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REFLECTION OF PLANE WAVES AT A PLANAR VISCOELASTIC MICROPOLAR INTERFACE

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The problem of the reflection and refraction of plane waves (P- and SV-waves) at an interface between viscoelastic solid half-space and micropolar elastic solid half-space with stretch is considered. The amplitude ratios for the different reflected and refracted waves have been calculated. Numerical values of amplitude ratios have been computed and plotted against the angle of emergence (θ_0^*) as well as with the angle between the propagation vector and attenuation vector (γ_0).

Key Words : Reflection and Refraction; Micropolar Elastic Solid; Axial Stretch; Viscoelastic Solid; Amplitude Ratios

INTRODUCTION

The observed attenuation of the seismic waves in the earth, an important source of information regarding the composition and state of the deep interior cannot be explained by assuming the earth to be an elastic solid. Keeping this fact in mind several problems on reflection and refraction in a linear viscoelastic solid have been discussed by many research workers[4-15].

The discrepancy between the results of classical theory of elasticity and the experiments appears in all cases when the microstructure of the body is significant, i.e., in the neighbourhood of cracks and notches where the stress gradients are considerable. The discrepancies also appear in granular media and multimolecular bodies such as polymers. The influence of the microstructure is particularly evident in the case of elastic vibrations of high frequency and small wavelength.

Eringen and Suhubi[1] introduced theory of micropolar elastic solids in which the micromotions of the particles contained in a macrovolume element with respect to its centroid are considered. Materials which are affected by such micromotions and macrodeformations are known as micromorphic materials. Eringen[2] developed a theory for a subclass of micromorphic materials which are called micropolar media and these materials show microrotation's effect and microrotational inertia. Several researchers[17-22] have discussed some problems on reflection and refraction in micropolar media.

A special micropolar material was fabricated in which uniformly distributed "rigid" aluminium

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shot was cast in an elastic epoxy matrix. Gauthier[16] found this aluminium-epoxy composite to be micropolar material and investigated the values of the relevant parameters based on specimen of aluminium-epoxy composite.

Eringen[3] extended his work to include the effect of axial stretch during the rotation of molecules and developed the theory of micropolar elastic solid with stretch. Composite materials reinforced with chopped elastic fibres, porous media whose pores are filled with gas or inviscid liquid, asphalt, or other elastic inclusions and 'solid-liquid' crystals, etc. should be characterizable by microstretch solids.

To relate macroscopic responses of matter to effects taking place on the microscale involves consideration of microstructural entities ranging from atoms through crystal lattice defects and on to cracks and gross inhomogeneities. In some cases, microstructural features play a passive role such as that of scattering of waves. In other cases, these features play a more active role, with their evolution giving rise to flow and fracture, change in chemical composition of matter, or other inelastic effects.

In the present problem, we consider the reflection and refraction of plane waves (P- and SV-waves) which are obliquely incident at an interface between the linear viscoelastic solid half-space and micropolar elastic solid half-space with stretch. As such a model may be found in the earth's crust, so the results of our problem can be applicable to the earth's crust, to a water-mud-rock boundary, or to some other specific problem in engineering or seismology like bedrock-soil interface or mantle-crust interface.

Basic Equations and Their Solutions

Following Borcherdt[11], the equation governing the small motions in a linear viscoelastic solid may be written as

$$(K^* + 4M^*/3)\ \nabla(\nabla \cdot \boldsymbol{u}^* - M^*\ \nabla \times (\nabla \times \boldsymbol{u}^*) = \rho_l\, \ddot{\boldsymbol{u}}^*, \qquad \dots (1)$$

where symbols K^* is the complex bulk modulus, M^* is the complex shear modulus, ρ_l is the density of linear viscoelastic solid and $\mathbf{u}^*$ is the displacement vector. Superposed dots on right hand side of eq. (1) stand for second partial derivative with respect to time.

The stresses in the viscoelastic solid are given by

$$\sigma^*_{kl} = (K^* - 2M^*/3)\ \theta\ \delta_{kl} + 2M^*\ e_{kl}, \qquad \dots (2)$$

where

$$e_{kl} = \frac{1}{2}\left(\frac{\partial u^*_k}{\partial x_l} + \frac{\partial u^*_l}{\partial x_k}\right), \quad \theta = \nabla \cdot \boldsymbol{u}^*, \qquad \dots (3)$$

Using Helmholtz's theorem

$$\boldsymbol{u}^* = \nabla\phi^* + \nabla x\, \boldsymbol{\psi}^*,\ \ \nabla \cdot \boldsymbol{\psi}^* = 0, \qquad \dots (4)$$

we can show that ϕ^* and ψ^* satisfy

$$\alpha^2 \nabla^2 \phi^* = \ddot{\phi}^* \text{ and } \beta^2 \nabla^2 \psi^* = \ddot{\psi}^*, \qquad \ldots (5)$$

where $$\alpha^2 = (K^* + 4M^*/3)/\rho_1,\ \beta^2 = M^*/\rho_1,\ \psi^* = -(\boldsymbol{\psi}^*)_y. \qquad \ldots (6)$$

Following Eringen[23], the constitutive and field equations in a micropolar elastic solid with stretch without body forces and body couples can be written as

$$\sigma_{kl} = \lambda\, u_{r,r}\, \delta_{kl} + \mu(u_{k,l} + u_{l,k}) + \kappa\,(u_{1,k} - \varepsilon_{klr}\, \phi_r), \qquad \ldots (7)$$

$$m_{kl} = \beta_0\, \varepsilon_{rkl}\, \Phi^*_{,r} + \alpha\, \phi_{r,r}\, \delta_{kl} + \beta\, \phi_{k,1} + \gamma\, \phi_{l,k}, \qquad \ldots (8)$$

$$\beta_k = \alpha_0\, \Phi^*_{,k} + (\beta_0/3)\, \varepsilon_{rkl}\, \phi_{r,l}, \qquad \ldots (9)$$

$$(c_1^2 + c_3^2)\ \nabla(\nabla \cdot \boldsymbol{u}) - (c_2^2 + c_3^2)\ \nabla \times (\nabla \times \boldsymbol{u}) + c_3^2\ \nabla \times \boldsymbol{\phi} = \ddot{\boldsymbol{u}}, \qquad \ldots (10)$$

$$(c_4^2 + c_5^2)\ \nabla(\nabla \cdot \boldsymbol{\phi}) - c_4^2\ \nabla \times (\nabla \times \boldsymbol{\phi}) + \omega_0^2\ \nabla \times \boldsymbol{u} - 2\, \omega_0^2\, \boldsymbol{\phi} = \ddot{\boldsymbol{\phi}}, \qquad \ldots (11)$$

and $$c_6^2\ \nabla^2\, \Phi^* - r_1\, \Phi^* = \ddot{\phi}^*, \qquad \ldots (12)$$

where

$$c_1^2 = (\lambda + 2\mu)/\rho,\ c_2^2 = \mu/\rho,\ c_3^2 = \kappa/\rho,$$

$$c_4^2 = \gamma/\rho j,\ c_5^2 = (\alpha + \beta)/\rho j,\ \omega_0^2 = c_3^2/j = \kappa/\rho j,$$

$$c_6^2 = 2\alpha_0/\rho j,\ r_1 = 2\eta_0/\rho j, \qquad \ldots (13)$$

where symbols $\lambda, \mu, \kappa, \alpha, \beta, \gamma, \alpha_0, \beta_0, \eta_0, \rho, j$ have their usual meanings. $\boldsymbol{u}, \boldsymbol{\phi}$ and Φ^* are displacement vector, microrotation vector and scalar microstretch respectively. δ_{kl} is the kronecker delta.

Parfitt and Eringen[22] have shown that there exists four basic waves propagating in an infinite micropolar solid, namely, a longitudinal displacement wave travelling with speed V_1, a longitudinal microrotational wave travelling with speed V_2 and two sets of two coupled transverse displacement and microrotational waves with velocities V_3 and V_4.

In order to solve eq. (12), we assume that wave propagates in a positive direction of unit vector $\boldsymbol{n}$ as

$$\Phi^* = D \exp\,[ik\,(\boldsymbol{n} \cdot \boldsymbol{r} - V(t))], \qquad \ldots (14)$$

where D is the amplitude of the wave, V is the phase velocity $\boldsymbol{r}$ is the position vector, k is the wave number and $\omega = kV$.

Substituting eq. (14) in eq. (12), we get

$$\{(kV/c_6)^2 - k^2 - r_2\}\ D \exp\ [ik\ (\boldsymbol{n} \cdot \boldsymbol{r} - Vt] = 0, \qquad \text{... (15)}$$

where $\qquad r_2 = \eta_0/\alpha_0.$

From eq. (15), we have

$$V^2 = c_6^2/\{1 - (r_2\, c_6^2/\omega^2)\}, \qquad \text{... (16)}$$

Eq. (16) shows the speed of the wave given by eq. (12) and we call this wave as longitudinal microstretch wave.

V will be real and finite only if

$$1 - (r_2\, c_6^2/\omega^2) > 0,$$

i.e. $\qquad$ if $\omega > 2^{1/2}\ \omega_0, \qquad \text{... (17)}$

where $\omega_0^2 = \eta_0/\rho j.$

This is the condition for the existence of the longitudinal microstretch wave.

Thus, in an unbounded micropolar elastic solid with stretch there exists five basic waves travelling with distinct speeds.

Since we are discussing a two dimensional problem in x-z plane, we have

$$\boldsymbol{u} = (u,\ 0,\ w),\ \boldsymbol{\phi} = (\mathbf{0},\ \phi_2,\ \mathbf{0}), \qquad \text{... (18)}$$

$$\boldsymbol{u}^* = (u^*, 0, w^*), \qquad \text{... (19)}$$

where

$$u = \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial z},\ w = \frac{\partial \phi}{\partial z} - \frac{\partial \psi}{\partial x},$$

$$u^* = \frac{\partial \phi^*}{\partial x} + \frac{\partial \psi^*}{\partial z},\ w = \frac{\partial \phi^*}{\partial z} - \frac{\partial \psi^*}{\partial x}, \qquad \text{... (20)}$$

where ϕ, ψ; ϕ^* and ψ^* are potentials satisfying certain wave equations in micropolar media and linear viscoelastic media respectively.

Boundary Conditions

For two-dimensional motion in xz-plane, the appropriate boundary conditions at the interface $z = 0$ are the continuity of stress and displacement components, vanishing of the shear couple stress and vector first moment, i.e.,

$$\sigma_{zz}^* = \sigma_{zz},\ \sigma_{zx}^* = \sigma_{zx},\ u^* = u,\ w^* = w,\ 0 = m_{zy},\ 0 = \beta_z,\ \text{at } z = 0. \qquad \text{... (21)}$$

REFLECTION AND REFRACTION

We consider a linear viscoelastic solid and a micropolar elastic solid with stretch as half-spaces in welded contact along a plane interface $z = 0$. We introduce rectangular Cartesian co-ordinates (x, y, z) and place the origin at the interface separating the two half-spaces as shown in Fig. 1. The micropolar elastic solid then occupies the region $z < 0$ and the region $z > 0$ is occupied by linear viscoelastic solid.

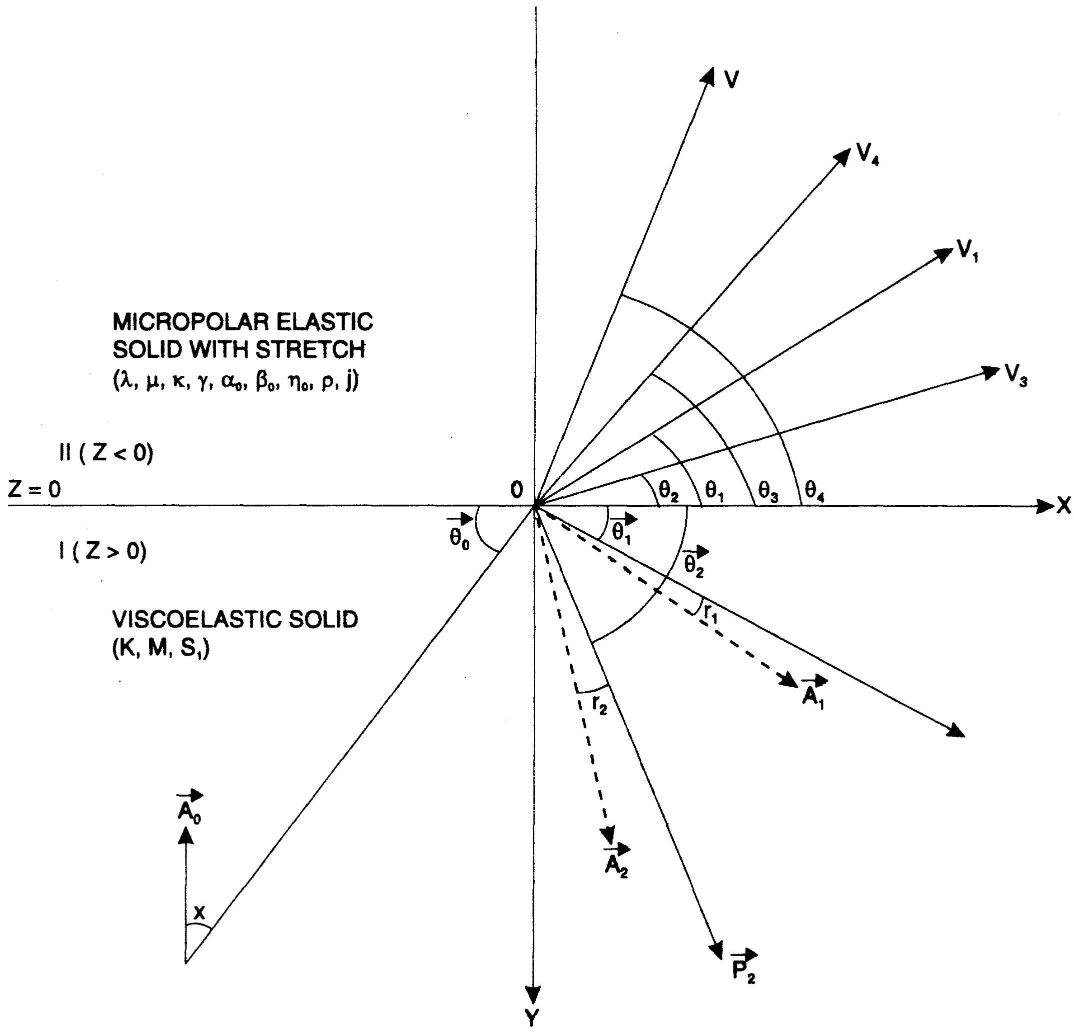

FIG. 1. Geometry of the problem

We now consider a plane harmonic seismic body wave (P or SV) with time dependence proportional to exp $(i\omega t)$, propagating through the viscoelastic medium $(z > 0)$ and incident at the interface, $z = 0$, with the direction of propagation making an angle θ_0 with the interface. Corresponding to this incident wave, we get waves in the viscoelastic solid as reflected P- and reflected SV-waves and four waves (longitudinal displacement or LD-wave, two sets of two coupled waves or CD I, II waves and a longitudinal microstretch wave) transmitted to the micropolar elastic solid half-space $(z < 0)$ as shown in Fig. 1.

For linear viscoelastic solid ($z > 0$), the potential functions ϕ^* and ψ^* for incident and reflected waves are as follows :

$$\phi^* = B_0^* \exp(A_0 \cdot r) \exp[i(\omega t + \boldsymbol{P_0} \cdot \boldsymbol{r})]$$

$$+ B_1^* \exp(\boldsymbol{A_1} \cdot \boldsymbol{r}) \exp[i(\omega t + \boldsymbol{P_1} \cdot \boldsymbol{r})] \quad \text{... (22)}$$

and

$$\psi^* = B_0^* \exp(\boldsymbol{A_0} \cdot r) \exp[i(\omega t + \boldsymbol{P_0} \cdot \boldsymbol{r})]$$

$$+ B_2^* \exp(\boldsymbol{A_2} \cdot \boldsymbol{r}) \exp[i(\omega t + \boldsymbol{P_2} \cdot \boldsymbol{r})], \quad \text{... (23)}$$

where

(a) for an incident P-wave

$$\left.\begin{aligned} \boldsymbol{P_0} &= k_{Re}\, x - d\alpha_{Re}\, z,\ \boldsymbol{A_0} = -k_{Im}\, x + d\alpha_{Im\, z}, \\ \boldsymbol{P_1} &= k_{Re}\, x + d\alpha_{Re}\, z,\ \boldsymbol{A_1} = -k_{Im}\, x - d\alpha_{Im\, z}, \\ \boldsymbol{P_2} &= k_{Re}\, x + d\beta_{Re}\, Z,\ \boldsymbol{A_2} = -k_{Im}\, x - d\beta_{Im\, z}, \end{aligned}\right] \quad \text{... (24)}$$

(b) for an incident SV-wave

$$\left.\begin{aligned} \boldsymbol{P_0} &= k_{Re}\, x - d\beta_{Re}\, z,\ \boldsymbol{A_0} = -k_{Im}\, x + d\beta_{I\mu\, z}, \\ \boldsymbol{P_1} &= k_{Re}\, x + d\alpha_{Re}\, z,\ \boldsymbol{A_1} = -k_{\mathrm{Im}}\, x - d\alpha_{Im\, z}, \\ P_2 &= k_{Re}\, x + d\beta_{Re}\, z,\ \boldsymbol{A_2} = -k_{Im}\, x - d\beta_{Im\, z}, \end{aligned}\right] \quad \text{... (25)}$$

where

$$d\alpha = p \cdot v \cdot [((\omega/\alpha)^2 - k')^{1/2}] = d\alpha_{Re} + d\alpha_{Im}$$

and

$$d\beta = p \cdot v \cdot [((\omega/\beta)^2 - k^2)^{1/2}] = d\beta_{Re} + d\beta_{Im}, \quad \text{... (26)}$$

and $k = k'_{Re} + i\, k'_{\mathrm{Im}}$ is a complex wave number ($P.\ V.$ stands for the principal value of the complex number) and $k'_{Re} \geq 0$ to ensure the propagation in the positive direction.

For micropolar elastic solid, we have the potentials functions as follows :

$$\phi = B_1 \exp[ik_0 (x \cos\theta_I - z \sin\theta_I) + i\omega_I t], \quad \text{... (27)}$$

$$\psi = B_2 \exp[i\delta_I (x \cos\theta_2 - z \sin\theta_2) + i\omega_2 t]$$

$$+ B_3 \exp[i\delta_2 (x \cos\theta_3 - z \sin\theta_3) + i\omega_3 t], \quad \text{... (28)}$$

$$\phi_2 = EB_2 \exp [i\delta_1 (x \cos \theta_2 - z \sin \theta_2) + i\omega_2 t]$$

$$+ \; FB_3 \exp [i\delta_2 (x \cos \theta_3 - z \sin \theta_3) + i\omega_3 t], \qquad \text{... (29)}$$

$$\Phi^* = GB_4 \exp [ik_4 (x \cos \theta_4 - z \sin \theta_4) + i\omega_4 t], \qquad \text{... (30)}$$

where

$$E = \delta_1^2 \left(\delta_1^2 - \frac{\omega^2}{c_2^2 + c_3^2} - pq \right) \Bigg/ p \left(2q - \frac{\omega^2}{c_4^2} \right)$$

and

$$F = \delta_2^2 \left(\delta_2^2 - \frac{\omega^2}{c_2^2 + c_3^2} - pq \right) \Bigg/ p \left(2q - \frac{\omega^2}{c_4^2} \right), \qquad \text{... (31)}$$

where

$$p = \mu/(\mu + \kappa), \;\; q = \kappa/\gamma,$$

$$\delta_1^2 = \lambda_1 \omega^2, \;\; \delta_2^2 = \lambda_2^2 \omega^2,$$

$$\lambda_{1,2}^2 = [B - (B^2 - 4AC)^{1/2}]/2,$$

$$B = \frac{q(p-2)}{\omega^2} + \frac{1}{(c_2^2 + c_3^2)} + \frac{1}{c_4^2}$$

and

$$C = \frac{1}{(c_2^2 + c_3^2)} \left(\frac{1}{c_4^2} - \frac{2q}{\omega^2} \right) \qquad \text{... (32)}$$

and G is the constant of dimension L^{-2}.

When a general type of inhomogeneous plane wave travelling in a linear viscoelstic medium is incident at the interface, following Borcherdt[13], we have

$$k' = |\boldsymbol{P_0}| \cos \theta_0^* - i \, |\boldsymbol{A_0}| \cos (\theta_0^* + \gamma_0), \qquad \text{... (33)}$$

In order to satisfy the boundary conditions for all values of x at $z = 0$, it is necessary to assume $k' = k$ (real) and hence from eq. (33) $\theta_0^* = 90^\circ - \gamma_0$ ($|A_0| \neq 0$); that is, the incident wave field is that in which the general plane wave is attenuating perpendicular to the interface. Following Borcherdt[13], the extension of Snell's law will be

$$k = k_{Re} = |\boldsymbol{P_i}| \cos \theta_i^* = V_1^{-1} \cos \theta_1 = V_3^{-1} \cos \theta_2 = V_4^{-1} \cos \theta_3 = V^{-1} \cos \theta_4, \; (i = 0, 1, 2) \qquad \text{... (34)}$$

and

$$- k_{\text{Im}} = |\boldsymbol{A_0}| \cos (\theta_0^* + \gamma_0), \qquad \text{... (35)}$$

and

$$\omega_1 = \omega_2 = \omega_3 = \omega_4 = \omega \text{ (say) at } z = 0. \quad \text{... (36)}$$

Making use of the potentials given by equations (22), (23), (27) to (30) in the boundary conditions (21), after using the equations (2), (3), (7) to (9), (18), to (20), (24), (25) and (31), to (36), we obtain a system to six nonhomogeneous equations as

$$\sum_{j=1}^{6} a_{ij} Z_j = b_i \; (i = 1, 2, ..., 6) \quad \text{... (37)}$$

where

$$Z_1 = B_1^*/B_0^*, \; Z_2 = B_2^*/B_0^*, \; Z_3 = B_1/B_0^*$$

$$Z_4 = B_2/B_0^*, \; Z_5 = B_3/B_0^*, \; Z_6 = B_4/B_0^*. \quad \text{... (38)}$$

are the amplitude ratios for reflected P - wave, reflected SV-wave, refracted longitudinal displacement wave, two refracted sets of two coupled waves (CD I and CD II) and refracted longitudinal microstretch wave (LMS-wave) respectively,

$$a_{11} = -2M^* + (K^* + 4M^*/3)(1 + d\alpha^2/k^2), a_{12} = -2M^* (d\beta/k),$$

$$a_{13} = -[\lambda + (2\mu + \kappa)\sin^2\theta_1](k_0/k)^2, \; a_{14} = -[(2\mu + \kappa)\cos\theta_2 \sin\theta_2](\delta_1/k)^2,$$

$$a_{15} = -[(2\mu + \kappa)\cos\theta_3 \sin\theta_3](\delta_2/k)^2, a_{16} = 0,$$

$$a_{21} = 2M^* (d\alpha/k), a_{22} = M^* [(d\beta^2/k^2) - 1],$$

$$a_{23} = (2\mu + \kappa)\cos\theta_1 \sin\theta_1 (k_0/k)^2,$$

$$a_{24} = (\mu\cos 2\theta_2 - \kappa\sin^2\theta_2)(\delta_1/k)^2 - \kappa E/k^2,$$

$$a_{25} = (\mu\cos 2\theta_3 - \kappa\sin^2\theta_3)(\delta_2/k)^2 - kF/k^2, a_{26} = 0,$$

$$a_{31} = -1, \; a_{32} = -(d\beta/k), \; a_{33} = (k_0/k)\cos\theta_1,$$

$$a_{34} = -(\delta_1/k)\cos\theta_2, \; a_{35} = -(\delta_2/k)\cos\theta_3, \; a_{36} = 0,$$

$$a_{41} = -(d\alpha/k), \; a_{42} = -1, \; a_{43} = (k_0/k)\sin\theta_1,$$

$$a_{44} = -(\delta_1/k)\cos\theta_2, \; a_{45} = -(\delta_2/k)\cos\theta_3, \; a_{46} = 0,$$

$$a_{51} = a_{52} = a_{53} = 0, \; a_{54} = (\delta_1/k)\sin\theta_2,$$

$$a_{55} = (F/E)\,(\delta_2/k)\sin\theta_3,\; a_{56} = (\beta_0\,G/\gamma E)\,(k_4/k)\cos\theta_4,$$

$$a_{61} = a_{62} = a_{63} = 0,\; a_{64} = -(\beta_0/3\alpha_0)\,(\delta_1/k)\cos\theta_2,$$

$$a_{65} = -(\beta_0\,F/3\alpha_0\,E)\,(\delta_2/k)\cos\theta_3,\; a_{66} = (k_4G/kE)\sin\theta_4, \quad \ldots (39)$$

and b_i (i = 1, 2, ..., 6) are given as follows :

(a) For an incident longitudinal displacement wave

$$b_1 = -a_{11},\; b_2 = a_{21},\; b_3 = -a_{31},\; b_4 = a_{41},\; b_5 = a_{51},\; b_6 = a_{61}, \quad \ldots (40)$$

(b) For an incident set of coupled transverse displacement and microrotational waves

$$b_1 = a_{12},\; b_2 = -a_{22},\; b_3 = a_{32},\; b_4 = -a_{42},\; b_5 = a_{52},\; b_6 = a_{62}, \quad \ldots (41)$$

NUMERICAL RESULTS AND DISCUSSION

Following Gauthier[16], we take the following values of relevant parameters for aluminium-epoxy composite (micropolar elastic solid) as

$$\rho = 2.19 \text{ gm/cm}^3,\; \lambda = 7.59\times10^{11}\text{dyne/cm}^2,\; \mu = 1.89\times10^{11} \text{ dyne/cm}^2,$$

$$\gamma = 0.268\times10^{11} \text{ dyne},\; j = 0.0196 \text{ cm}^2,\; \kappa = 0.0149\times10^{11} \text{ dyne/cm}^2,$$

$$\omega^2/\omega_0^2 = 10,\; \alpha_0 = 0.02 \text{ dyne},\; \beta_0\; 0.04 \text{ dyne},\; \eta_0 = 0.05 \text{ dyne}.$$

Following Silva[23], the physical parameters representing the crust as a linear viscoelastic solid are as follows

$$Q_p = 100,\; Q_s = 45,\; \rho_1 = 2.6 \text{ gm/cm}^3,\; V_p = 6.1 \text{ km/s},\; V_s = 3.5 \text{ km/s}.$$

For the above values of the relevant parameters, the system of eqs. (37) are solved for amplitude ratios by using the Gauss elimination method for different values of θ_0^* and γ_0 varying from 0° to 90° when $\omega^2/\omega_0^2 = 10$. The variations of the modulus of the amplitude ratios $|Z_i|$, (i = 1, 2, ..., 6) for reflected P - and SV – waves and refracted LD-CD-I, CD II - and LMS- waves with the angle of emergence θ_0^* as well as angle γ_0 from incident P - and SV-waves have been shown by solid and dashed lines respectively in Figs. 2 to 13 (waves).

(a) Incident P-wave

The variations of the amplitude ratios $|Z_1|$ for reflected P- waves with the angle θ_0^* as well as with angle γ_0 have been shown in Fig. 2 by solid and dashed line respectively. The variations of amplitude ratios as shown in Fig. 2 by solid and dashed lines are found to be monotonic in nature. It has its

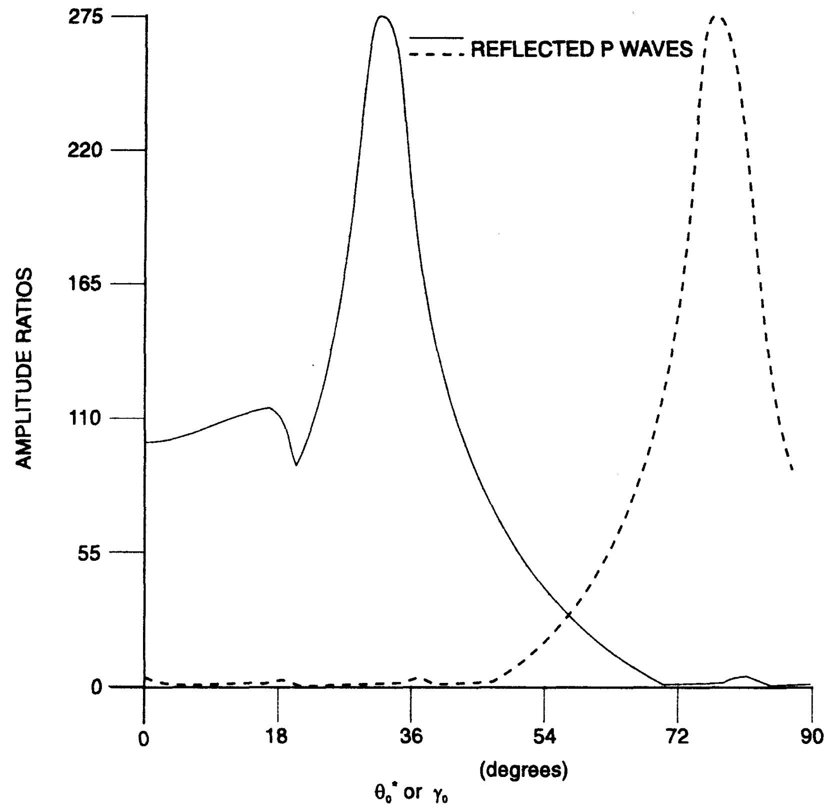

FIG. 2. Variations of the amplitude ratios with the angle of emergence (β_0') as well as with gamma (γ_0')

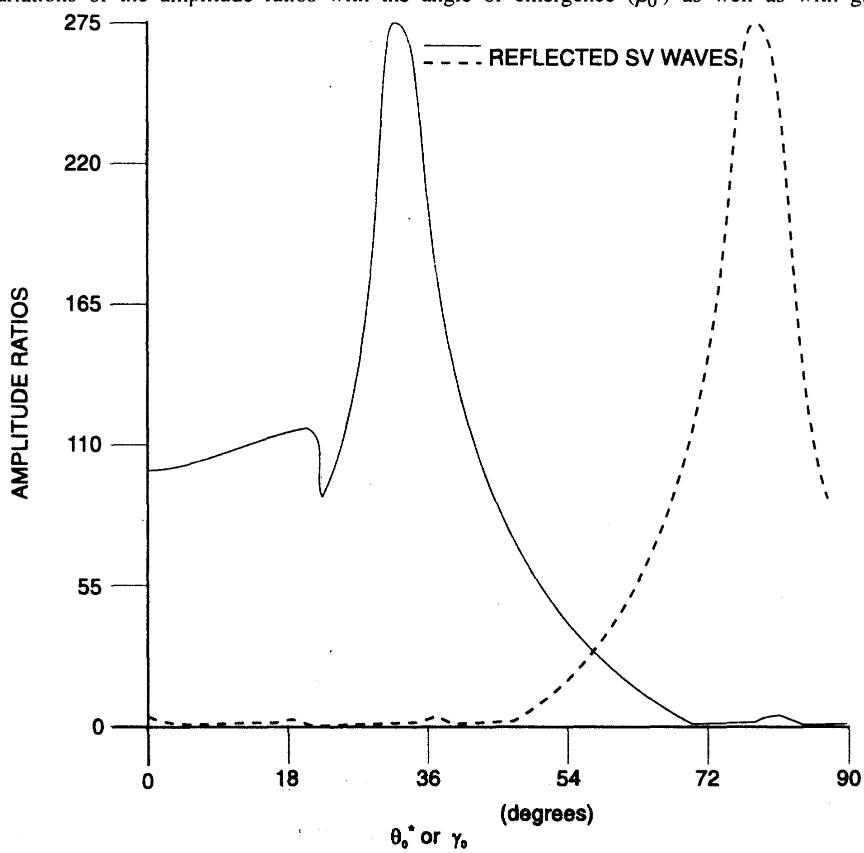

FIG. 2. Variations of the amplitude ratios with the angle of emergence (θ_0') as well as with gamma (γ_0')

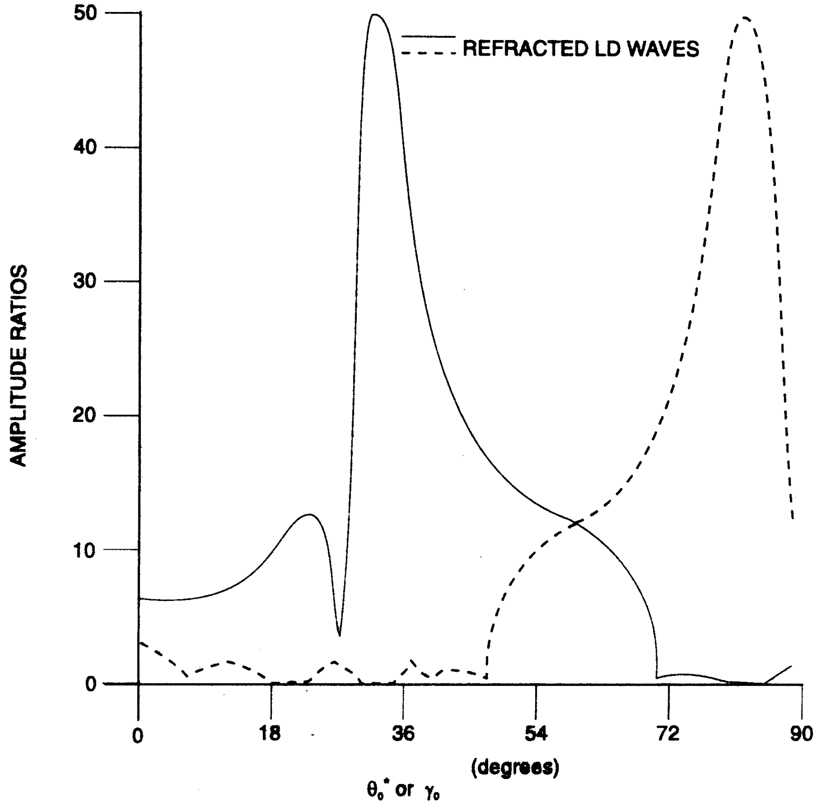

FIG. 4. Variations of the amplitude ratios with the angle of emergence (θ_0') as well as with gamma (γ_0')

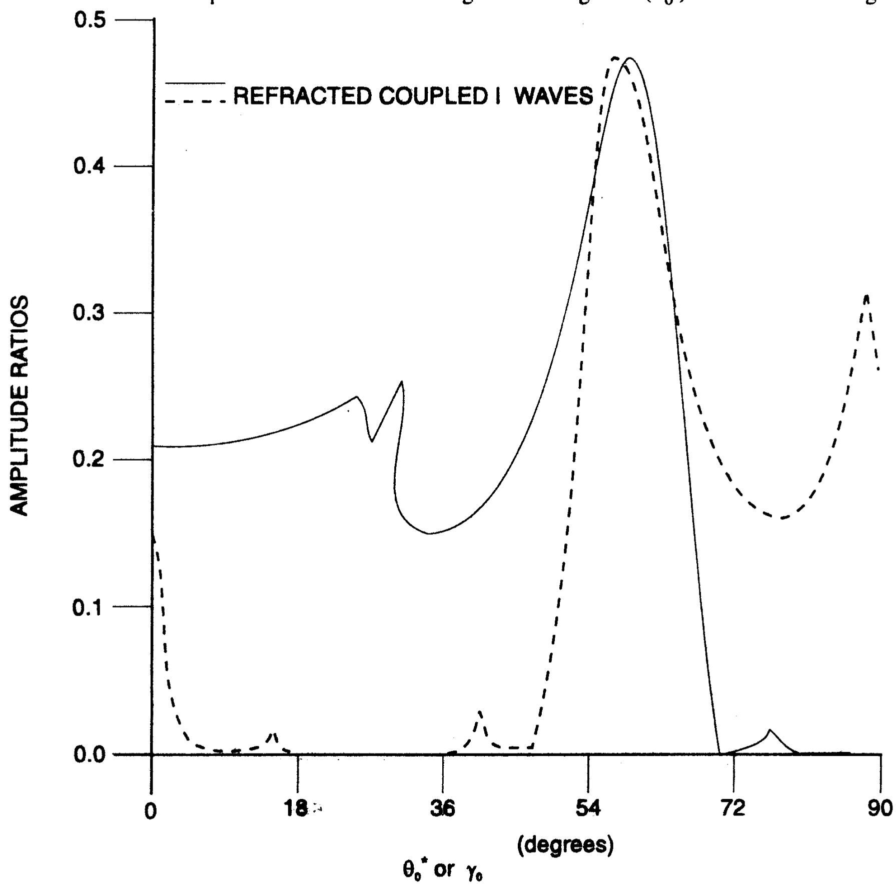

FIG. 5. Variations of the amplitude ratios with the angle of emergence (θ_0') as well as with gamma (γ_0')

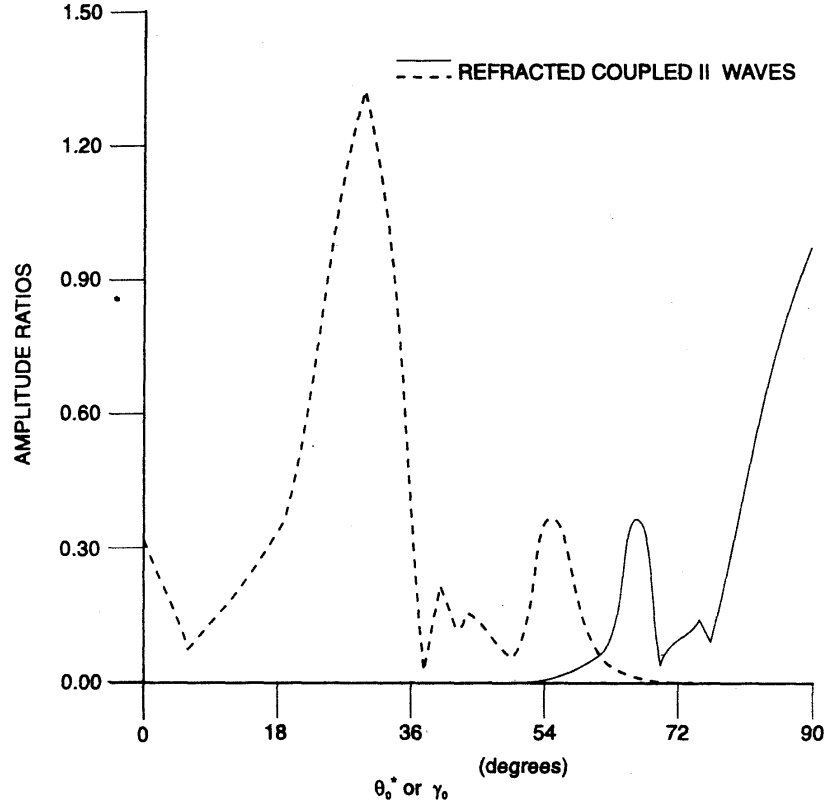

FIG. 6. Variations of the amplitude ratios with the angle of emergence (θ_0') as well as with gamma (γ_0')

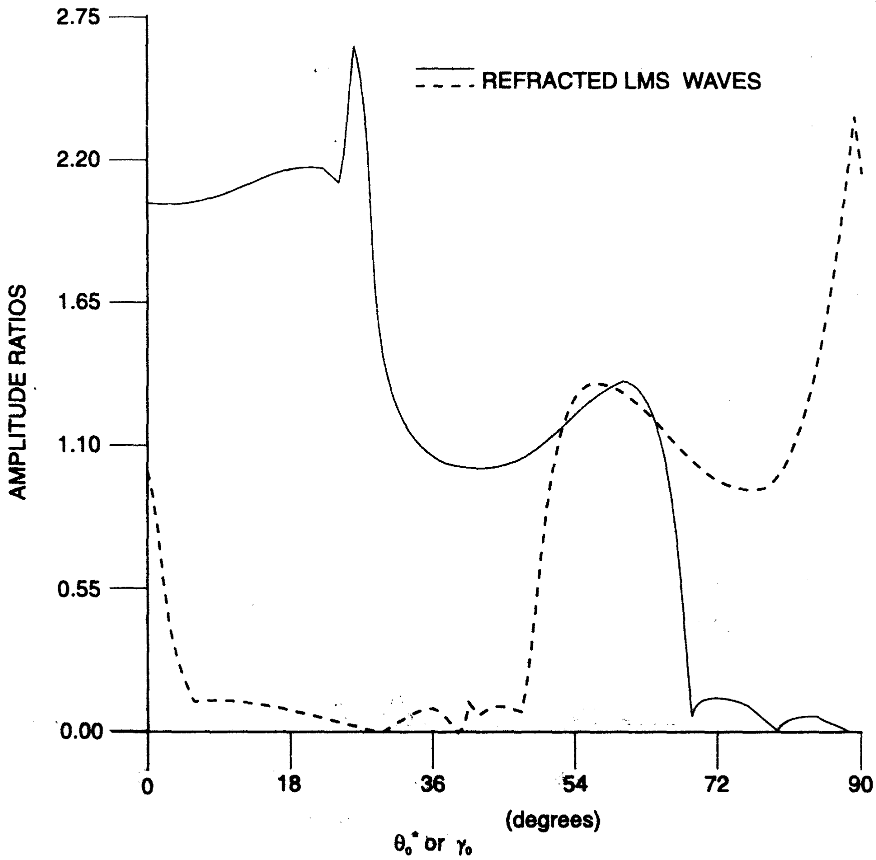

FIG. 5. Variations of the amplitude ratios with the angle of emergence (θ_0') as well as with gamma (γ_0')

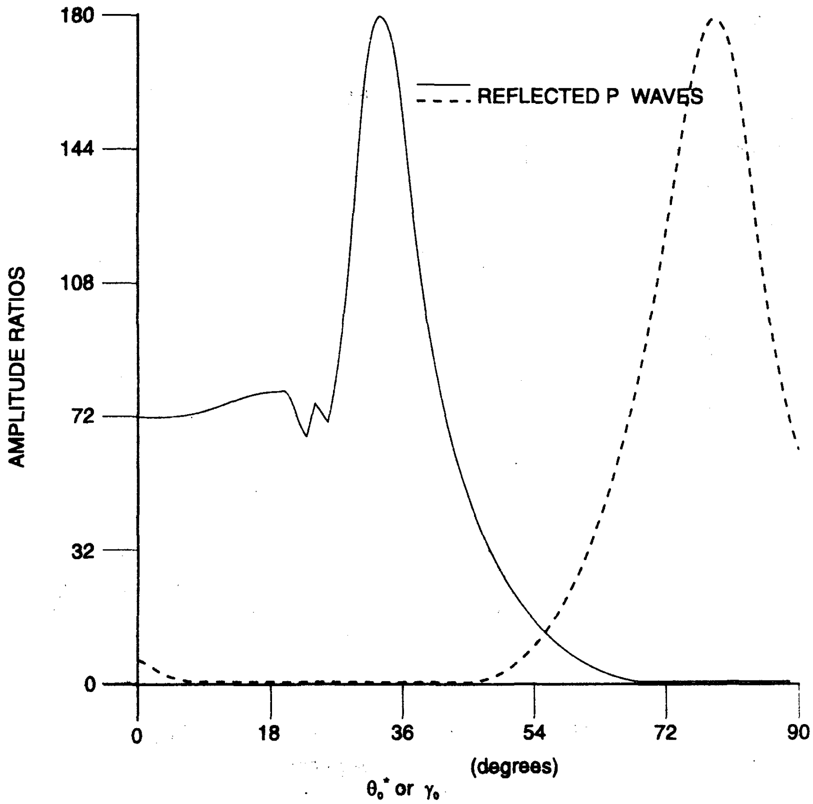

FIG. 8. Variations of the amplitude ratios with the angle of emergence (θ_0') as well as with gamma (γ_0')

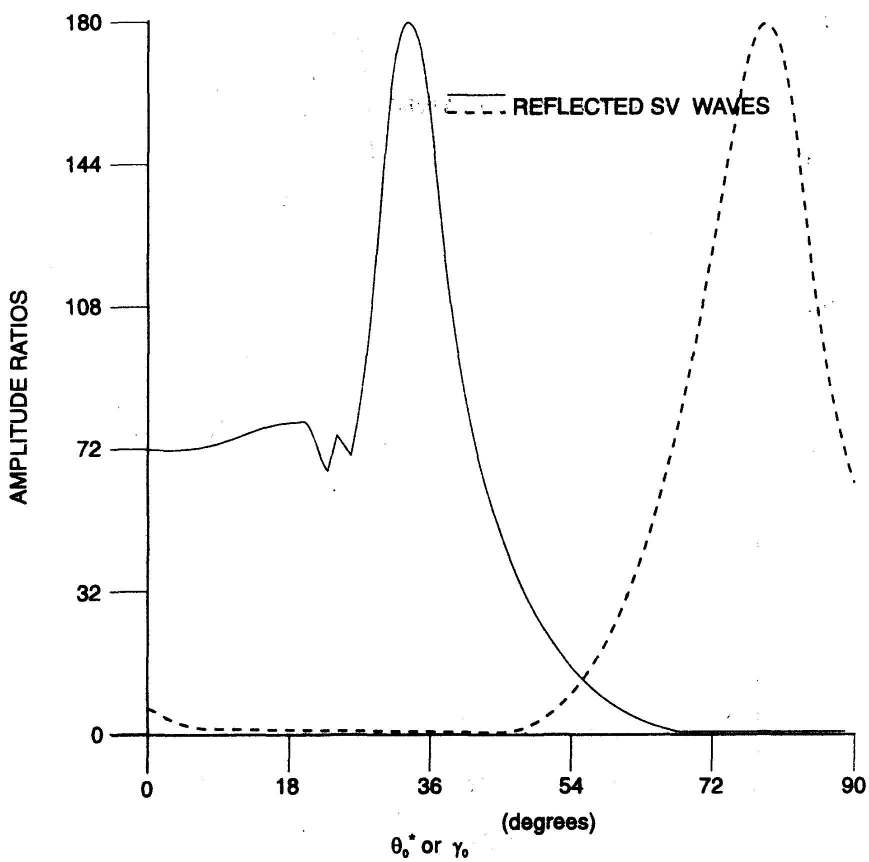

FIG. 9. Variations of the amplitude ratios with the angle of emergence (θ_0') as well as with gamma (γ_0')

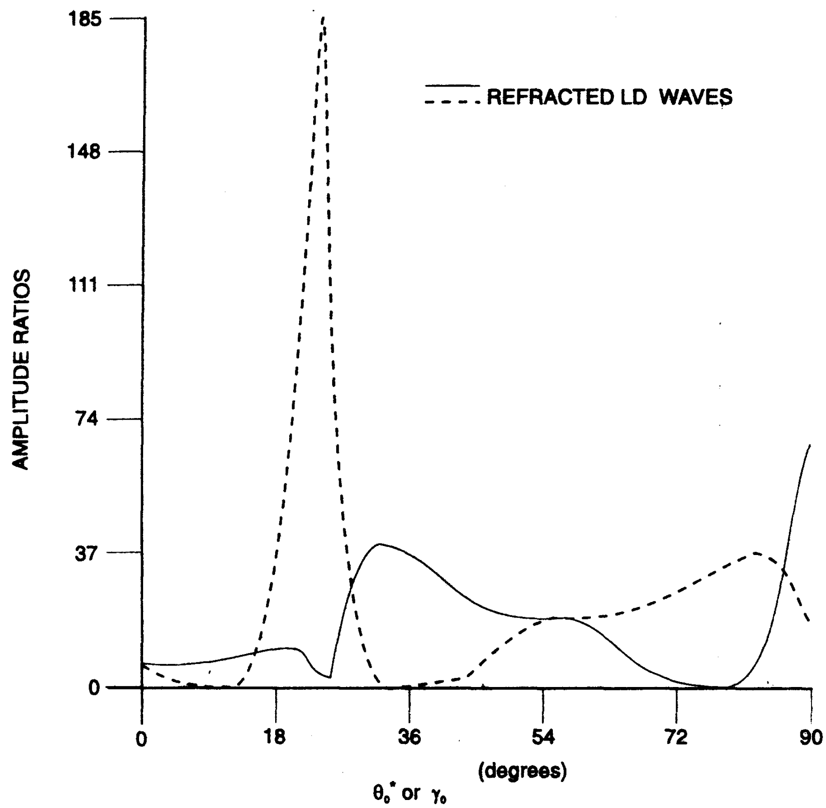

FIG. 10. Variations of the amplitude ratios with the angle of emergence (θ_0') as well as with gamma (γ_0')

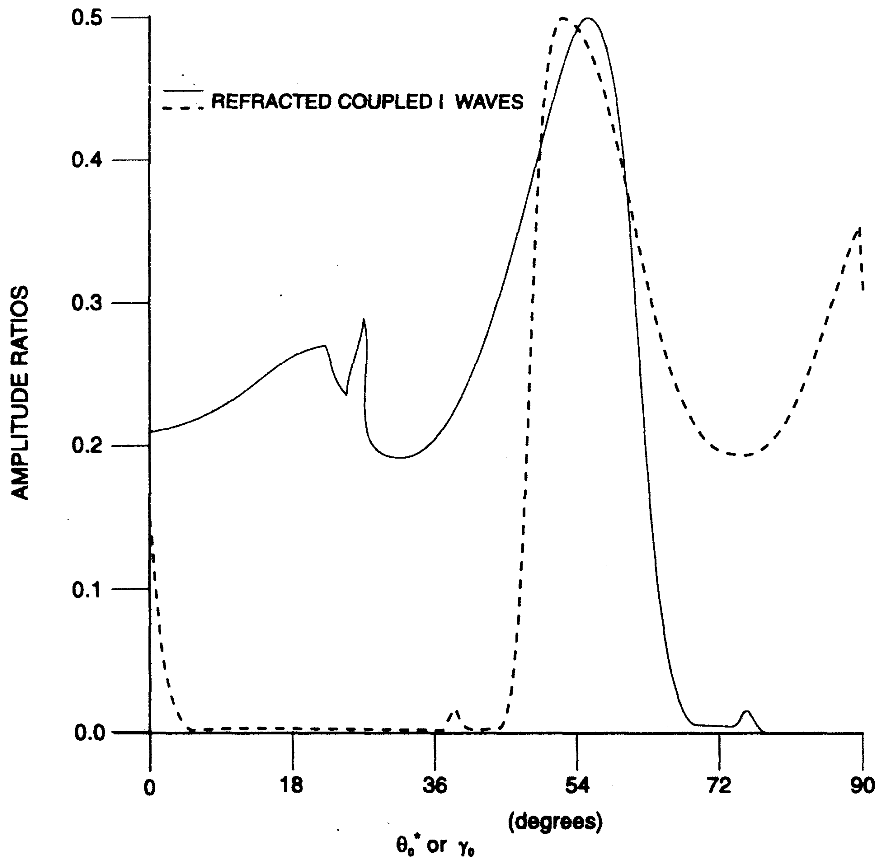

FIG. 11. Variations of the amplitude ratios with the angle of emergence (θ_0') as well as with gamma (γ_0')

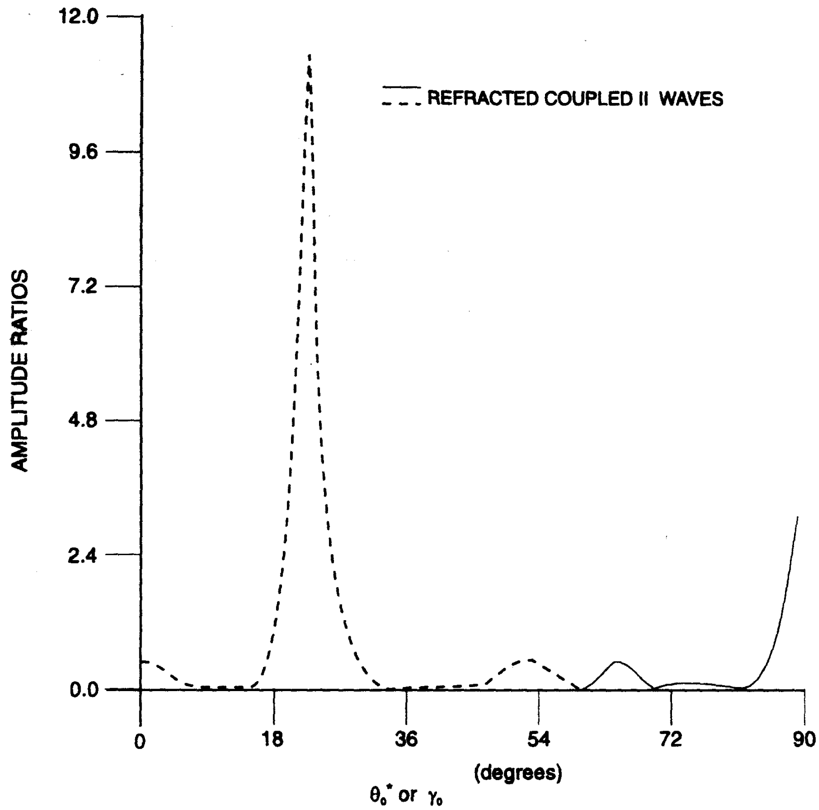

FIG. 12. Variations of the amplitude ratios with the angle of emergence (θ_0') as well as with gamma (γ_0')

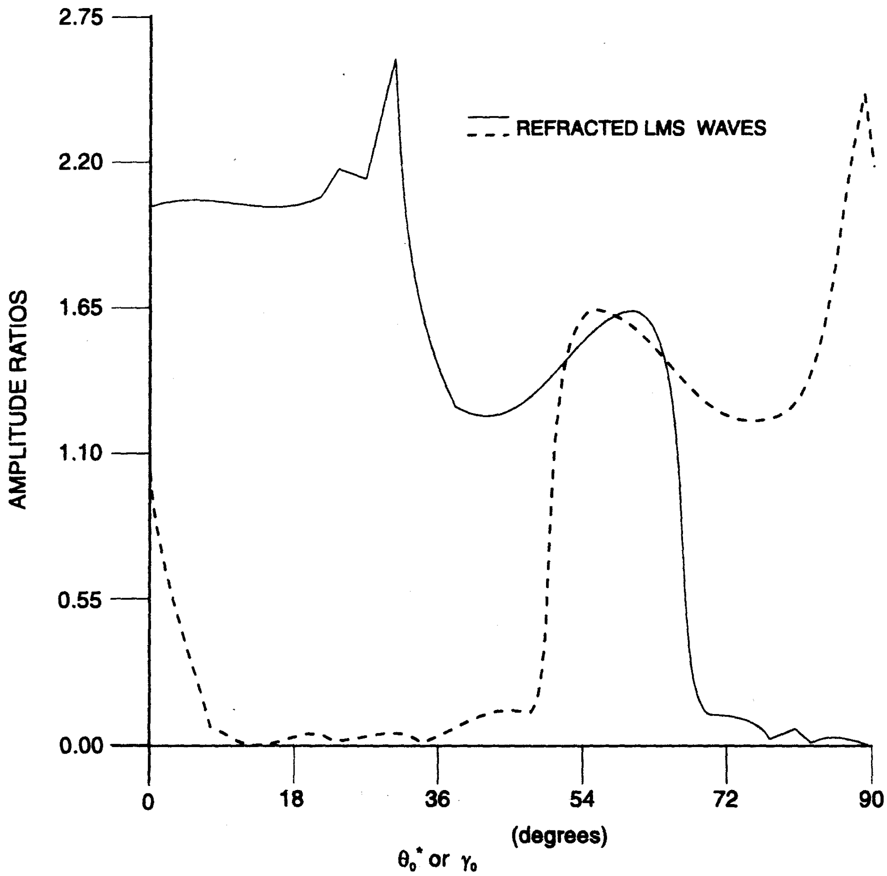

FIG. 13. Variations of the amplitude ratios with the angle of emergence (θ_0') as well as with gamma (γ_0')

maxima at $\theta_0^* = 32^o$ and minima at $\theta_0^* = 90^o$ whereas it is maximum near $\gamma_0 = 85^o$ and minimum near $\gamma_0 = 20^o$. The variations of the amplitude ratios $|Z_2|$ for reflected SV-waves with the angle θ_0^* as well as with angle γ_0 are similar to those for reflected p-waves and have been shown in Fig. 3 by solid and dashed line respectively.

The variations of the amplitude ratios $|Z_3|$ for refracted LD-waves with the angle θ_0^* as well as with angle γ_0 are also monotonic in nature as those of reflected P- and SV-waves and have been shown in Fig. 4 by solid and dashed lines respectively.

The amplitude ratios $|Z_4|$ and $|Z_5|$ for refracted coupled waves (CD I and CD II) are of oscillating behaviour. They attain their maxima at $\theta_0^* = 61^o$ and $\theta_0^* = 90^o$ respectively as shown by solid lines in Figs. 5 and 6. The variations of these amplitude ratios with angle γ_0 are also oscillatory and have been shown by dashed lines in Figs. 5 and 6 respectively. They attain their respective maxima at $\gamma_0 = 58^o$ and $\gamma_0 = 26^o$.

The amplitude ratios $|Z_6|$ for refracted LMS-wave have their maxima at $\theta_0^* = 27^o$ and at $\gamma_0^* = 88^o$. The variations of the amplitude ratios $|Z_6|$ with the angle θ_0^* as well as with angle γ_0 are also monotonic in nature and have been shown in Fig. 7.

(*b*) Incident SV-wave

The variations of the amplitude ratios $|Z_j|$, ($j = 1, 2, \ldots 6$) for various reflected and refracted waves with the angle of emergence θ_0^* as well as with angle γ_0 have been depicted graphically in Figs. 8 to 13. If we compare the variations of the amplitude ratios of various reflected and refracted waves for incident SV-wave with those of incident P-wave, it is noticed that they are similar in nature but differ in their magnitudes, minima and maxima to some extent.

CONCLUSIONS

Detailed numerical calculations have been presented for the cases of both P and SV waves incident at the interface between a linear viscoelastic solid as crust and an aluminium-epoxy composite as micropolar elastic solid. Theory and numerical results indicate the dependence of amplitude ratios of various reflected and refracted waves on angle of emergence as well as on the angle between the propagation vector and attenuation vector. The problem may be of physical interest in the fields of seismology, geophysics, earthquake engineering, etc.

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