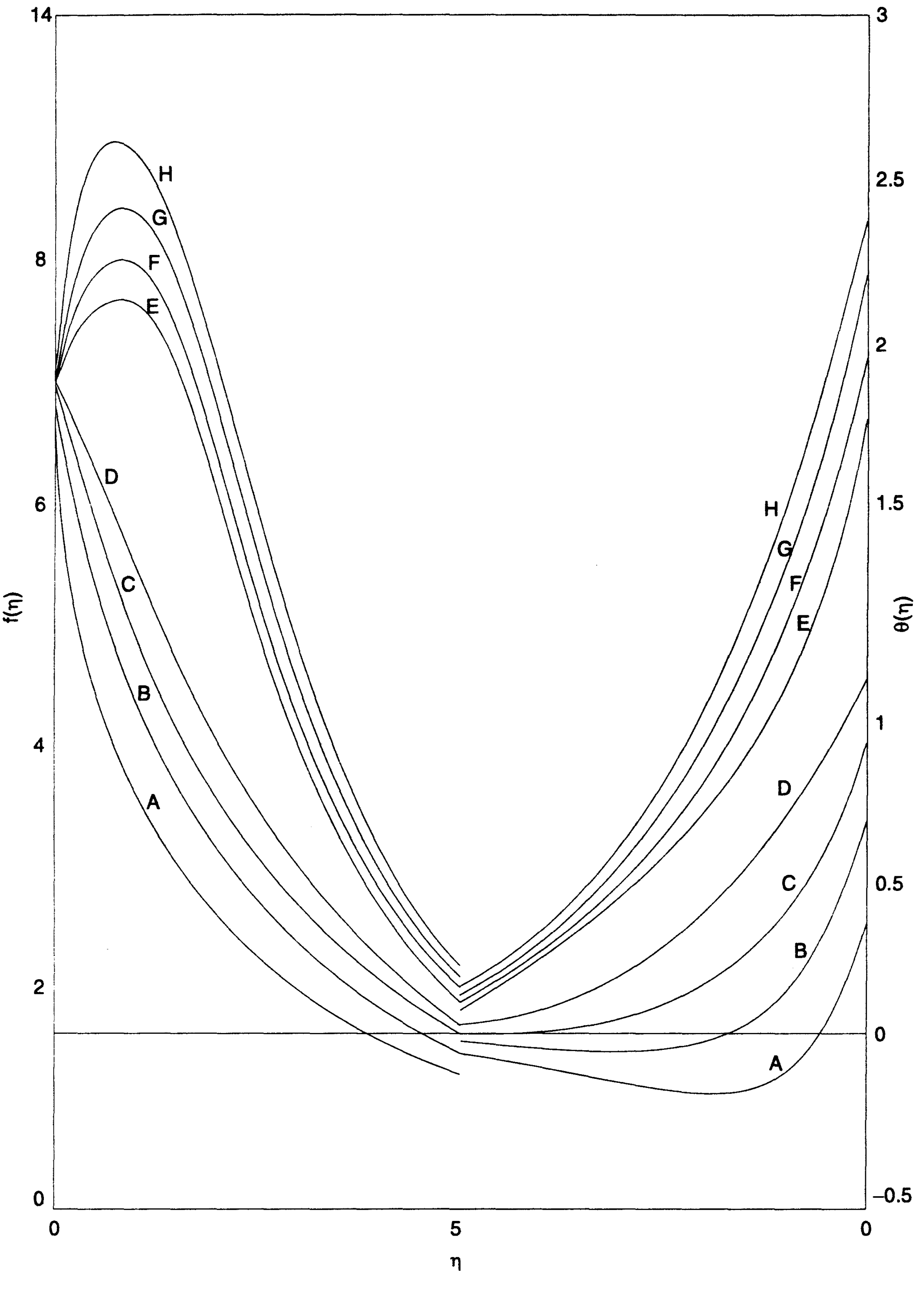

FIG. 4. Velocity and temperature profiles against η for various E. (Gr = 10.0, externally cooled plate), Pr = 0.71, Sc = 0.6, Gm = 2.0, α = 1.0, M = 1.0, values of E for curve A = – 0.04, B = – 0.03, C = – 0.02, D = – 0.01, E = 0.01, F = 0.02, G = 0.03, H = 0.04

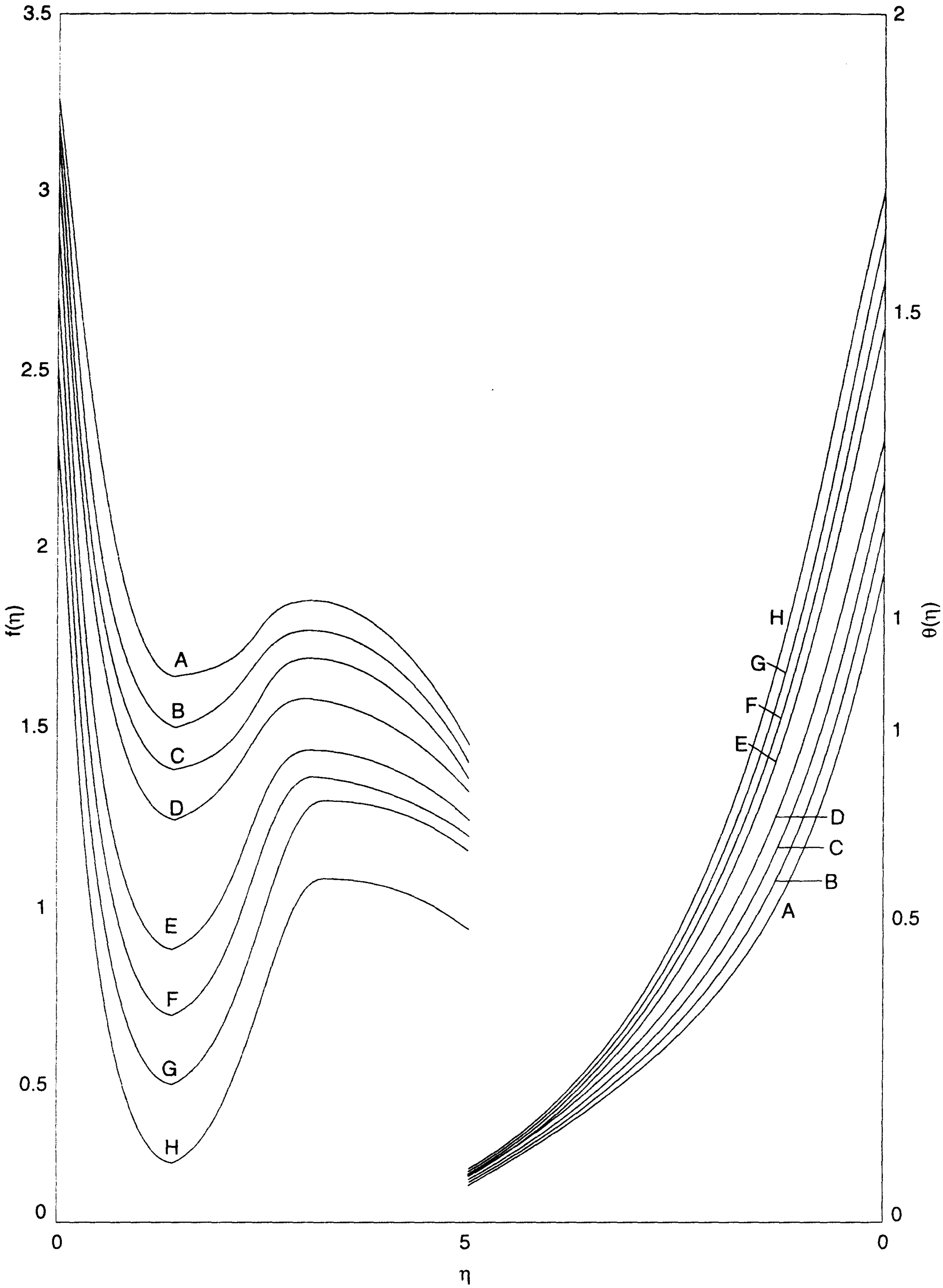

FIG. 5. Velocity and temperature profiles against η for various E. (Gr = − 10.0, externally cooled plate), Pr = 0.71, Sc = 0.6, Gm = 2.0, α = 1.0, M = 1.0, values of E for curve A = − 0.04, B = − 0.03, C = − 0.02, D = − 0.01, E = 0.01, F = 0.02, G = 0.03, H = 0.04

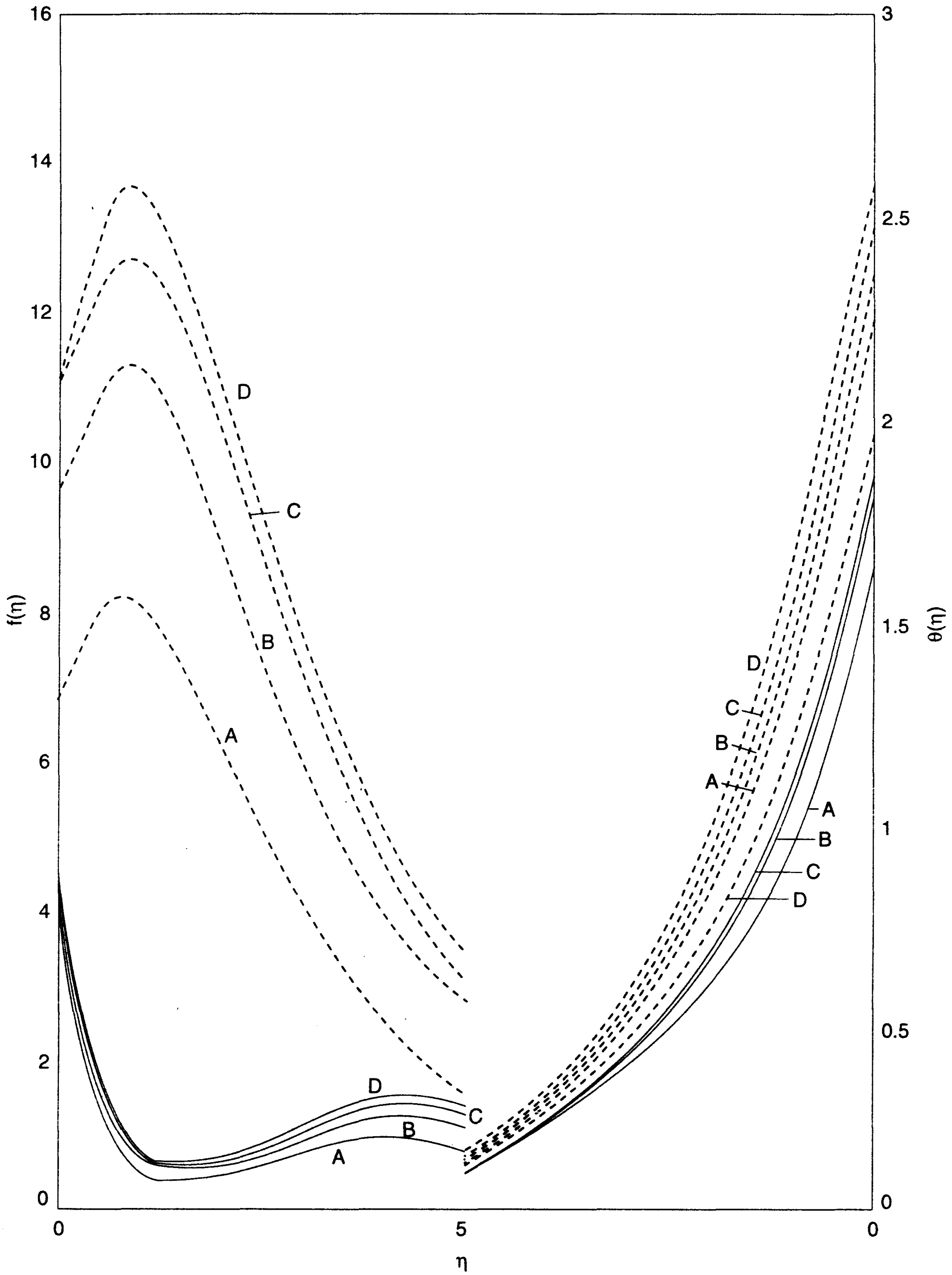

FIG. 6. Velocity and temperature profiles against η for various values of α. Pr = 0.71, Sc = 0.6, Gm = 2.0, M = 1.0, E = 0.02, --- Gr = 10.0 (externally cooled plate), — Gr = − 10.0 (externally cooled plate). Values of α for curve A = 1.0, B = 2.0, C = 3.0, D = 4.0

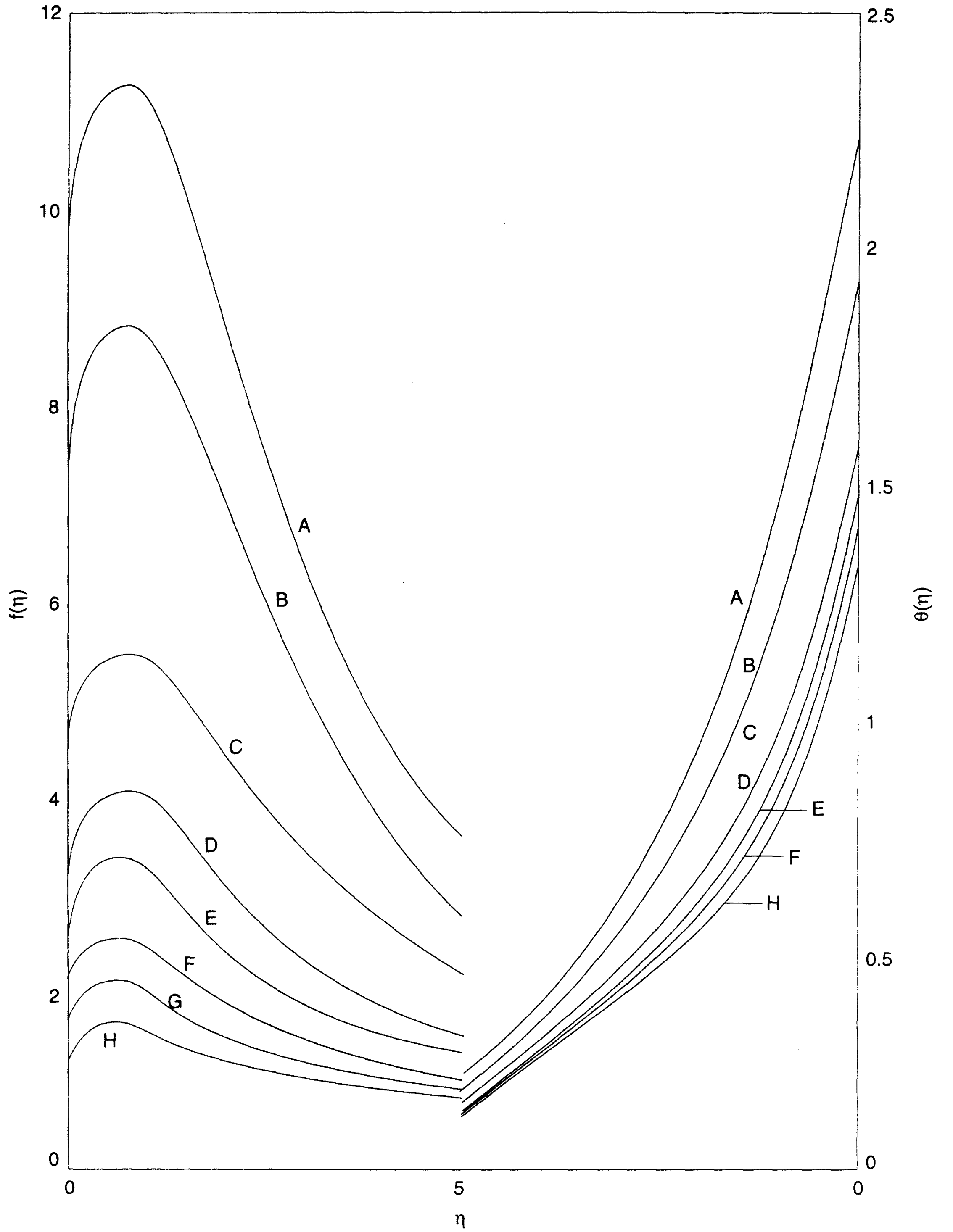

FIG. 7. Velocity and temperature profiles against η for various values of M. Pr = 0.71, Sc = 0.60, Gm = 2.0, M = 1.0, E = 0.02, Gr = 10.0 (externally cooled plate). Values of α for curve A = 1.0, B = 2.0, C = 3.0, D = 4.0

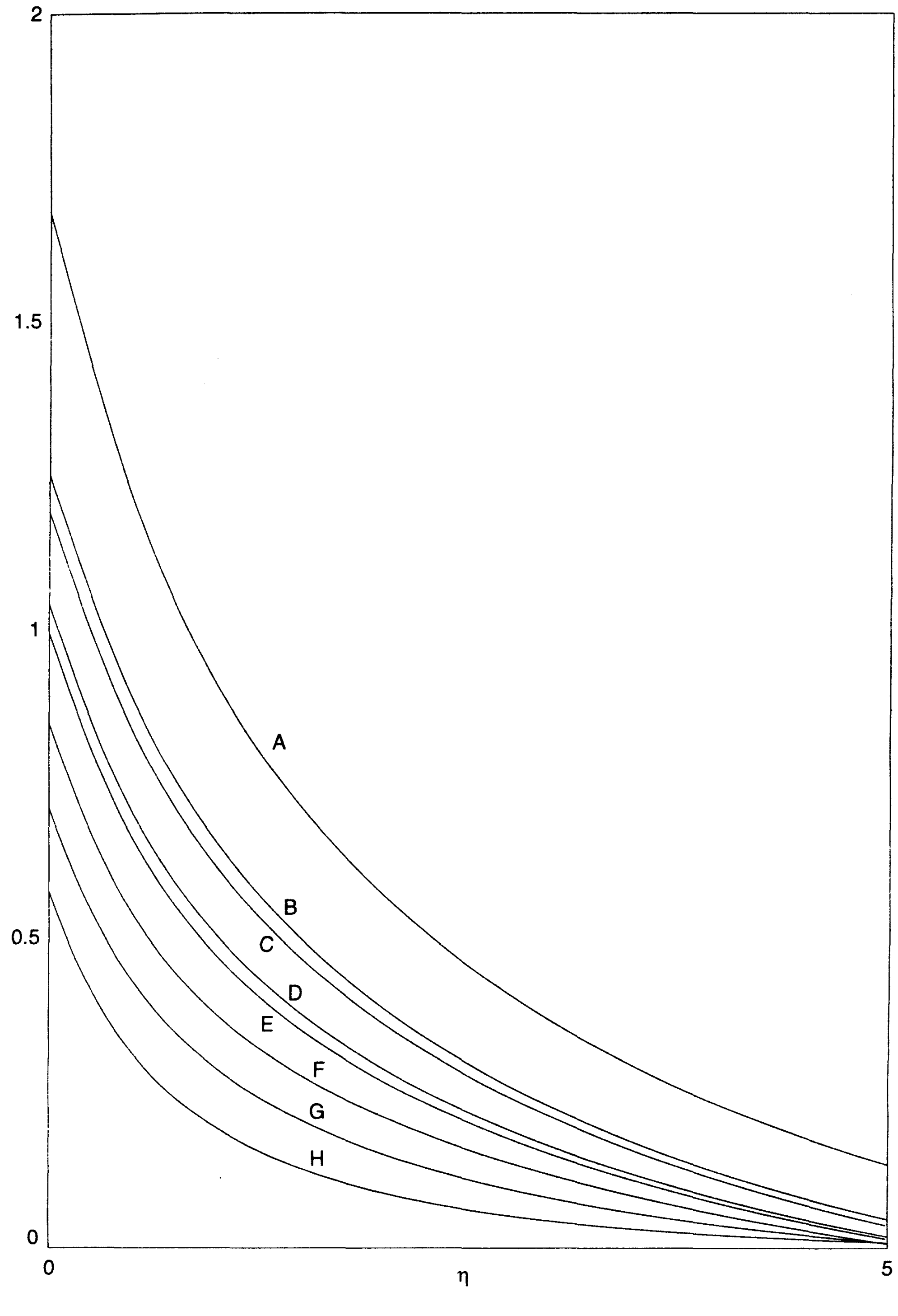

FIG. 8. Variation of concentration profiles against η for different values of Schmidt number (Sc.) Values of Sc for curve $A = 0.60$, $B = 0.75$, $C = 0.78$, $D = 0.94$, $E = 0.97$, $F = 1.17$, $G = 1.30$, $H = 1.60$

coefficient increases for externally heated plate with increasing E (Table III). The porosity of the medium has considerable effect on skin friction, it increases for externally cooled plate and decreases for externally heated plate (Table IV). Finally increasing magnetic field skin friction coeffcient decreases (Table V).

TABLE I

Variation of skin friction coefficient at the plate with Schmidt Number for Pr = 0.71

	$(\tau)_0$	
Sc	Gr= 10.0	Gr = − 10.0
0.22	3.3964	− 10.7219
0.60	3.9948	− 9.6559
0.78	3.7537	− 9.8539
2.62	9.1924	− 4.1922

TABLE II

Variation of skin friction coefficient at the plate with Gm for Pr = 0.71

	$(\tau)_0$	
Gm	Gr = 10.0	Gr = − 10.0
0.1	4.8034	− 8.8100
0.2	3.3964	− 10.7219
0.3	2.1577	− 12.8023
0.4	1.0874	− 15.0508

TABLE III

Variation of the skin friction at the plate with E, where Pr = 0.71 and Sc = 0.22

	$(\tau)_0$	
E	Gr = 10.0	Gr =− 10.0
− . 04	− 2.5132	− 1.4761
− . 03	− 1.5283	− 3.0171
− . 02	− 0.5433	− 4.5581
− . 01	+ 0.4416	− 6.0991
+ . 01	2.4115	− 9.1810
+ . 02	3.3964	− 10.7219
+ . 03	4.3814	− 12.2629
+ . 04	5.3663	− 13.8039

TABLE IV

Variation of skin friction at the plate with permeability parameter α, where $Pr = 0.71$, $Sc = 0.22$ and $Gm = 2.0$

	$(\tau)_0$	
α	$Gr = 10.0$	$Gr = -10.0$
1.0	3.3964	− 10.7219
2.0	4.1612	− 15.1262
3.0	4.5002	− 17.8309
4.0	4.6767	− 19.6268

TABLE V

Variation of skin friction coefficient at the plate with magnetic number M, where $Pr = 0.71$, $Sc = 0.22$, $Gm = 2.0$, $Gr = 10$, $\alpha = 1.0$ and $E = 0.02$

M	$(\tau)_0$
0.5	4.1612
1.0	3.3964
2.0	2.6556
3.0	2.3278
4.0	2.1406
6.0	1.9167
7.0	1.8385
8.0	1.7725

6. CONCLUSIONS

We now summarise some important observations :

1. Presence of foreign species reduces the velocity as well as thermal boundary layer and further reduction occurs with increasing Schmidt number in case of externally cooled plate.
2. Greater cooling results in increase in velocity and thermal boundary layer thickness.
3. Velocity of fluid layer decreases and thickness of thermal boundary layer increases with increasing Schmidt number in case of externally heated plate.
4. Greater heating causes reduction in fluid velocity and increase in thermal boundary layer thickness.
5. Increase in velocity and temperature occurs for externally cooled plate whereas velocity increases but temperature decreases for externally heated plate as Gm increases.
6. Velocity and temperature increases with Eckert number in case of externally cooled plate.

7. For externally heated plate ($Gr = -10.0$) velocity decreases and temperature increases with increasing Eckert number and fluctuation in velocity distribution curve is observed.
8. Porosity of the medium has considerable effect on velocity and temperature distribution. Both the profiles increase with increases in permeability parameter
9. Application of magnetic field causes decrease in both velocity and temperature.
10. In presence of thicker diffusing species thickness of concentration layer decreases both in magnitude and extent.

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