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THERMOELASTIC INTERACTIONS DUE TO PRESCRIBED PRESSURE INSIDE A SPHERICAL CAVITY IN AN UNBOUNDED MEDIUM

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Temperature induced, stresses and displacements in an infinite medium due to the application of (*i*) unit step-stress or (*ii*) impulsive pressure applied on the inner wall of a spherical cavity have been determined for four different problems of generalized thermoelasticity. The results are finally presented by means of graphs for a few cases.

Key Words : Thermoelastic Interactions; Spherical Cavity; Unbounded Medium; Unite Step-Stress; Impulsive Pressure

INTRODUCTION

Exact solution to the transient coupled thermoelastic problem has not so far been obtained. Boley and Tolins[1] presented approximate solutions to some transient coupled thermoelastic boundary value problems in the half space. Soler and Bull[2] used perturbation technique to solve such problems while the iteration technique was applied by Wilms[3] to obtain the induced temperature due to application of unit step-stress on the inner wall of a spherical cavity in an infinite medium. It is pertinent to refer to the researches of Erbray *et al.*[4], Mukhopadhyay *et al.*[5], Chatterjee *et al.*[6], Noda *et al.*[7], Chandrasekharaiah *et al.*[8] etc. in this context.

In this paper, we have considered four different problems on generalized coupled transient thermoelastic problems for an unbounded body with a spherical cavity, and solved by the eigenvalue approach. The solutions for the cases of conventional thermoelasticity (CTE), extended thermoelasticity (ETE) and the temperature rate dependent thermoelasticity (TRDTE) have been compared and presented in graphs.

Nomenclature

u = Radial displacement component

T = Temperature

ρ = Density

α_t = Coefficient of linear expansion

λ, μ = Lamé constants

$\beta = [(\lambda + 2\mu)/\mu]^{1/2}$

C_v = Heat capacity at constant volume

$C_1 = [(\lambda + 2\mu)/\rho]^{1/2}$ = Speed of the dilatational wave in the solid

K = Thermal diffusivity

$C = C_1^2/K$

α, α_0 = Material constants

τ = Thermal relaxation parameter

$$\varepsilon = \frac{\beta^2 T}{\rho C_v (\lambda + 2\mu)}$$

p = Laplace transform parameter

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}$$

$\beta_1 = \beta(1 + \alpha p)$

$C_2 = C(1 + \tau p)$

$\varepsilon_1 = \alpha_0 p + \varepsilon(1 + \alpha p)(1 + \tau p).$

Basic Equations

We consider a homogeneous and isotropic thermoelastic unbounded body with a spherical cavity and analyse the thermoelastic interactions that are spherically symmetric. Then the displacement has only the radial component $u = u(r, t)$ and the three principal stresses in the radial, cross radial and transverse directions are σ_r, σ_θ and σ_ϕ respectively. In this case we consider two normal stresses σ_r and $\sigma_\phi (= \sigma_\theta)$ in the radial and transverse directions. In the case of spherical symmetry the displacement equation of motion and heat conduction equation in the absence of body forces and heat sources are given by[8]

$$\rho C_1^2 \frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r}+\frac{2u}{r}\right)-\beta\left(\frac{\partial T}{\partial r}+\alpha\frac{\partial^2 T}{\partial t \partial r}\right)=\rho\frac{\partial^2 u}{\partial t^2} \quad \dots (1)$$

and

$$K\nabla^2 T=\rho C\left(\frac{\partial T}{\partial t}+\alpha_0\frac{\partial^2 T}{\partial t^2}\right)+\beta T_0\left(\frac{\partial}{\partial t}+\tau\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u}{\partial r}+\frac{2u}{r}\right). \quad \dots (2)$$

The constitutive relation for the stresses σ_r and σ_ϕ can be written as

$$\sigma_r=\rho C_1^2\frac{\partial u}{\partial r}+\frac{2\rho C_1^3 u}{r(1-C_1)}-\beta\left(\theta+\alpha\frac{\partial T}{\partial t}\right), \quad \dots (3)$$

and

$$\sigma_\phi=\frac{\rho C_1^3}{(1-C_1)}\frac{\partial u}{\partial r}+\frac{\rho C_1^3 u}{r(1-C_1)}-\beta\left(\theta+\alpha\frac{\partial T}{\partial t}\right). \quad \dots (4)$$

From the general equations (1) and (2), we now classify the problem into three classes, *vide*, Chandrasekharaiah[8], for our further reference and for the comparison of our numerical computations of the results.

i) If $\alpha_0=\alpha=\tau=0$, then the problem reduces to the problem of classical thermoelasticity (CTE).

ii) In the case when $\alpha = 0$, but $\alpha_0=\tau\neq 0$, we called the problem as a problem of extended thermoelasticity (ETE).

iii) When $\alpha\neq 0$, $\alpha_0\neq 0$, but $\tau=0$, then the equations reduces to a problem of temperature rate dependent thermoelasticity (TRDTE)

We now apply the transformations defined by

$$r'=\frac{r}{a},\quad t'=\frac{Kt}{\rho Ca^2},\quad u'=\frac{\rho C_1^2 u}{\beta a T_0},\quad T'=\frac{T-T_0}{T_0}$$

$$\alpha'=\frac{k\alpha}{\rho Ca^2},\quad \alpha_0'=\frac{k\alpha_0}{\rho Ca^2},\quad \tau'=\frac{k\tau}{\rho Ca^2},$$

$$\sigma_r'=\frac{\rho C_1^2\sigma_r}{2\mu\beta T_0},\quad \sigma_\phi'=\frac{\rho C_1^2\sigma_\phi}{2\mu\beta T_0}. \quad \dots (5)$$

As such, suppressing the primes, from (1-4) we get the following equations in dimensionless form :-

$$\frac{\partial}{\partial r}\left[\frac{\partial u}{\partial r}+\frac{2u}{r}-\left(1+\alpha\frac{\partial}{\partial t}\right)T\right]=\delta^2\frac{\partial^2 u}{\partial t^2}, \quad \text{... (6)}$$

$$\left[\frac{\partial}{\partial r}+\frac{2}{r}\right]\left[\frac{\partial T}{\partial r}-\varepsilon\left(1+\tau\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t}\right]=\left[1+\alpha_0\frac{\partial}{\partial t}\right]\frac{\partial T}{\partial t}, \quad \text{... (7)}$$

$$\sigma_r=\frac{1}{1-2\upsilon}\left[(1-\upsilon)\frac{\partial u}{\partial r}+\frac{2\upsilon u}{r}-(1-\upsilon)\left(1+\alpha\frac{\partial u}{\partial t}\right)T\right], \quad \text{... (8)}$$

and

$$\sigma_\phi=\frac{1}{1-2\upsilon}\left[\upsilon\frac{\partial u}{\partial r}+\frac{u}{r}-(1-\upsilon)\left(1+\alpha\frac{\partial}{\partial t}\right)T\right]. \quad \text{... (9)}$$

Here, $\delta=\dfrac{k}{\rho C C_1 a}$ ... (10)

is a dimensionless inertial parameter, and υ is the poisson ratio of the material.

METHOD OF SOLUTION : FORMULATION OF A VECTOR MATRIX DIFFERENTIAL EQUATIONS

We now apply Laplace transform of the form

$$\bar{u}(r,p)=\int_0^\infty u(r,t)\exp(-pt)\,dt$$

and

$$\bar{T}(r,p)=\int_0^\infty T(r,t)\exp(-pt)dt \quad \text{... (11)}$$

to the eqs. (6) and (7) to obtain

$$L(\bar{u})=\delta^2p^2\,\bar{u}+(1+\alpha p)\frac{d\bar{T}}{dr} \quad \text{... (12)}$$

and

$$L\left(\frac{d\bar{T}}{dr}\right)=\varepsilon p^3\delta^2(1+\tau p)\,\bar{u}+[\varepsilon p(1+\tau p)(1+\alpha p)+p(1+\alpha_0 p)]\frac{d\bar{T}}{dr}, \quad \text{... (13)}$$

where

$$L=\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}-\frac{2}{r^2}. \quad \text{... (14)}$$

Equations (12) and (13) can be written in the form of a vector matrix differential equation as :

$$L\underset{\sim}{V} = \underset{\sim}{A}\underset{\sim}{V}, \quad \ldots (15)$$

where

$$\underset{\sim}{V} = \left[u, \frac{d\bar{T}}{dr} \right]^T, \quad \ldots (16)$$

$$\underset{\sim}{A} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \quad \ldots (17)$$

and

$$C_{11} = \delta^2 p^2, \quad C_{12} = (1 + \alpha p), \quad C_{21} = \varepsilon p^3 \delta^2 (1 + \tau p),$$

$$C_{22} = p[\varepsilon(1 + \alpha p)(1 + \tau p) + \alpha_0 p + 1]. \quad \ldots (18)$$

To solve eq. (15), we substitute

$$\underset{\sim}{V} = \underset{\sim}{X}(\lambda)\, \omega(r, \gamma), \quad \ldots (19)$$

where λ is a scalar, $\underset{\sim}{X}$ is a vector independent of r and $\omega(r, \gamma)$ is a non-trivial solution of the scalar differential equation

$$L\omega = 0 \quad \ldots (20)$$

The solution of the above equation can be written as

$$\omega = \frac{1}{r^2} e^{-\gamma r} + \frac{\gamma}{r^2} e^{-\gamma r}. \quad \ldots (21)$$

Using (19) and (20) in (15) and simplifying the result, we obtain the algebraic eigenvalue problem

$$\underset{\sim}{A}\,\underset{\sim}{X}(\gamma) = \gamma^2 \underset{\sim}{X}(\gamma), \quad \ldots (22)$$

where $\underset{\sim}{X}(\gamma)$ is the eigenvector corresponding to the eigenvalue γ^2. The characteristic equation corresponding the matrix $\underset{\sim}{A}$ can be written as

$$\gamma^4 - \gamma^2 (C_{11} + C_{22}) + (C_{11} C_{22} - C_{12} C_{21}) = 0. \quad \ldots (23)$$

The roots of the characteristic eq. (23) are of the form $\gamma = \gamma_1^2$ and $\gamma = \gamma_2^2$

where,

$$\gamma_1^2 + \gamma_2^2 = C_{11} + C_{22} \quad \text{and} \quad \gamma_1^2 \gamma_2^2 = C_{11} C_{22} - C_{12} C_{21}. \qquad \dots (24)$$

The eigenvectors $X(\gamma_j^2), j = 1, 2$ corresponding to the eigenvalues $\gamma_j^2, j = 1, 2$ can be calculated as

$$\underset{\sim}{X}_j(\gamma_j) = \begin{bmatrix} X_1 & (\gamma_j^2) \\ X_2 & (\gamma_j^2) \end{bmatrix} = \begin{bmatrix} -C_{12} \\ C_{11} - \gamma_j^2 \end{bmatrix}_{j=1,2} \qquad \dots (25)$$

So, the solution of (15) can now be written as

$$\underset{\sim}{V}(r, p) = A \underset{\sim}{X}(\gamma_1^2)\left(\frac{e^{-\gamma_1 r}}{r^2} + \frac{\gamma_1}{r} e^{-\gamma_1 r}\right) + B \underset{\sim}{X}(\gamma_2^2)\left(\frac{e^{-\gamma_2 r}}{r^2} + \frac{\gamma_2}{r} e^{-\gamma_2 r}\right), \qquad \dots (26)$$

where the constants A and B are to be determined from the boundary conditions. The components of the space vector $\underset{\sim}{V}$ in (26) can be written as :

$$\bar{u}(r, p) = -AC_{12}\left[\frac{e^{-\gamma_1 r}}{r^2} + \frac{\gamma_1}{r} e^{-\gamma_1 r}\right] - BC_{12}\left[\frac{e^{-\gamma_2 r}}{r^2} + \frac{\gamma_2}{r} e^{-\gamma_2 r}\right], \qquad \dots (27)$$

and

$$\frac{dT}{dr} = A(C_{11} - \gamma_1^2)\left[\frac{e^{-\gamma_1 r}}{r^2} + \frac{\gamma_1}{r} e^{-\gamma_1 r}\right] + B(C_{11} - \gamma_2^2)\left[\frac{e^{-\gamma_2 r}}{r^2} + \frac{\gamma_2}{r} e^{-\gamma_2 r}\right]. \qquad \dots (28)$$

From (28) we get,

$$T(r, p) = -A(C_{11} - \gamma_1^2)\frac{e^{-\gamma_1 r}}{r} - B(C_{11} - \gamma_2^2)\frac{e^{-\gamma_2 r}}{r}. \qquad \dots (29)$$

Taking Laplace transform of (8) and (9) and using (27) and (29) we get,

$$\bar{\sigma}_r = \frac{1}{1-2\upsilon}\left[Ae^{-\gamma_1 r}\left\{(1-\upsilon)\left(\frac{\gamma_1^2}{r} + \frac{2\gamma_1}{r^2} + \frac{2}{r^3}\right)C_{12} - 2C_1\,\upsilon\left(\frac{1}{r^3} + \frac{\gamma_1}{r^2}\right) + (1-\upsilon)(1+\alpha p)\frac{C_{11} - \gamma_1^2}{r}\right\}\right.$$

$$+ Be^{-\gamma_2 r}\left\{(1-\upsilon)\left(\frac{\gamma_2^2}{r}+\frac{2\gamma_2}{r^2}+\frac{2}{r^3}\right)C_{12}-2\upsilon C_{12}\left(\frac{1}{r^3}+\frac{\gamma_2}{r^2}\right)+(1-\upsilon)(1+\alpha p)\frac{C_{11}-\gamma_2^2}{r}\right\}\Bigg], \quad \text{... (30)}$$

$$\bar{\sigma}_\phi=\frac{C_{12}}{1-2\upsilon}\Bigg[Ae^{-\gamma_1 r}\left\{\left(\upsilon\frac{\gamma_1^2}{r}+\frac{2\gamma_1}{r^2}+\frac{2}{r^3}\right)-\left(\frac{1}{r^3}+\frac{\gamma_1}{r^2}\right)+(1-\upsilon)\frac{C_{11}-\gamma_1^2}{r}\right\}$$

$$+ Be^{-\gamma_2 r}\left\{\upsilon\left(\frac{\gamma_2^2}{r}+\frac{2\gamma_2}{r^2}+\frac{2}{r^3}\right)-\left(\frac{1}{r^3}+\frac{\gamma_2}{r^2}\right)+(1-\upsilon)\frac{C_{11}-\gamma_2^2}{r}\right\}\Bigg]. \quad \text{... (31)}$$

We now consider four different problems of thermoelastic interactions when the boundary of the cavity is either maintained at zero temperature or insulated with the radial stress which is either step type or impulsive in nature.

The boundary conditions at the surface of the cavity $r = 1$ for different cases are taken as

(a) $\sigma_r(1, t) = H(t)$

$T(1, t) = 0$

(b) $\sigma_r(1, t) = H(t)$

$\frac{\partial T}{\partial r}(1, t) = 0$

(c) $\sigma_r(1, t) = \delta(t)$

$T(1, t) = 0$

(d) $\sigma_r(1, t) = \delta(t)$

$\frac{\partial T}{\partial r}(1, t) = 0,$ ... (32)

where $H(t)$ and $\delta(t)$ are respectively the Heaviside unit step and Dirac {italic delta} function of t. All field variables tend to zero when $r \to \infty$.

Solving these problems, we present the expressions of the quantities u, T, σ_r and σ_ϕ in Laplace transform domain for different cases in the following Table I :

TABLE - I

Quantities u, T, σ_r and σ_0 in Laplace transform domain

Quantity / Cases	$\bar{u}$	$\bar{T}$	$\bar{\sigma}_r$	$\bar{\sigma}_0$
(a)	$\frac{C_{12}}{p(M_3N_4-M_4N_3)} \times \left[-N_4\left(\frac{1}{r^2}+\frac{\gamma_1}{r}\right)e^{-\gamma_1(r-1)} + N_3\left(\frac{1}{r^2}+\frac{\gamma_2}{r}\right)e^{-\gamma_2(r-1)}\right]$	$\frac{1}{p(M_3N_4-M_4N_3)r} \times [-N_4(C_{11}-\gamma_1^2)e^{-\gamma_1(r-1)} + N_3(C_{11}-\gamma_2^2)e^{-\gamma_2(r-1)}]$	$\frac{1}{p(M_3N_4-M_4N_3)r} \times [N_4M_1e^{-\gamma_1(r-1)} - N_3M_2e^{-\gamma_2(r-1)}]$	$\frac{1}{p(M_3N_4-M_4N_3)} [N_4T_1e^{-\gamma_1(r-1)} - N_3T_2e^{-\gamma_2(r-1)}]$
(b)	$\frac{C_{12}}{p[(C_{11}-\gamma_2^2)Q_4M_3-(C_{11}-\gamma_1^2)Q_3M_4]} \times \left[-Q_4(C_{11}-\gamma_2^2)\left(\frac{1}{r^2}+\frac{\gamma_1}{r}\right)e^{-\gamma_1(r-1)} + Q_3(C_{11}-\gamma_1^2)\left(\frac{1}{r^2}+\frac{\gamma_2}{r}\right)e^{-\gamma_2(r-1)}\right]$	$\frac{(C_{11}-\gamma_1^2)(C_{11}-\gamma_2^2)}{rp[(C_{11}-\gamma_2^2)Q_4M_3-(C_{11}-\gamma_1^2)Q_3M_4]} \times [-Q_4e^{-\gamma_1(r-1)} + Q_3e^{-\gamma_2(r-1)}]$	$\frac{1}{p[(C_{11}-\gamma_2^2)Q_4M_3-(C_{11}-\gamma_1^2)Q_3M_4]} \times [M_1Q_4(C_{11}-\gamma_2^2)e^{-\gamma_1(r-1)} - M_2Q_3(C_{11}-\gamma_1^2)e^{-\gamma_2(r-1)}]$	$\frac{1}{p[(C_{11}-\gamma_2^2)Q_4M_3-(C_{11}-\gamma_1^2)Q_3M_4]} \times [Q_4T_1(C_{11}-\gamma_2^2)e^{-\gamma_1(r-1)} - Q_3T_2(C_{11}-\gamma_1^2)e^{-\gamma_2(r-1)}]$
(c)	$\frac{C_{12}}{(M_3N_4-M_4N_3)} \times \left[-N_4\left(\frac{1}{r^2}+\frac{\gamma_1}{r}\right)e^{-\gamma_1(r-1)} + N_3\left(\frac{1}{r^2}+\frac{\gamma_2}{r}\right)e^{-\gamma_2(r-1)}\right]$	$\frac{1}{(M_3N_4-M_4N_3)r} \times [-N_4(C_{11}-\gamma_1^2)e^{-\gamma_1(r-1)} + N_3(C_{11}-\gamma_2^2)e^{-\gamma_2(r-1))}]$	$\frac{1}{(M_3N_4-M_4N_3)} \times [N_4M_1e^{-\gamma_1(r-1)} - N_3M_2e^{-\gamma_2(r-1)}]$	$\frac{1}{(M_3N_4-M_4N_3)} \times [N_4T_1e^{-\gamma_1(r-1)} - N_3T_2e^{-\gamma_2(r-1)}]$
(d)	$\frac{C_{12}}{[(C_{11}-\gamma_2^2)Q_4M_3-(C_{11}-\gamma_1^2)Q_3M_4]} \times \left[-Q_4(C_{11}-\gamma_2^2)\left(\frac{1}{r^2}+\frac{\gamma_1}{r}\right)e^{-\gamma_1(r-1)} + Q_3(C_{11}-\gamma_1^2)\left(\frac{1}{r^2}+\frac{\gamma_2}{r}\right)e^{-\gamma_2(r-1)}\right]$	$\frac{(C_{11}-\gamma_1^2)(C_{11}-\gamma_2^2)}{r[(C_{11}-\gamma_2^2)Q_4M_3-(C_{11}-\gamma_1^2)Q_3M_4]} \times [-Q_4e^{-\gamma_1(r-1)} + Q_3e^{-\gamma_2(r-1)}]$	$\frac{1}{[(C_{11}-\gamma_2^2)Q_4M_3-(C_{11}-\gamma_1^2)Q_3M_4]} \times [M_1Q_4(C_{11}-\gamma_2^2)e^{-\gamma_1(r-1)} - M_2Q_3(C_{11}-\gamma_1^2)e^{-\gamma_2(r-1)}]$	$\frac{1}{[(C_{11}-\gamma_2^2)Q_4M_3-(C_{11}-\gamma_1^2)Q_3M_4]} \times [Q_4T_1(C_{11}-\gamma_2^2)e^{-\gamma_1(r-1)} - Q_3T_2(C_{11}-\gamma_1^2)e^{-\gamma_2(r-1)}]$

where

$$M_1 = \frac{C_{12}}{1-2\upsilon}\left\{(1-\upsilon)\left(\frac{C_{11}}{r}+\frac{2\gamma_1}{r^2}+\frac{2}{r^3}\right)-2\upsilon\left(\frac{1}{r^3}+\frac{\gamma_1}{r^2}\right)\right\},$$

$$M_2 = \frac{C_{12}}{1-2\upsilon}\left\{(1-\upsilon)\left(\frac{C_{11}}{r}+\frac{2\gamma_2}{r^2}+\frac{2}{r^3}\right)-2\upsilon\left(\frac{1}{r^3}+\frac{\gamma_2}{r^2}\right)\right\},$$

$$M_3 = \frac{C_{12}}{1-2\upsilon}\left\{(1-\upsilon)(C_{11}+2\gamma_1+2)-2\upsilon(1+\gamma_1)\right\},$$

$$M_4 = \frac{C_{12}}{1-2\upsilon}\left\{(1-\upsilon)(C_{11}+2\gamma_2+2)-2\upsilon(1+\gamma_2)\right\},$$

$$N_1 = \frac{C_{11}-\gamma_1^2}{r}, \qquad N_2 = \frac{C_{11}-\gamma_2^2}{r},$$

$$N_3 = C_{11}-\gamma_1^2, \qquad N_4 = C_{11}-\gamma_2^2,$$

$$Q_1 = \frac{1}{r^2}+\frac{\gamma_1}{r}, \qquad Q_2 = \frac{1}{r^2}+\frac{\gamma_2}{r},$$

$$Q_3 = 1+\gamma_1, \qquad Q_4 = 1+\gamma_2,$$

$$T_1 = \frac{C_{12}}{1-2\upsilon}\left[\upsilon\left(\frac{\gamma_1^2}{r}+\frac{2\gamma_1}{r^2}+\frac{2}{r^3}\right)-\left(\frac{1}{r^3}+\frac{\gamma_1}{r^2}\right)+(1-\upsilon)N_1\right],$$

$$T_2 = \frac{C_{12}}{1-2\upsilon}\left[\upsilon\left(\frac{\gamma_2^2}{r}+\frac{2\gamma_2}{r^2}+\frac{2}{r^3}\right)-\left(\frac{1}{r^3}+\frac{\gamma_2}{r^2}\right)+(1-\upsilon)N_2\right]. \qquad \text{... (34)}$$

NUMERICAL SOLUTION

The inversion of Laplace transform of the expressions $\bar{u}(r,p)$, $\bar{T}(r,p)$, $\bar{\sigma}_r(r,p)$ and $\bar{\sigma}_\phi(r,p)$ in space time domain for different cases as mentioned in (32) are very complex and we prefer to develop an efficient computer programme for the purpose of inversion of Laplace transforms. As such we follow the method of Bellman[9] and choose a time span by seven values of the time t_i, $i = 1$ to 7 (specified later), at which u, T, σ_r and σ_ϕ have been determined.

We furnish our results in the form of graphs for different cases as in (32), to compare the quantities $[u, T, \sigma_r, \sigma_\phi](r,t)$ for the cases CTE, ETE and TRDTE. The time variables t are vide, [9]. $t_1 = 0.025775$, $t_2 = 0.138382$, $t_3 = 0.352509$, $t_4 = 0.693147$, $t_5 = 1.21376$, $t_6 = 2.04612$ and $t_7 = 3.67119$ are labelled in the abscissa.

For the computations of our results we consider the data for steel for which

$$\upsilon = 0.25, \; \varepsilon = 2.97 \times 10^{-4}$$

As mentioned earlier the values of other parameters α, α_0 and τ for different cases are chosen as follows, vide [8]

i) CTE : $\alpha = \alpha_0 = \tau = 0$

ii) ETE : $\alpha = 0, \alpha_0 = \tau = 0.05$, and for

iii) TRDTE : $\alpha = 0.1, \alpha_0 = 0.05, \tau = 0$.

We see from (33) and (34) that the solution of u, T, σ_r and σ_ϕ for different cases as considered in (32) depend also on the inertial parameter δ. As in Sternberg and Chakraborty[10], we choose δ = 0.20, for a qualitative analysis and $\delta = 7.3 \times 10^{-9}$ for a realistic quantitative analysis.

Choosing the boundary of the cavity at a distance one unit i.e, a = 1, several graphs for temperature, displacement and stresses have been drawn (using cubic spline formalism) for two values of δ, and for increasing values of r in the time span as mentioned earlier.

To study the characteristic of each quantity graphs are drawn for four different cases of (32). The differences in values for the cases of CTE, ETE and TRDTE for any particular quantity occur only after five or six decimal places. A typical graph has been shown in Fig. 1. As such, we now

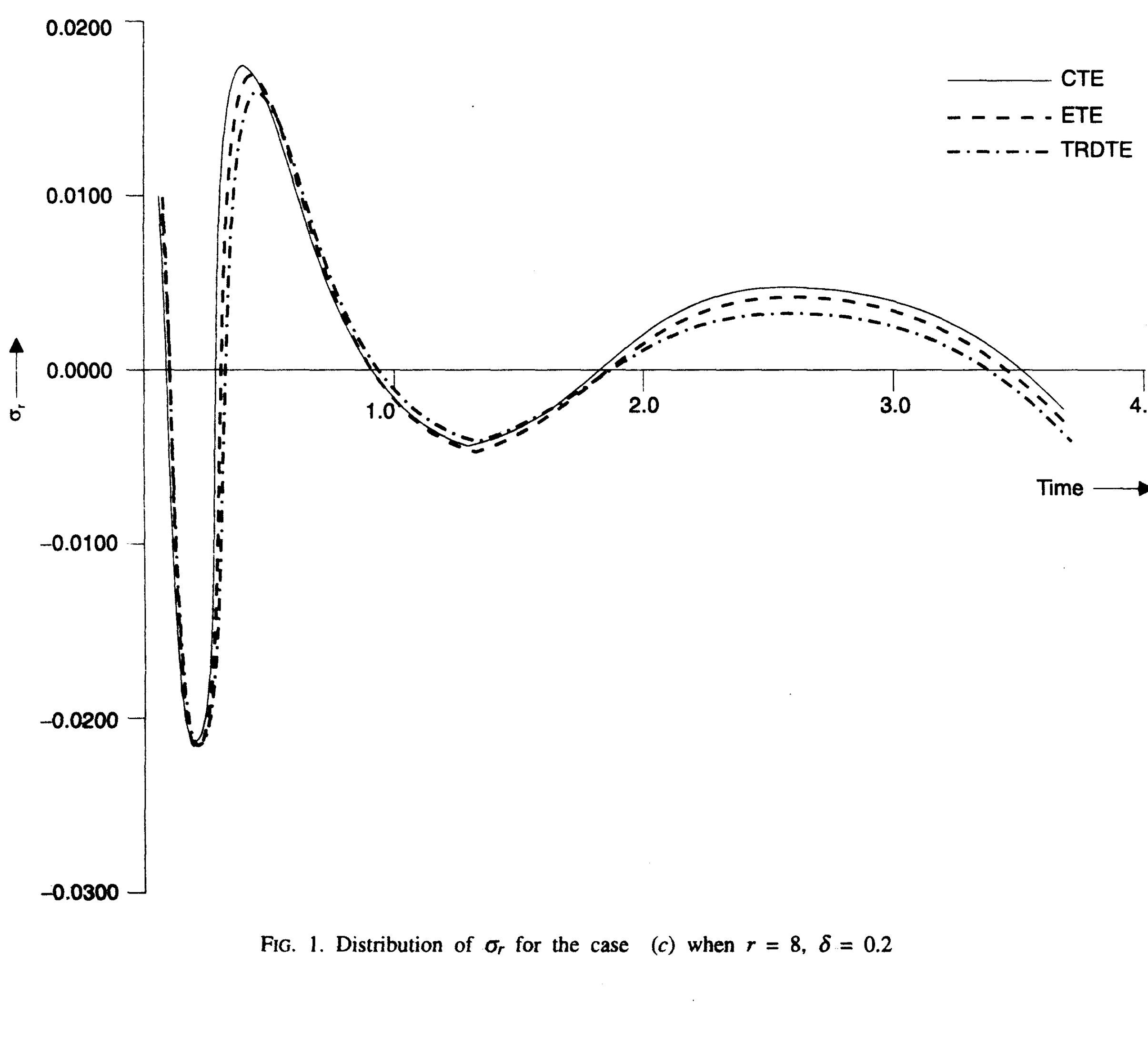

FIG. 1. Distribution of σ_r for the case (c) when r = 8, δ = 0.2

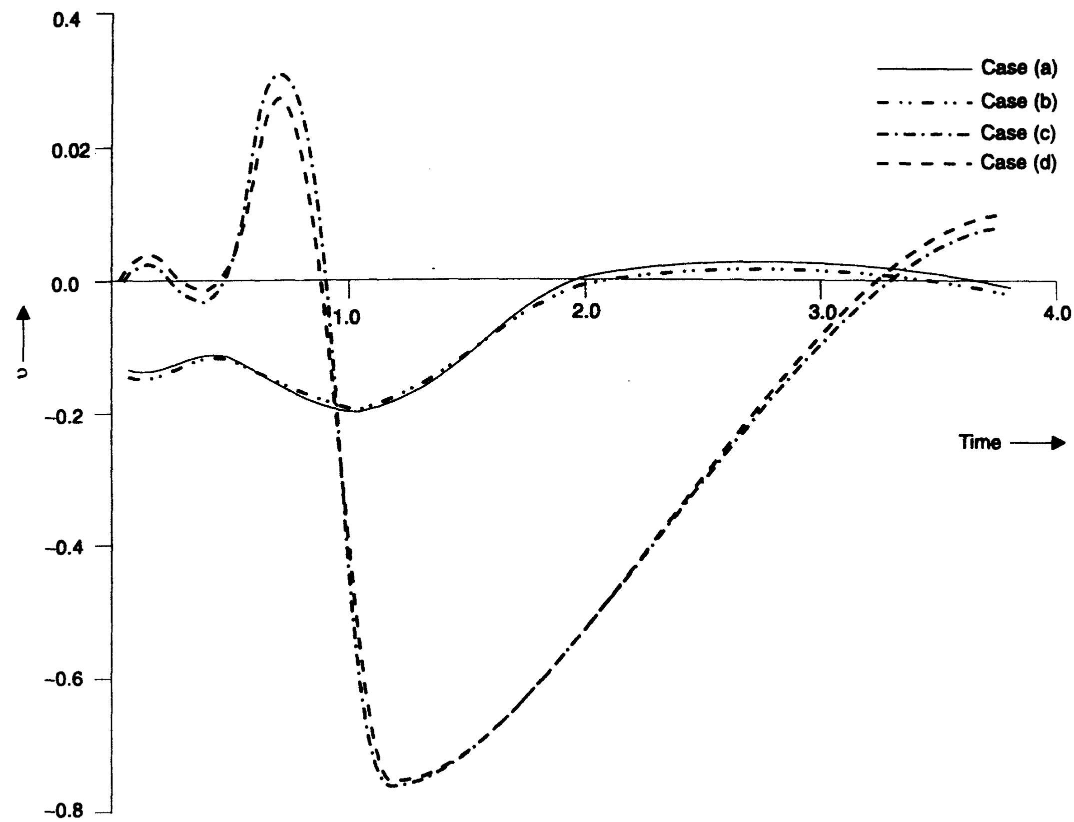

FIG. 2. Distribution of u for the cases (a) – (d) when $0.2\delta = 0.2$

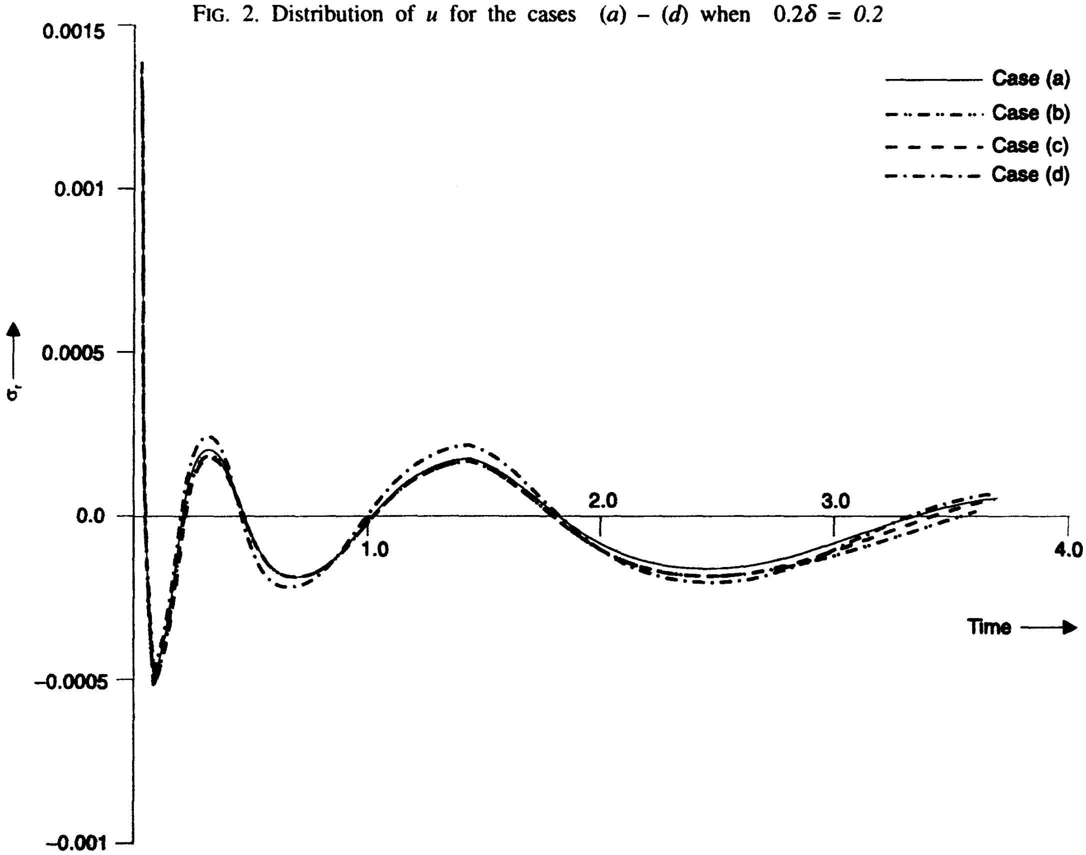

FIG. 3. Distribution of σ_r for the cases (a) - (d) when $\delta = 7.3 \times 10^{-9}$

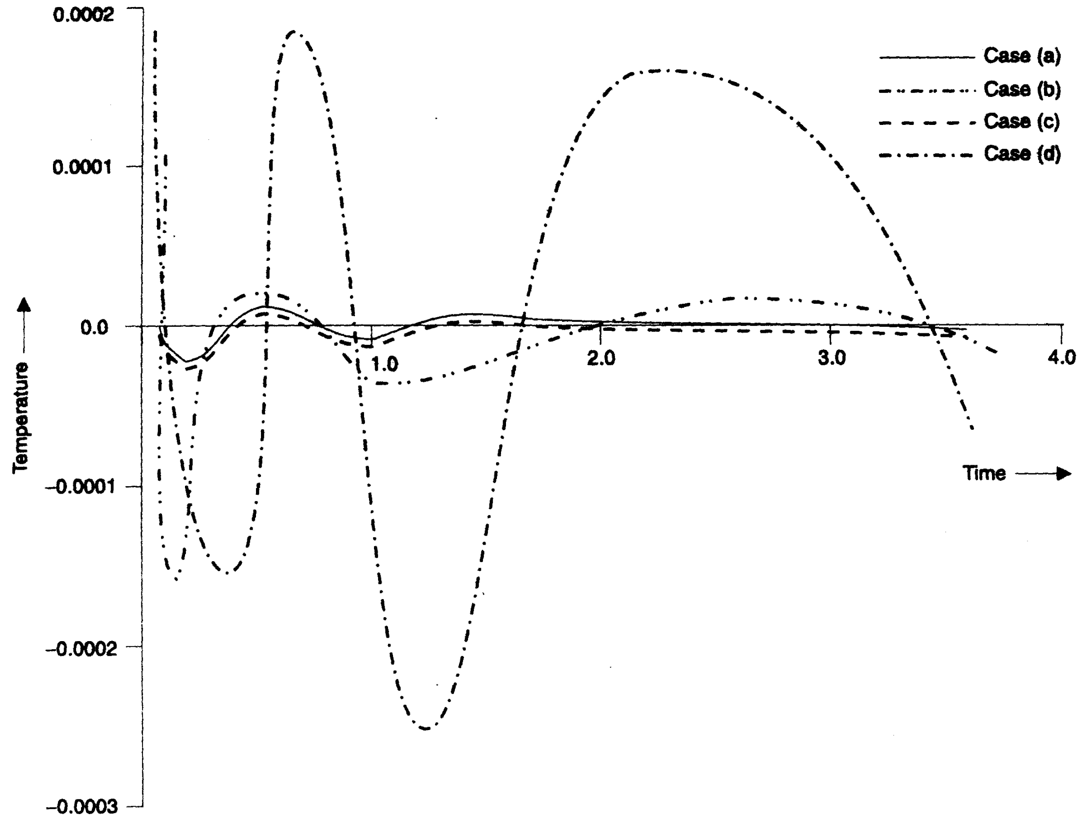

FIG. 4. Distribution of Temperature for the case (*a*) - (*d*) when $\delta = 7.3 \times 10^{-9}$

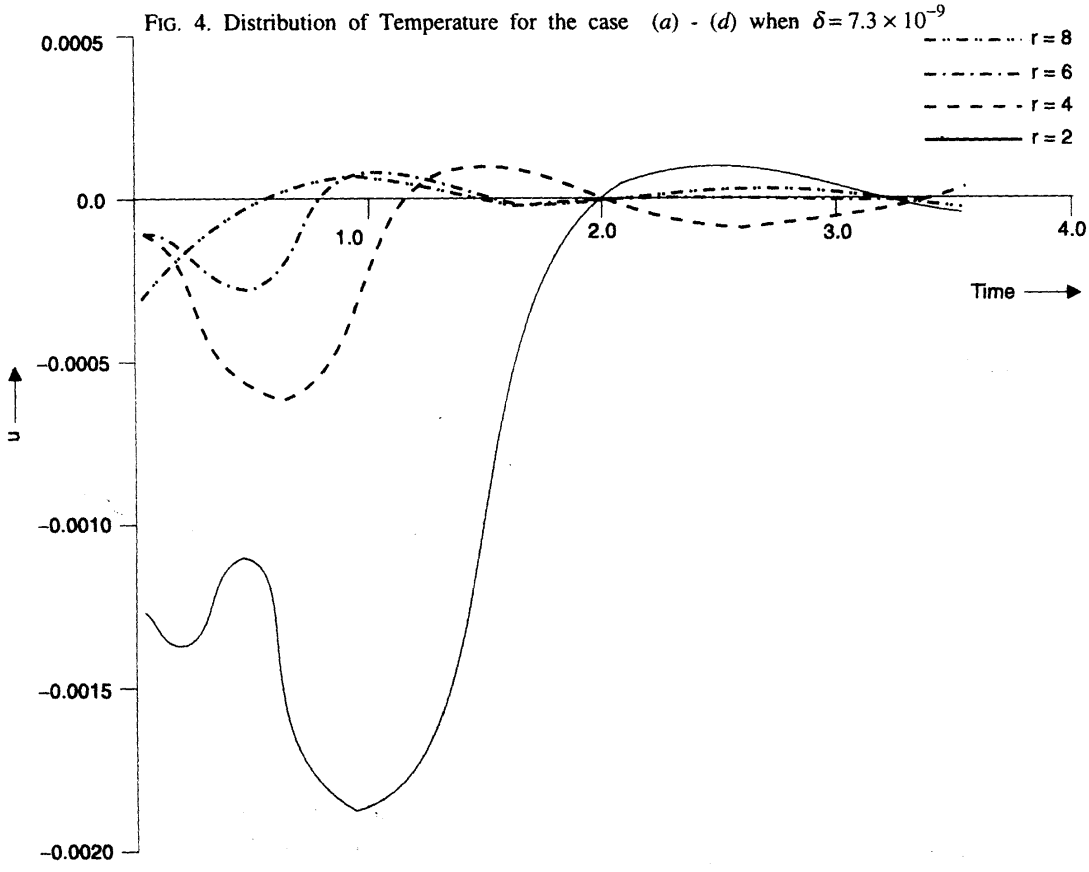

FIG. 5. Distribution of *u* for the case (*a*) when $\delta = 0.2$

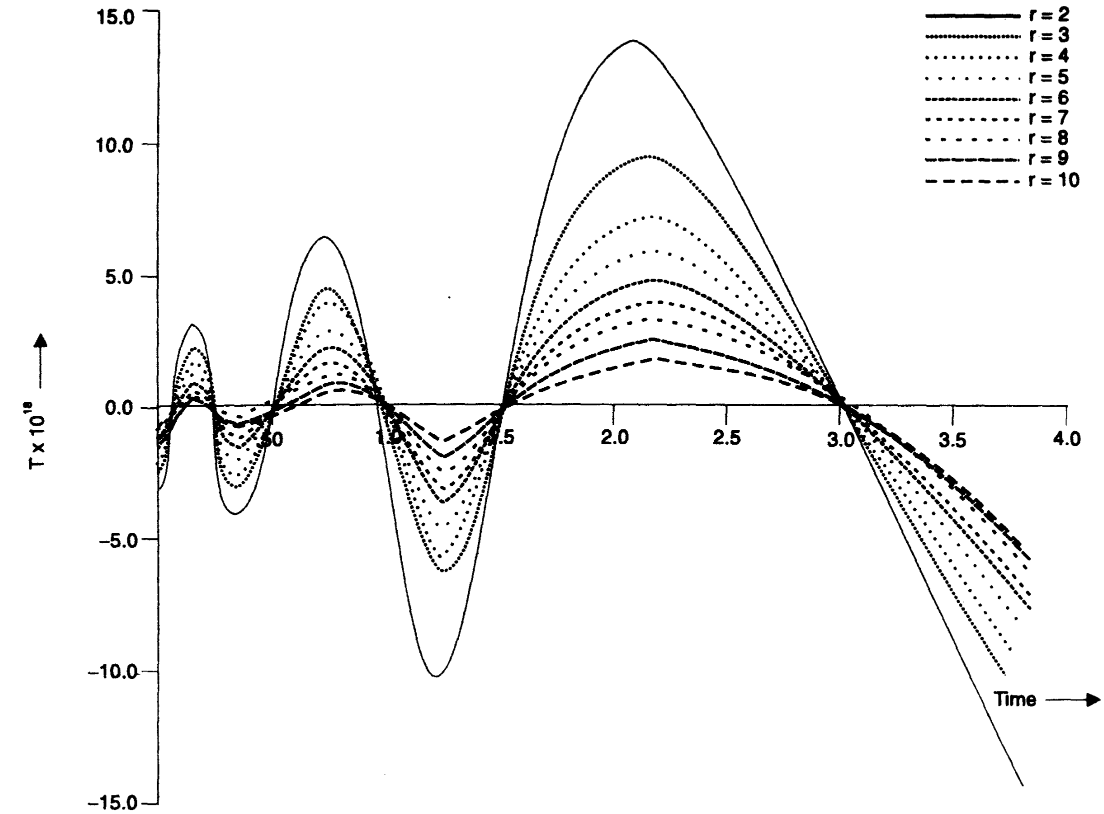

FIG. 6. Distribution of Temperature (T) for different values of r

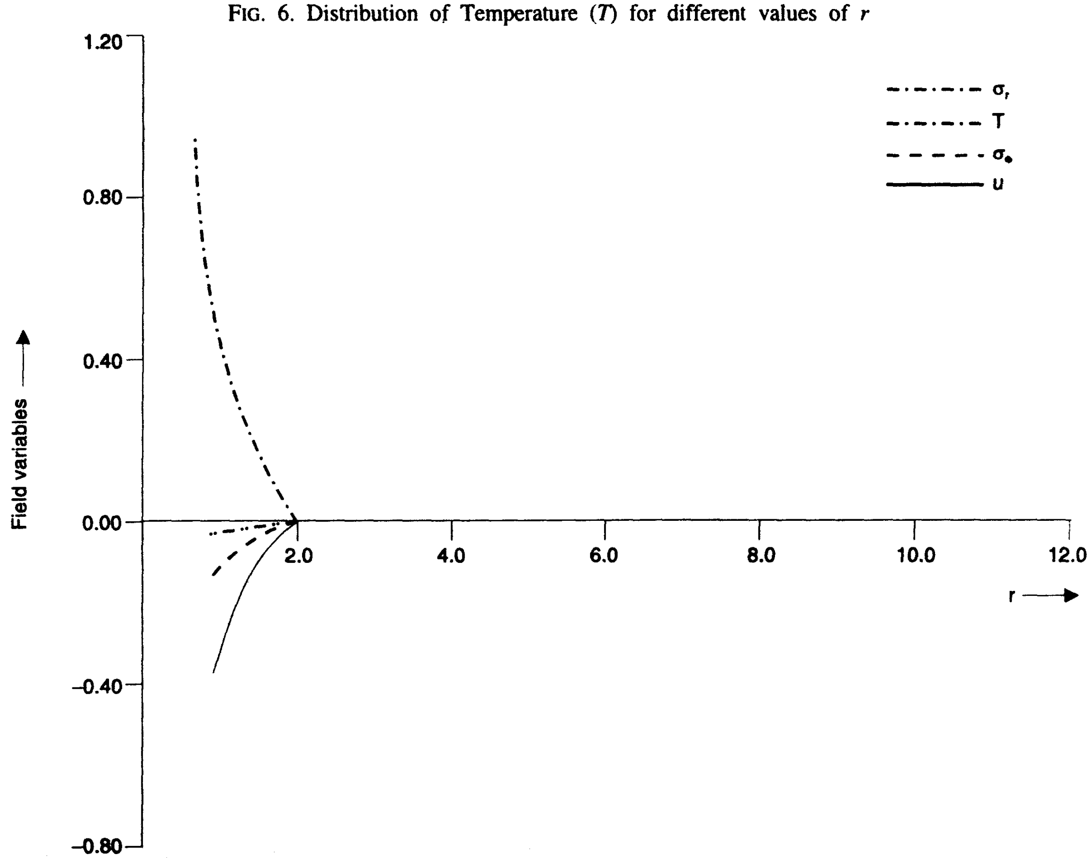

FIG. 7. Distribution of u, T, σ_r and σ_Φ when time = 0.025775

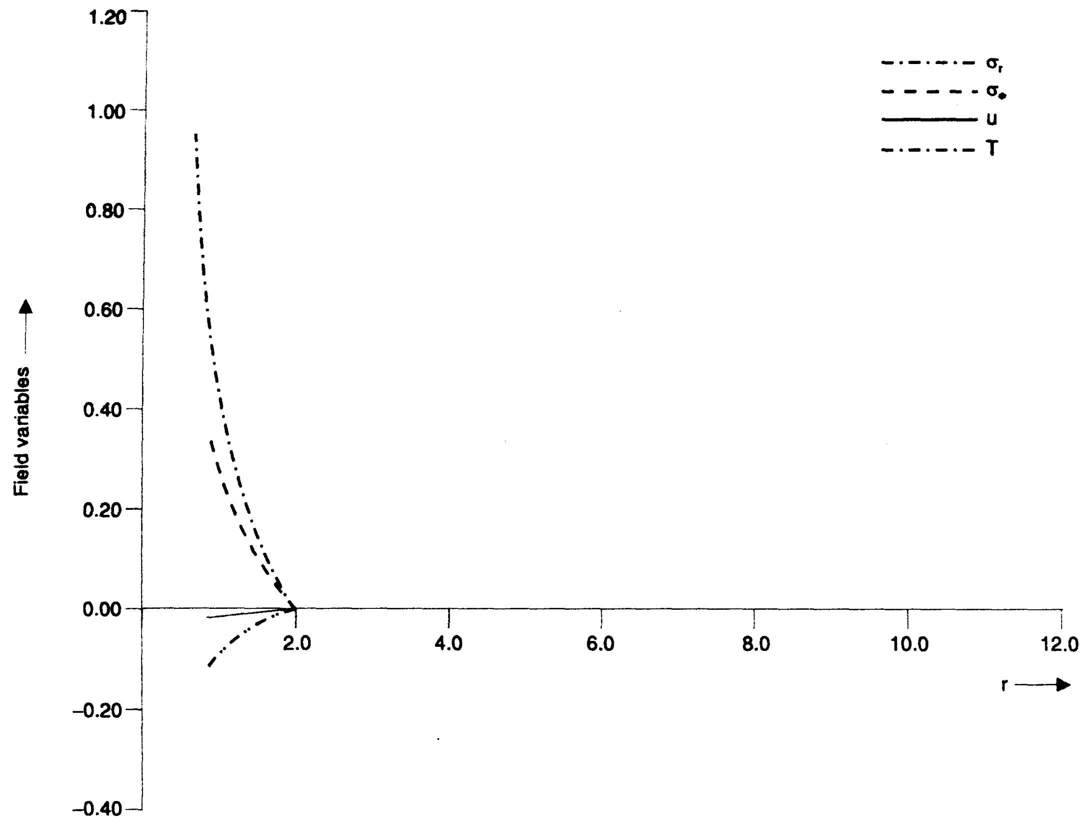

FIG. 8. Distribution of u, T, σ_r and σ_Φ when time = 3.67119

restrict ourselves to draw graph only for the cases of TRDTE for different quantities. In the cases of TRDTE, when r = 2 and for δ= 0.2 or 7.3 $\times 10^{-9}$ several graphs for different problems in (32) have been drawn, (Fig. 2 to 4). It is found that all these quantities tend to zero when r increases. A typical graph has been shown in Fig. 5. It is noticed that the field quantities represent almost the same graph for the cases of (a) and (b) or (c) and (d), (Figs. 2 to 3) except for the case of temperature when r = 2 and δ = 7.3 $\times 10^{-9}$ (Fig. 4).

It is interesting to note that the temperature induced on the surface r = 2, 3, 4, 5, 6, 7, 8, 9, 10 for the case of (c) when δ = 7.3 $\times 10^{-9}$ is the same at times t = 0.04, 0.24, 0.96, 1.58, 3.1. Fig. 6.

We now proceed to find out the location of the wave front at $r = r^*$ for a particular time $t = t^*$ (say) by drawing the graphs. Such studies are made for problem (a) in the case of TRDTE when δ = 0.2. It is seen from the Figs. (7 & 8) that when t^* = 0.025775, the wave-front can be located approximately at r^* = 1.9 and when t^* = 3.67119, then r^* = 2.1. Similar studies can be made for other cases also.

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Appendix - I

Theory

Let the vector-matrix differential equation be of the form

$$L \underset{\sim}{x} = t^2 \underset{\sim}{A} \underset{\sim}{x} \qquad \ldots (1)$$

where L is the linear second order differential operator and

$$L = t^2 \frac{d^2}{dt^2} + tp(t) \frac{d}{dt} + q(t),$$

$\underset{\sim}{A}$ is an $n \times n$ constant matrix and $p(t)$, $q(t)$ are two real valued continuous functions on [0, 1]. The initial conditions are assumed as

$$\underset{\sim}{x}(1) = \underset{\sim}{a} \text{ and } \underset{\sim}{x}'(1) = \underset{\sim}{b} \qquad \ldots (2)$$

where $\underset{\sim}{x}, \underset{\sim}{a}, \underset{\sim}{b}$ are n-vectors.

Assume that $\underset{\sim}{x}(t) = \underset{\sim}{X}(\lambda)\, \omega(t, \lambda)$ be a solution of the equation (1), where λ is scalar, $\underset{\sim}{X}$ is an n-vector independent of t and $\omega(t, \lambda)$ is a non-trivial solution of the scalar differential equation

$$t^2 \frac{d^2 y}{dt^2} + tp(t) \frac{dy}{dt} + q(t) y = t^2 \lambda y \quad \text{i.e.,} \quad Ly = \lambda t^2 y \qquad \ldots (3)$$

Applying the operator L on x, we get

$$L \underset{\sim}{x} = L(\underset{\sim}{X}\, \omega) = \underset{\sim}{X}\, L\omega = \underset{\sim}{X}\, (t^2 \lambda \omega) = \lambda t^2 \underset{\sim}{X}\, \omega$$

Thus the equation (1) becomes

$$\lambda t^2 \underset{\sim}{X}\, \omega = t^2 \underset{\sim}{A}\, (\underset{\sim}{X}\, \omega) = t^2 (\underset{\sim}{A}\, \underset{\sim}{X})\, \omega \quad \text{or,} \quad t^2 (\lambda \underset{\sim}{X} - \underset{\sim}{A}\, \underset{\sim}{X})\, \omega = 0$$

Since $\omega(t, \lambda)$ is non-trivial and $(\lambda \underset{\sim}{X} - \underset{\sim}{A}\, \underset{\sim}{X})$ is independent of t, it follows that $(\underset{\sim}{A}\, \underset{\sim}{X}) = \lambda \underset{\sim}{X}$.

This gives rise to the algebraic eignevalue problem where λ is the eigenvalue of the matrix A and X is the corresponding eigenvector.

Appendix - II

NUMERICAL INVERSION OF THE LAPLACE TRANSFORM

Let the Laplace transform $F(p)$ of $u(t)$ be given by

$$F(p) = \int_0^\infty e^{-pt}\, u(t) dt \qquad p \geq 0 \qquad \ldots (1)$$

We assume that $u(t)$ is sufficiently smooth to permit the approximate method we employ.

putting $\qquad x = e^{-t} \qquad \ldots (2)$

in (1) we get

$$F(p) = \int_0^1 xp^{-1}\, g(x) dx, \qquad \ldots (3)$$

where $\qquad u\,(-\log x) = g(x)$

Applying the Gaussian quadrature formula in (3) we get

$$\sum_{i=1}^{N} W_i x_i^{p-1}\, g(x_i) = F(p), \qquad \ldots (4)$$

where x_i are the roots of the shifted Legendre polynomial $P_N(x) = 0$ and W_i are the corresonding coefficients. Thus x_i and W_i are known.

Eq. (4) can be written as

$$W_1 x_1^{p-1}\, g(x_1) + W_2 x_2^{-1}\, g(x_2) + \ldots. + W_N x_N^{p-1}\, g(x_N) = F(p) \qquad \ldots (5)$$

We now put $p = 1, 2, \ldots, N$ in (5), then the resulting equations become

$$W_1\, g(x_1) + W_2\, g(x_2) + \ldots.. + W_N\, g(x_N) = F(1)$$

$$W_1 x_1 g(x_1) + W_2 x_2 g(x_2) + \ldots\ldots \_ W_N x_N g(x_N) = F(2)$$

....

$$W_1 x_1^{N-1} g(x_1) + W_2 x_2^{N-1} g(x_2) + \ldots\ldots + W_N x_N^{N-1} g(x_N) = (N) \qquad \ldots (6)$$

Thus

$$\begin{bmatrix} g(x_1) \\ g(x_2) \\ \ldots\ldots \\ g(x_N) \end{bmatrix} = \begin{bmatrix} W_1 & W_2 & .. & W_N \\ W_1 x_1 & W_2 x_2 & .. & W_N x_N \\ \ldots & \ldots & \ldots & \ldots \\ W_1 x_1^{N-1} & W_2 x_2^{N-1} & .. & W_n x_N^{N-1} \end{bmatrix}^{-1} \begin{bmatrix} F(1) \\ F(2) \\ \ldots \\ F(N) \end{bmatrix}$$

Hence, $g(x_1), g(x_2) \ldots\ldots g(x_N)$ are known.

Now

$$u(-\log x_1) = g(x_1),\ u(-\log x_2) = g(x_2),\ \ldots\ldots,\ u(-\log x_N) = g(x_N)$$

For $N = 7$

Roots of the shifted Legendre Polynomial x_i	$u(-\log x_i) = g(x_i)$
$x_1 = -0.94910791$	3.671194951
$x_2 = -0.74153119$	2.046127431
$x_3 = -0.40584515$	1.213762484
$x_4 = 0$	0.69314718
$x_5 = 0.40584515$	0.352508528
$x_6 = 0.74153119$	0.138382
$x_7 = 0.94910791$	0.025775394