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CYLINDRICALLY SYMMETRIC MATTER DISTRIBUTION WITH MAGNETIC FIELD

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A solution for Cylindrically symmetric matter distribution is presented here with magnetic field along Z axis.

Key Words : Cylindrically Symmetric Matter Distribution; Magnetic Field; Eistein-Maxwell Equation

1. INTRODUCTION

Banerjee[1] found a solution for cylindrically symmetric matter distribution with magnetic energy. Paul[2] also found a solution for cylindrically symmetric fluid distribution with magnetic energy. But the solution has singularity at $r = 0$. Here in this paper, the author tried the same problem with certain modification of the assumption and found that gravitational effect of matter balances the pressure gradient force. The solution is singularity free in all respect. It differs from that of Banerjee in the sense that magnetic energy and matter density are independent from each other. Here both mass density and pressure vanish at the boundary which is some finite distance away from the source and the magnetic field is found to be sourceless here.

2. FIELD EQUATIONS AND THEIR SOLUTION

The cylindrically symmetric line element according to Marder[3] is

$$ds^2 = e^{2(\alpha - \beta)} (dt^2 - dr^2) - r^2 e^{-2\beta} d\phi^2 - e^{2(\beta + \gamma)} dZ^2, \qquad \dots (1)$$

where t, r, ϕ and z are numbered as 0, 1, 2, & 3 respectively. Here, α, β and γ are functions of r alone.

Einstein-Maxwell equations are

$$R^{\gamma}_{\mu} - \frac{1}{2} R g^{\gamma}_{\mu} = - 8\pi T^{\nu}_{\mu} \qquad \dots (2)$$

$$T^{\gamma}_{\mu} = (\rho + p)\, V_{\mu}\, v^{\gamma} - \delta^{\gamma}_{\mu} p + E^{\gamma}_{\mu} \qquad \dots (3)$$

$$E^{\gamma}_{\mu} = -\frac{1}{4\pi}\left[F^{\gamma\alpha} F_{\mu\alpha} - \frac{1}{4}\delta^{\gamma}_{\mu} F^{\alpha\beta} F_{\alpha\beta} \right] \qquad \dots (4)$$

$$F^{\mu\gamma}_{i\gamma} = 4\pi J^{\mu} \qquad \dots (5)$$

and $$F_{[\mu\gamma,\alpha]} = 0, \qquad \dots (6)$$

Where $\rho \rightarrow$ mass density, $p \rightarrow$ pressure and J^{μ} is the conduction current density along ϕ direction.

The matter is at rest in the coordinate system of (r) so that

$$V^{\mu} = \delta^{\mu}_{0} (g_{00})^{-1/2}$$

Eqs. (2) are written out as follows :

$$e^{2(\beta-\alpha)}\left(-\beta_1^2 - 2\beta_1\gamma_1 + \alpha_1\gamma_1 + \frac{\alpha_1}{r} + \frac{\gamma_1}{r} \right) = 8\pi p + H^2, \qquad \dots (7)$$

$$e^{2(\beta-\alpha)}(2\beta_1\gamma_1 + \alpha_{11} + \beta_1^2 + \gamma_{11} + \gamma_1^2) = 8\pi p + H^2, \qquad \dots (8)$$

$$e^{2(\beta-\alpha)}\left(-2\beta_{11} - \frac{2\beta_1}{r} + \alpha_{11} + \beta_1^2 \right) = 8\pi p - H^2 \qquad \dots (9)$$

and $$e^{2(\beta-\alpha)}\left(-\alpha_1\gamma_1 + 2\beta_1\gamma_1 - \frac{\alpha_1}{r} + \beta_1^2 + \gamma_{11} + \gamma_1^2 \right) = -(8\pi\rho + H^2) \qquad \dots (10)$$

Where the subscript (1) indicates differentiation with respect to r and $F^{12} F_{12} = H^2$.

The magnetic field F^{12} acts along Z-axis.

Now since there are four equations and six variables let us now write.

$$\gamma = \text{constant} \qquad \dots (11)$$

and $$2\beta - \alpha = -Cr^2, \qquad \dots (12)$$

where C is constant.

As a result, trhe equations (7) & (8) give

$$\frac{\alpha_1}{r} - \alpha_{11} - 2\beta_1^2 = 0 \qquad \dots (13)$$

using eqs. (12) & (13) we have

$$\beta = \log(1 + Ar^2) \qquad \dots (14)$$

and $$\alpha = 2\log(1 + Ar^2) + Cr^2, \qquad \dots (15)$$

where A is constant.

Equations (7)-(10) with the help of (14) & (15), give

$$H^2 = 4A\,(1 + Ar^2)^{-4} - e^{-2cr^2}, \qquad \dots (16)$$

$$8\pi\rho = 8\pi p = 2c(1 + Ar^2)^{-2}\, e^{-2cr^2} \qquad \dots (17)$$

and
$$J^2 = 0. \qquad \dots (18)$$

3. Discussion of The Results

With $C = 0$, $\rho = p = 0$, $H^2 = 4A\,(1 + Ar^2)^{-4}$ and hence Vacuum solution is obtained. $J^2 = 0$ means that magnetic field is source-less

At $r = 0$, $\rho = p = 2c$, $H^2 = 4A$.

With $A = 0$, $H^2 = 0$, $\rho = p = 2C\, e^{-2Cr^2}$.

From the above, it is evident that C and A are all positive. The Values of ρ, p, H^2 are all decreasing with the increase of r and will be zero each at a certain finite distance away from the source.

References

1. S. Banerjee, *J. Phys. S*, **1** (1968) 520.
2. B. B. Paul. *Indian J. pure appl. Math.*, **7** (1976) 350.
3. L. Marder. *Proc. Roy. Soc.*, **A244** (1968) 524.