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HALL EFFECT ON THERMAL INSTABILITY OF RIVLIN-ERICKSEN FLUID

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The thermal convection in Rivlin-Ericksen elastico-viscous fluid is considered in the presence of uniform horizontal magnetic field to include the Hall currents. For the case of stationary convection, the Hall currents hasten the onset of convection, the magnetic field postpones the onset of convection, whereas the kinematic viscoelasticity has no effect on the onset of convection. The Hall currents, kinematic viscoelasticity and the magnetic field introduce oscillatory modes in the system, which were non-existent in their absence. The case of overstability is also considered wherein the sufficient conditions for the non-existence of overstability are obtained.

Key Words : Hall Effect; Thermal Instability; Rivlin-Ericksen Fluid; Viscoelasticity

1. INTRODUCTION

A detailed account of the theoretical and experimental results of the onset of thermal instability (Bénard convection) in an incompressible, viscous (Newtonian) fluid layer, under varying assumptions of hydrodynamics and hydromagnetics, has been given in the celebrated monograph by Chandrasekhar[1]. If an electric field is applied at right angles to the magnetic field, the whole current

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will not flow along the electric field. This tendency of the electric current of flow across an electric field in the presence of a magnetic field is called Hall effect. The Hall effect is likely to be important in many geophysical and astrophysical situations as well as in flows of laboratory plasma. Sherman and Sutton[2] have considered the effect of Hall current on the efficiency of a magneto-fluid- dynamic generator while Sato[3] and Tani[4] have studied the incompressible viscous flow of an ionised gas with tensor conductivity in channels. The effect of Hall current on the thermal instability of electrically conducting fluid in the presence of a uniform vertical magnetic field has been studied by Gupta[5]. Sharma and Sunil[6] have considered the effect of Hall current and finite Larmor radius on the thermal instability of a compressible plasma in porous medium. The fluid is considered to be Newtonian in all the above studies.

There is growing importance of non-Newtonian fluids in geophysical fluid dynamics, chemical technology and petroleum industry. Bhatia and Steiner[7] have studied the problem of thermal instability of Maxwellian viscoelasticfluid in the presence of rotation and have found that the rotation has a destabilizing influence in contrast to the stabilizing effect on an ordinary viscous (Newtonian) fluid. The thermal instability of an Oldroydian viscoelastic fluid acted on by a uniform rotation has been studied by Sharma[8]. An experimental demonstration by Toms and Strawbridge[9] has revealed that a dilute solution of methyl methacrylate in *n*-butyl acetate agrees well with the theoretical model of Oldroyd[10]. There are many elastico-viscous fluids that cannot be characterized by such *b* and constitutive relations. Two such classes of fluids are Rivlin-Ericksen[11] and Walters[12] (model B) fluid. Walters[12] has proposed the constitutive equations for such elastico-viscous fluids. The mixture of polymethyl methacrylate and pyridine at 25 °C containing 30.5 g of polymer per litre behaves very nearly as the Walters (model B′) elastico-viscous fluid and has been reported by Walters[13]. Rivlin and Ericksen[11] have proposed a theoretical model for such another elastico-viscous fluid. Such and other polymers are used in agriculture, communication appliances and in bio-medical applications. Sharma and Kumar[14] have studied the effect of rotation on thermal instability in Rivlin-Ericksen elastico- viscous fluid whereas the thermal convection in electrically conducting Rivlin-Ericksen fluid in presence of magnetic field has been studied by Sharma and Kumar[15].

The Hall current is likely to be important in many geophysical situations and in industry (e.g., MHD generator). The Rivlin- Ericksen elastico-viscous fluid has relevence and importance in geophysical fluid dynamics, chemical technology and industry (e.g., manufacture of various items mentioned above). The present paper, therefore, deals with the effect of Hall current on the thermal instability of a Rivlin-Ericksen elastico-viscous fluid in the presence of a uniform horizontal magnetic field.

2. Formulation of the Problem and Perturbation Equations

Here we consider an infinite, horizontal, incompressible Rivlin-Ericksen elastico-viscous fluid layer of thickness d, heated and soluted from below so that the temperatures, densities at the bottom surface $z = 0$ are T_0, ρ_0 and at the upper surface $z = d$ are T_d, ρ_d, respectively, and that a uniform temperature gradient $\beta (= |dT/dz|)$ is maintained. The gravity field $\mathbf{g}(0, 0, -g)$ and a uniform horizontal magnetic field $\mathbf{H}(H, 0, 0)$ pervade the system.

Let $p, \rho, T, \alpha, g, \eta, \mu_e, N, e$ and $\mathbf{q}(u, v, w)$ denote, respectively, the fluid pressure, density, temperature, thermal coefficient of expansion, gravitational acceleration, resistivity, magnetic permeability, electron number density, charge of an electron and fluid velocity. Then the equations expressing the conservation of momentum, mass, energy and the equation of state (Rivlin and Ericksen[11], Chandrasekhar[1], Sharma and Kumar[15]) are

$$\left[\frac{\partial \boldsymbol{q}}{\partial t}+(\boldsymbol{q}\cdot\nabla)\,\boldsymbol{q}\right]=-\left(\frac{1}{\rho_0}\right)\nabla p+\boldsymbol{g}\left(1+\frac{\delta\rho}{\rho_0}\right)+\left(\nu+\nu'\frac{\partial}{\partial t}\right)\nabla^2\boldsymbol{q}+\frac{\mu_e}{4\pi\rho_0}(\nabla\times\boldsymbol{H})\times\boldsymbol{H}, \quad \ldots (1)$$

$$\nabla\cdot\boldsymbol{q}=0, \quad \ldots (2)$$

$$\frac{\partial T}{\partial t}+(\boldsymbol{q}\cdot\nabla)\,T=\kappa\nabla^2 T, \quad \ldots (3)$$

and
$$\rho=\rho_0\,[1-\alpha(T-T_0)], \quad \ldots (4)$$

where the suffix zero refers to values at the references level $z = 0$ and in writing eq. (1), use has been made of the Boussinesq approximation. The magnetic permeability μ_e, the kinematic viscosity ν, the Kinematic viscoelasticity ν', the thermal diffusivity κ are all assumed to be constants.

The Maxwell's equations yield

$$\frac{d\boldsymbol{H}}{dt}=(\boldsymbol{H}\cdot\nabla)\boldsymbol{q}+\eta\nabla^2\boldsymbol{H}-\frac{c}{4\pi Ne}\nabla\times[(\nabla\times\boldsymbol{H})\times\boldsymbol{H}\,], \quad \ldots (5)$$

and
$$\nabla\cdot\boldsymbol{H}=0, \quad \ldots (6)$$

where $\frac{d}{dt}=\frac{\partial}{\partial t}+\boldsymbol{q}\cdot\nabla$ stands for the convective derivative.

The steady state solution is

$$\boldsymbol{q}=(0,0,0), \qquad T=-\beta z+T_0,$$

$$\rho=\rho_0(1+\alpha\beta z). \quad \ldots (7)$$

Here we use linearized stability theory and normal mode analysis method. Consider a small perturbation on the steady state solution and let δp, $\delta\rho$, θ, $\boldsymbol{h}(h_x, h_y, h_z)$ and $\boldsymbol{q}(u, v, w)$ denote, respectively, the perturbations in pressure p, density ρ, temperature T, magnetic field $\boldsymbol{H}(H, 0, 0)$ and velocity (0, 0, 0). The change in density $\delta\rho$, caused mainly by the perturbation θ in temperature, is given by

$$\delta\rho=-\rho_0\alpha\theta. \quad \ldots (8)$$

Then the linearized perturbation equations become

$$\frac{\partial \boldsymbol{q}}{\partial t}=-\frac{1}{\rho_0}(\nabla\delta p)-\boldsymbol{q}\alpha\theta+\left(\nu+\nu'\frac{\partial}{\partial t}\right)\nabla^2 q+\frac{\mu_e}{4\pi\rho_0}(\nabla\times\boldsymbol{h})\times\boldsymbol{H}, \quad \ldots (9)$$

$$\nabla\cdot\boldsymbol{q}=0, \quad \ldots (10)$$

$$\frac{\partial \theta}{\partial t} = \beta w + \kappa \nabla^2 \theta, \quad \text{... (11)}$$

$$\nabla \cdot \boldsymbol{h} = 0, \quad \text{... (12)}$$

and

$$\frac{\partial \boldsymbol{h}}{\partial t} = (\boldsymbol{H} \cdot \nabla)\boldsymbol{q} + \eta \nabla^2 \boldsymbol{H} - \frac{c}{4\pi Ne} \nabla \times [(\nabla \times \boldsymbol{h}) \times \boldsymbol{H}]. \quad \text{... (13)}$$

3. The Dispersion Relation

Analysing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$$[w, h_z, \theta, \zeta, \xi] = [W(z), K(z)\ \Theta(z), Z(z), X(z)] \exp(ik_x x + ik_y y + nt), \quad \text{... (14)}$$

where k_x, k_y are the wave numbers along the x- and y- directions respectively, $k = \sqrt{(k_x^2 + k_y^2)}$ is the resultant wave number and n is the growth rate which is, in general, a complex constant. $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ and $\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$ stand for the z-components of vorticity and current density, respectively.

Expressing the coordinates x, y, z in the new unit of length d and letting $a = kd$,

$$\sigma = \frac{nd^2}{\nu}, \quad p_1 = \frac{\nu}{\kappa}, \quad p_2 = \frac{\nu}{\eta}, \quad F = \frac{\nu'}{d^2} \quad \text{and} \quad D = \frac{d}{dz},$$

eqs. (9)-(13), using (14), yield

$$[(1 + \sigma F)(D^2 - a^2) - \sigma](D^2 - a^2) W - \frac{g\alpha a^2 d^2}{\nu} \Theta + \frac{ik_x \mu_e H d^2}{4\pi\rho_0 \nu}(D^2 - a^2)K = 0, \quad \text{... (15)}$$

$$[(1 + \sigma F)(D^2 - a^2) - \sigma] Z = -\frac{ik_x \mu_e H d^2}{4\pi\rho_0 \nu} X, \quad \text{... (16)}$$

$$(D^2 - a^2 - p_2\sigma) K = -\left(\frac{ik_x H d^2}{\eta}\right) W + \left(\frac{ik_x c H d^2}{4\pi Ne\ \eta}\right) X, \quad \text{... (17)}$$

$$(D^2 - a^2 - p_2\sigma) X = -\left(\frac{ik_x H d^2}{\eta}\right) Z - \left(\frac{ik_x c H}{4\pi Ne\ \eta}\right)(D^2 - a^2) K, \quad \text{... (18)}$$

and

$$(D^2 - a^2 - p_1\sigma) \Theta = -\left(\beta \frac{d^2}{\kappa}\right) W. \quad \text{... (19)}$$

Eliminating $\Theta, K, X,$ and Z between eqs. (15)-(19), we obtain

$$\{[(1+\sigma F)(D^2-a^2)-\sigma](D^2-a^2-p_2\sigma)^2+Qk_x^2d^2(D^2-a^2-p_2\sigma)$$

$$-Mk_x^2d^2(D^2-a^2)[(1+\sigma F)(D^2-a^2)-\sigma]\}$$

$$\times\ \{[(1+\sigma F)(D^2-a^2)-\sigma](D^2-a^2)(D^2-a^2-p_1\sigma)+Ra^2\}\ W$$

$$+\ Qk_x^2d^2(D^2-a^2)(D^2-a^2-p_1\sigma)$$

$$\{[(1+\sigma F)(D^2-a^2)-\sigma](D^2-a^2-p_2\sigma)+Qk_x^2d^2\}W=0, \quad \ldots (20)$$

where $Q=\left(\frac{\mu_e H^2 d^2}{4\pi\rho_0 \nu\eta}\right)$ is the Chandrasekhar number, $R=\left(\frac{g\alpha\beta d^4}{\nu\kappa}\right)$ is the Rayleigh number and $M=\left(\frac{cH}{4\pi Ne\,\eta}\right)^2$ is the non-dimensional number accounting for Hall currents.

Consider the case where both boundaries are free as well as maintained at constant temperatures while the adjoining medium is perfectly conducting. The case of two free boundaries is a little artificial but it enable us to find analytical solutions and to make some qualitative conclusions. The appropriate boundary conditions, with respect to which eqs. (15)-(19) must be solved are (Chandrasekhar[1]):

$$W=D^2W=0,\quad \Theta=0,\quad DZ=0,\quad \text{at } z=0 \text{ and } 1$$

$$DX=0,\quad K=0 \text{ on the perfectly conducting boundaries.} \quad \ldots (21)$$

The case of two free boundaries, though little artificial, is the most appropriate for steller atmospheres (Spiegel[16]). Using the above boundary conditions, it can be shown that all the even order derivatives of W must vanish for $z=0$ and 1 and hence the proper solution of (20) characterizing the lowest mode is

$$W=W_0\sin \pi z, \quad \ldots (22)$$

where W_0 is a constant. Substituting (22) in (20) and letting $a^2=\pi^2 x$, $R_1=\frac{R}{\pi^4}$, $Q_1=\frac{Q}{\pi^2}$, $i\sigma_1=\frac{\sigma}{\pi^2}$, $k_x=k\cos\theta$, we obtain the dispersion relation

$$R_1=\left(\frac{1+x}{x}\right)[(1+i\sigma_1\pi^2F)(1+x)+i\sigma_1](1+x+ip_1\sigma_1)$$

$$+Q_1\cos^2\theta(1+x)(1+x+ip_1\sigma_1)\times\{[(1+i\sigma_1\pi^2F)(1+x)+i\sigma_1](1+x+ip_2\sigma)$$

$$+Q_1x\cos^2\theta\}\ [(1+x+ip_2\sigma_1)^2\ \{(1+i\sigma_1\pi^2F)(1+x)+i\sigma_1\}$$

$$+ \; Q_1 x \cos^2\theta\,(1+x+ip_2\sigma_1) + Mx\cos^2\theta\,(1+x)\;\{(1+i\sigma_1\pi^2 F)(1+x)+i\sigma_1\}]^{-1}. \qquad \text{... (23)}$$

Eq. (23) is the required dispersion relation including the effects of magnetic field, kinematic viscoelasticity and Hall currents on the thermal instability of Rivlin-Ericksen fluid. In the absence of Hall current ($M = 0$), the dispersion relation (23) reduces to that of Sharma and Kumar[15] whereas in the absence of viscoelasticity ($V = 0$), the dipersion relation (23) reduces to that of Gupta[5]. Here the magnetic field is considered to be horizontal and so there is corresponding change in magnetic field and Hall current terms.

4. The Stationary Convection

When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma = 0$. Putting $\sigma = 0$, the dispersion relation (23) reduces to

$$R_1 = (1+x)\left[\frac{(1+x)^2}{x} + \frac{Q_1\cos^2\theta\,[(1+x)^2 + Q_1 x\cos^2\theta]}{(1+x)^2 + Q_1 x\cos^2\theta + Mx\cos^2\theta\,(1+x)}\right], \qquad \text{... (24)}$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x and the parameters Q_1 and M. For stationary convection, the parameter F accounting for the kinematic viscoelasticity effect vanishes and the dispersion relation (24), as expected, resembles to that of viscous, Newtonian fluid (Gupta[5]).

To investigate the effects of magnetic field and Hall currents, we examine the behaviour of $\frac{dR_1}{dQ_1}$ and $\frac{dR_1}{dM}$ analytically. Eq. (24) yields

$$\frac{dR_1}{dQ_1} = \frac{(1+x)\cos^2\theta\left\{\begin{array}{l} Mx(1+x)\cos^2\theta\,[(1+x)^2 + 2Q_1 x\cos^2\theta] \\ +\,Q_1 x\cos^2\theta\,[2(1+x)^2 + Q_1 x\cos^2\theta] + (1+x)^4 \end{array}\right\}}{[(1+x)^2 + Q_1 x\cos^2\theta + Mx(1+x)\cos^2\theta]^2}$$

and

$$\frac{dR_1}{dM} = -\frac{Q_1 x(1+x)^2\cos^4\theta\,[(1+x)^2 + Q_1 x\cos^2\theta]}{[(1+x)^2 + Q_1 x\cos^2\theta + Mx(1+x)\cos^2\theta]^2}. \qquad \text{... (26)}$$

Thus for stationary convection, magnetic field has a stabilizing effect whereas the Hall current has a destabilizing effect on thermal instability of Rivlin-Ericksen fluid. These results are in agreement with those of Gupta[5]. The kinematic viscoelasticity has no effect for stationary convection.

The dispersion relation (24) is analysed numerically. In Fig. 1, R_1 is plotted against x for $M = 10$, $\theta = 45^\circ$ and $Q_1 = 10$, 20 and 30. The stabilizing role of the magnetic field is clear from the increase of the Rayleigh number with increasing magnetic field parameter value Q_1. Fig. 2 gives R_1 plotted against x for $Q_1 = 20$, $\theta = 45^\circ$ and $M = 10$, 20 and 30. Here we find the destabilizing role of the Hall currents as the Rayleigh number decreases with the increase in Hall currents parameter M.

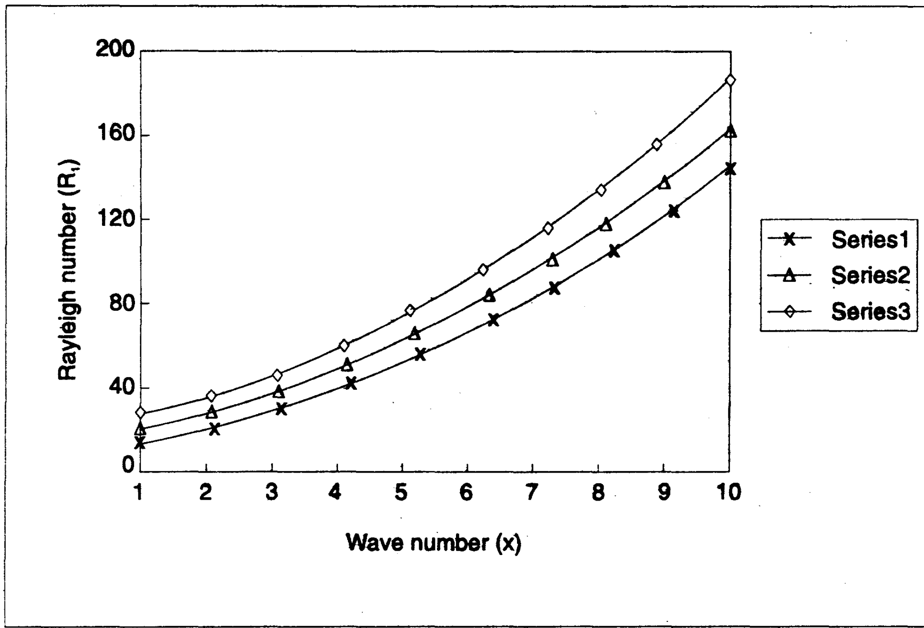

FIG. 1. The variation of Rayleigh number (R_1) with wavenumber (x) for $M = 10$, $\theta = 45^o$; $Q_1 = 10$, for Series 1, $Q_1 = 20$ for Series 2 and $Q_1 = 30$ for Series 3.

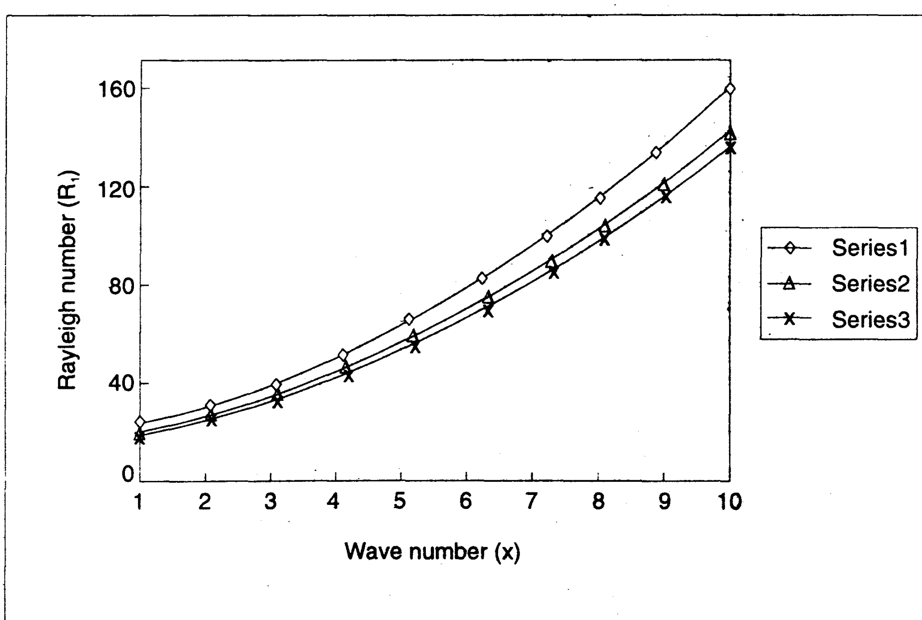

FIG. 2. The variation of Rayleigh number (R_1) with wavenumber (x) for $Q_1 = 20$, $\theta = 45^o$; $M = 10$, for Series 1, $M = 20$ for Series 2 and $M = 30$ for Series 3.

5. Stability of the System and Oscillatory Modes

Here we examine the possibility of oscillatory modes, if any, on stability problem due to the presence of kinematic viscoelasticity, Hall current and magnetic field. Multiplying (15) by W^*, the complex conjugate of W, and using (16)-(19) together with the boundary conditions (21), we obtain

$$(1+\sigma F)I_1+\sigma I_2+\left(\frac{\mu_e\eta}{4\pi\rho_0\nu}\right)[I_5+p_2\sigma^* I_6]+\left(\frac{\mu_e\eta d^2}{4\pi\rho_0\nu}\right)[I_7+p_2\sigma I_8]$$

$$+\ d^2[(1+\sigma^* F)I_9+\sigma^* I_{10}]-\left(\frac{g\alpha\kappa a^2}{\nu\beta}\right)[I_3+p_1\sigma^* I_4]=0, \qquad \dots (27)$$

where

$$\left.\begin{aligned}
I_1 &= \int_0^1 (|D^2W|^2+2a^2|DW|^2+a^4|W|^2)\,dz, & I_2 &= \int_0^1 (|DW|^2+a^2|W|^2)\,dz,\\
I_3 &= \int_0^1 (|D\Theta|^2+a^2|\Theta|^2)\,dz, & I_4 &= \int_0^1 (|\Theta|^2)\,dz,\\
I_5 &= \int_0^1 (|D^2K|^2+2a^2|DK|^2+a^4|K|^2)\,dz, & I_6 &= \int_0^1 (|DK|^2+a^2|K|^2)\,dz,\\
I_7 &= \int_0^1 (|DX|^2+a^2|X|^2)\,dz, & I_8 &= \int_0^1 (|X|^2)\,dz,\\
I_9 &= \int_0^1 (|DZ|^2+a^2|Z|^2)\,dz, & I_{10} &= \int_0^1 (|Z|^2)\,dz.
\end{aligned}\right\} \qquad \dots (28)$$

The integrals $I_1, \dots, \dots, I_{10}$ are all positive definite. Putting $\sigma=\sigma_r+i\sigma_i$ and equating the real and imaginary parts of eq. (27), we obtain

$$\sigma_r\left[(I_2+FI_1)+\left(\frac{\mu_e\eta}{4\pi\rho_0\nu}\right)p_2(I_6+d^2I_8)+d^2(I_{10}+FI_9)-\left(\frac{g\alpha\kappa a^2}{\nu\beta}\right)p_1I_4\right]$$

$$=-\left[I_1+\left(\frac{\mu_e\eta}{4\pi\rho_0\nu}\right)(I_5+d^2I_7)+d^2I_9-\left(\frac{g\alpha\kappa a^2}{\nu\beta}\right)I_3\right], \qquad \dots (29)$$

$$\sigma_i\left[(I_2+FI_1)-\left(\frac{\mu_e\eta}{4\pi\rho_0\nu}\right)p_2(I_6-d^2I_8)-d^2(I_{10}+FI_9)+\left(\frac{g\alpha\kappa a^2}{\nu\beta}\right)p_1I_4\right]=0. \qquad \dots (30)$$

It follows from eq. (29) that σ_r may be positive or negative which means that the system may be stable or unstable. It is clear from (30) that σ_i may be zero or non- zero, meaning that the modes may be non-oscillatory or oscillatory. The osciallatory modes are introduced due to the presence of kinematic viscoelasticity, Hall currents and magnetic field which were non-existent in their absence.

6. The Case of Overstability

Here we discuss the possibility of whether instability may occur as overstability. Since we wish to determine the Rayleigh number for the onset of instability *via* a state of pure oscillations, it suffices to find conditions for which (23) will admit of solutions with σ_1 real.

If we equate real and imaginary parts of (23) and eliminate R_1 between them, we obtain

$$A_3c_1^3 + A_2c_1^2 + A_1c_1 + A_0 = 0, \qquad \text{... (31)}$$

where we have put $c_1 = \sigma_1^2$, $b = 1 + x$ and

$$A_3 = b^2(b-1)\,p_2^4\,(1+\pi^2 bF)^2\,[p_1 + (1+\pi^2\,bF)] \qquad \text{... (32)}$$

and

$$A_0 = b^5\,(b-1)\,[p_1 + (1+\pi^2 bF)]\,[2b\,(b-1)\cos^2\theta\,(Mb + Q_1)$$
$$+ M(b-1)^2\cos^4\theta\,(Mb + 2Q_1) + b^3]$$
$$+\ b^2(b-1)^3\,Q_1^2\,(p_1 - p_2)\cos^4\theta\,[b(2b+1) + Q_1\,(b-1)\cos^2\theta]$$
$$+\ Q_1 b^5\,(b-1)^2\,(p_1 - 2p_2)\cos^2\theta + b^3(b-1)^4\,MQ_1^2\,[p_1 - (1+\pi^2 bF)]\cos^6\theta$$
$$+\ b^3(b-1)^3\,Q_1\cos^4\theta\,[Q_1^2\,(1+\pi^2 bF) + Mp_1 b]. \qquad \text{... (33)}$$

Since σ_1 is real for overstability, the three values of $c_1\ (= \sigma_1^2)$ must be positive. The product of the roots of (31) is $-\dfrac{A_0}{A_3}$ and this has to be positive.

It is clear from eq. (32) and (33) that A_0 and A_3 are always positive if

$$p_1 > p_2 \text{ and } p_1 > (1+\pi^2 bF),$$

which implies that $\kappa < \eta$ and $\kappa < \dfrac{\nu d^2}{d^2 + \pi^2\nu'\,(1+x)}$, ... (35)

i.e.
$$\kappa < \min\left(\eta, \frac{\nu d^2}{d^2 + \pi^2 \nu' (1+x)} \right). \qquad \text{... (36)}$$

$\kappa < \min\left(\eta, \frac{\nu d^2}{d^2 + \pi^2 \nu' (1+x)} \right)$ are, therefore, the sufficient conditions for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability.

The sufficient condition for the non-existence of overstability for thermal instability in Rivlin-Ericksen elastico-viscous fluid in presence of magnetic field is $\kappa < \eta$ (Sharma and Kumar[15]), whereas that in presence of rotation is $\kappa < \frac{\nu d^2}{d^2 + \pi^2 \nu' (1+x)}$ (Sharma and Kumar[14]). Since the presence of Hall currents induces a vertical component of vorticity (similar to that of rotational), the later condition $\kappa < \frac{\nu d^2}{d^2 + \pi^2 \nu' (1+x)}$ is found as the additional for the non-existence of overstability for thermal instability in Rivlin-Ericksen elastico-viscous fluid in presence of magnetic field to include Hall currents. Moreover, in the absence of viscoelasticity ($\nu' = 0$), sufficient condition for non-existence of overstability in viscous, Newtonian fluid (Gupta[5]), as expected, reduces to $\kappa < \min \{\eta, \nu\}$.

7. Discussion

The inclusion of Hall currents gives rise to a cross flow i.e., a flow at right angles to the primary flow in a channel in the presence of a transverse magnetic field, has been shown by Sato[3] and Tani[4] has found that Hall effect produces a cross-flow of double-swirl pattern in incompressible flow through a straight channel with arbitrary cross-section. This breakdown of the primary flow and formation of a secondary flow may be attributed to the inherent instability of the primary flow in the presence of Hall current. Sato[3] has pointed out that even if the distribution of the primary flow velocity be stable to external disturbances, the whole layer may become turbulent if the distribution of the cross-flow velocity is unstable. A similar situation occurs on the three dimensional boundary layer along a swept-back wing. Gupta[5] has found that the presence of Hall current induces a vertical component of vorticity and this may well the reason for the destabilizing influence.

The Hall current, therefore, has a destabilizing influence on the thermal instability of Rivlin-Ericksen elastico-viscous fluid, for the stationary convection. The Hall current, kinematic viscoelasticity and magnetic field introduce oscillatory modes in the system which were non-existent in their absence. The sufficient condition for the non-existence of overstability for thermal instability in Rivlin-Ericksen elastico-viscous fluid in presence of magnetic field is $\kappa < \eta$ (Sharma and Kumar[15]). Since Hall current induces a vertical component of vorticity (similar to that of rotation and the sufficient condition for the non-existence of overstability for thermal instability in Rivlin-Ericksen elastico-viscous fluid in presence of rotation is $\kappa < \frac{\nu d^2}{d^2 + \pi^2 \nu' (1+x)}$, the additional sufficient condition for non-existence of overstability for the thermal instability in Rivlin-Ericksen elastico-viscous fluid in presence of magnetic field to include Hall currents is $\kappa < \frac{\nu d^2}{d^2 + \pi^2 \nu' (1+x)}$. Hence, the sufficient conditions are $\kappa < \min\left(\eta, \frac{\nu d^2}{d^2 + \pi^2 \nu' (1+x)}\right)$. In the absence of viscoelasticity ($\nu' = 0$), the sufficient conditions reduce to $\kappa < \min\ \{\eta, \nu\}$, the same as those of Gupta[5].

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