Indian J. pure appl. Math., **31**(4) : 449-450, April 2000

A NOTE ON θ-IRRESOLUTE FUNCTIONS IN THE SENSE OF PARK

TAKASHI NOIRI

Department of Mathematics, Yatsushiro College of Technology, Yatsushiro, Kumamoto, 866-8501, *Japan*

(*Received* 29 *April* 1999; *Accepted* 23 *July* 1999)

The purpose of this note is to show that Example 2.2 of [8] is false and to point out that θ-irresoluteness due to Park and Park[8] is equivalent to quasi-irresoluteness defined by Di Maio and Noiri[4].

Key Words : Semi-open; Semi-regular; θ-Irresolute; Quasi-irresolute.

Let A be a subset of a topological space X. The closure of A and the interior of A are denoted by Cl(A) and Int(A), respectively. A subset A is said to be *semi-open*[7] if $A \subset \text{Cl}(\text{Int}(A))$. The family of all semi-open sets of X is denoted by SO(X). We set $\text{SO}(X, x) = \{U \in \text{SO}(X) : x \in U\}$. The complement of a semi-open set is said to be *semi-closed*. The intersection of all semi-closed sets containing a subset A is called the *semi-closure*[1] of A and is denoted by sCl(A). In 1972, Crossley and Hildebrand[2] defined a function $f : X \to Y$ to be *irresolute* if $f^{-1}(V) \in SO(X)$ for every $V \in \text{SO}(Y)$. Among several generalizations, the following are defined and investigated.

Definition 1 — A function $f : X \to Y$ is said to be

(1) *θ-irresolute*[6] if for each $x \in X$ and each $V \in \text{SO}(Y, f(x))$, there exists $U \in \text{SO}(X, x)$ such that $f(Cl\,(U)) \subset Cl(V)$; and

(2) *quasi-irresolute*[4] if for each $x \in X$ and each $V \in \text{SO}(Y, f(x))$, there exists $U \in \text{SO}\,(X, x)$ such that $f(U) \subset \text{sCl}(V)$.

It was shown by Dontchev and Ganster[5] that θ-irresoluteness and quasi-irresoluteness are independent of each other. Recently, Park and Park[8] have defined a function under the same term as follows :

Definition 2 — A function $f : X \to Y$ is said to be *θ-irresolute*[8] if for each $x \in X$ and each $V \in \text{SO}\,(Y, f(x))$, there exists $U \in \text{SO}\,(X, x)$ such that $f(\text{sCl}(U)) \subset \text{sCl}(V)$.

It is obvious that θ-irresoluteness (in the sense of Park) implies quasi-irresoluteness. By using the following example, Park and Park[8] showed that the converse is not true.

Example 1 — Let $X = \{a, b, c, d\}$, $\tau_1 = \{X, \phi, \{a\}, \{a, d\}, \{b, c\}, \{a, b, c\}\}$ and $\tau_2 = \{X, \phi, \{d\}, \{b, c\}, \{b, c, d\}\}$. Define $f : (X, \tau_1) \to (X, \tau_2)$ by $f(a) = d$, $f(b) = c$, $f(c) = b$ and $f(d) = a$. Then f is quasi-irresolute but not θ-irresolute.

However, unfortunately, this example is false. We have

$\text{SO}\,(X, \tau_1) = \{X, \phi, \{a\}, \{a, d\}, \{b, c\}, \{a, b, c\}\} = \tau_1$ and

$\text{SO}\,(X, \tau_2) = \{X, \phi, \{d\}, \{b, c\}, \{a, d\}, \{b, c, d\}, \{a, b, c\}\}$.

There exist $d \in X$ and $V = \{a, b, c\}$ such that $f(d) = a \in V$ and $V \in SO(X, \tau_2)$. Moreover, for every $U \in SO(X, \tau_1)$ containing d, $f(U)$ is not contained in $sCl(V) = V$. Because semi-open sets of (X, τ_1) containing d are only $\{a, d\}$ and X.

Actually, it is shown in [4, Prop. 3.3] that quasi-irresoluteness is equivalent to θ-irresoluteness in the sense of Park. Since this fact is very important and an unexpected result, we shall show this fact again. A subset A is said to be *semi-regular*[3] if A is semi-open and semi-closed. By SR(X), we denote the family of all semi-regular sets of a topological space X. A subset A is said to be *semi θ-open*[3] if for each $x \in X$ there exists $U \in SO(X)$ such that $x \in U \subset sCl(U) \subset A$.

Lemma 1 — (Di Maio and Noiri[3]) Let A be a subset of a topological space X.

(1) If $A \in SO(X)$, then $sCl(A) \in SR(X)$.

(2) If $A \in SR(X)$, then it is semi θ-open in X.

Lemma 2 — (Di Maio and Noiri[3]) A function $f: X \to Y$ is quasi-irresolute if and only if $f^{-1}(V) \in SO(X)$ for every semi θ-open set V of Y.

Theorem 1 — *The following are equivalent for a function $f: X \to Y$:*

(1) f is quasi-irresolute;

(2) For any $x \in X$ and $V \in SO(Y, f(x))$, there exists $U \in SO(X, x)$ such that $f(sCl(U)) \subset sCl(V)$;

(3) f is θ-irresolute in the sense of Park.

PROOF (1) $\Rightarrow$ (2) : Let $x \in X$ and $V \in SO(Y, f(x))$. There exists $U \in SO(X, x)$ such that $f(U) \subset sCl(V)$. Suppose that $y \notin sCl(V)$. There exists $W \in SO(Y, y)$ such that $W \cap V = \phi$. By Lemma 1, we have $sCl(W) \cap sCl(V) = \phi$. It follows from Lemmas 1 and 2 that $f^{-1}(sCl(W)) \in SO(X)$. Since $U \cap f^{-1}(sCl(W)) = \phi$, we obtain $sCl(U) \cap f^{-1}(sCl(W)) = \phi$ and $f(sCl(U)) \cap sCl(W) = \phi$. Since $y \in W$, we have $y \notin f(sCl(U))$. Consequently, we obtain $f(sCl(U)) \subset sCl(V)$.

Remark : By the above theorem, we realize the following :

(1) Theorems 2.2, 2.5 and 2.7 of [8] follow from Propositions 3.4, 4.1 and 2.7 of [4], respectively;

(2) It follows from [4, Prop. 4.2] that a topological space is semi-Urysohn if and only if it is semi-T_2. Therefore, Theorems 2.11, 2.13 and 2.12 of [8] follow from Propositions 4.3 and 4.4 and Corollary 4.1 of [4], respectively;

(3) Theorem 2.14 of [8] follows from Corollary 6.1 of [4].

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