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δ_p-ALMOST COMPACTNESS FOR FUZZY TOPOLOGICAL SPACES

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This article deals with a sort of covering property, called δ_p-almost compactness for fuzzy sets and fuzzy topological spaces. A number of characterizations of the concept and certain relevant investigations are done in this paper.

Key Words : Fuzzy δ-preopen Set; Fuzzy δ_p-closure and p-closure; Fuzzy δ_p-almost Compact Set and Space; Fuzzy p-adherence

1. INTRODUCTION AND PRELIMINARIES

It is found in literature that different types of fuzzy covering properties like fuzzy compactness, almost compactness, near compactness, S-closedness and s-closedness have been introduced and studied by many mathematicians. It is the purpose of this paper to introduce yet another kind of fuzzy covering axiom, termed δ_p-almost compactness.

In [2], we introduced a class of generalized type of fuzzy open sets, called fuzzy δ-preopen sets, for a fuzzy topological space X (henceforth fts X, for short) and used them to study two types of functions between fts's. The present paper is intended for an investigation of a sort of fuzzy covering property introduced in terms of fuzzy δ-preopen sets.

In Section 2, we define fuzzy δ_p-almost compactness for fuzzy sets and fts's, and derive a number of characterizations of the same via different approaches and appliances like p refilterbases, fuzzy nets and subnets etc.

In Section 3, we take up for consideration certain results relevant to fuzzy δ_p-almost compactness. Among other things, it is shown that a fuzzy δ_p-almost compact space is fuzzy almost compact but not conversely. Finally, a condition is laid down under which the converse holds.

In what follows, by (X, τ) or simply by X we shall mean a fuzzy topological space as prescribed by Chang [3]. The notations clA, int A and 1-A (or A') will stand respectively for the fuzzy closure, interior and complement of a fuzzy set A in an fts X. The support of a fuzzy set A in X will be denoted by supp A (i.e., supp A = $\{x \in X : A(x) \neq 0\}$). A fuzzy point [8] in X with the singleton support $\{x\} \subseteq X$ and the value $\alpha\,(0 < \alpha \leq 1)$ will be denoted by x_α. The fuzzy sets in X taking on respectively the constant values 0 and 1 on X are denoted by 0_X and 1_X respectively. For two fuzzy sets A and B in X, we write $A \leq B$ if $A(x) \leq B(x)$, for each $x \in X$ [9], while we write $A\ q\ B$ if A is quasi-coincident (q-coincident, for short) with B [8], i.e., if $A(x) + B(x) > 1$, for some

$x \in X$. The negation of these statements are written as $A \not\leq B$ and $A \not{q} B$ respectively. A fuzzy set B is called a quasi-neighbourhood (q-nbd, for short) of a fuzzy set A if there is a fuzzy open set U in X such that $A \, q \, U \leq B$[8]. If, in addition, B is fuzzy open, then B is called a fuzzy open q-nbd of A. A fuzzy nbd A of a fuzzy point x_a in an fts X is defined in the usual way, i.e., if $x_a \leq U \leq A$, for some fuzzy open set U; A is a fuzzy open nbd if A is fuzzy open, in addition. A fuzzy set A in X is said to be fuzzy regular open[1] if int(cl A) = A. A fuzzy point x_a is said to be a fuzzy δ-cluster point of a fuzzy set A in an fts X if every fuzzy regular open q-nbd U of x_a is q-coincident with A[5]. The union of all fuzzy δ-cluster points of A is called the fuzzy δ-closure of A and is denoted by $\delta cl \, A$. A fuzzy set A in X is said to be fuzzy δ-preopen if $A \leq \text{int}(\delta cl \, A)$[2]. The complement of a fuzzy δ-preopen set is called fuzzy δ-preclosed. A fuzzy δ-preopen set A in an fts X is called a fuzzy δ-preopen nbd (δ-preopen q-nbd) of a fuzzy point x_a if $x_\alpha \leq A$ (resp. $x_\alpha \, q \, A$). A fuzzy point x_α in an fts X is said to be a fuzzy δ-precluster point of a fuzzy set A in X if every fuzzy δ-preopen q-nbd of x_α is q-coincident with A[2]. The union of all fuzzy δ-precluster points of A is called the fuzzy δ-preclosure of A and will be denoted by δ-pcl A. The union of all fuzzy δ-preopen sets in an fts X, each contained in a fuzzy set A in X, is called the fuzzy δ-preinterior of A and is denoted by δ-pint A. A collection $\mathcal{F}$ of fuzzy sets in an fts (X, τ) is said to form a prefilterbase [7] in X if $0_X \notin \mathcal{F}$ and for any $F_1, F_2 \in \mathcal{F}$, there exists $F_3 \in \mathcal{F}$ such that $F_3 \leq F_1 \cap F_2$. A prefilterbase $\mathcal{F}$ such that each member of $\mathcal{F}$ is contained in a given fuzzy set A in X is called a prefilterbase in A. A fuzzy point x_a in X is said to be a fuzzy δ_p-cluster point of a prefilterbase in $\mathcal{B}$ iff $x_a \leq \delta$-pcl B for all $B \in \mathcal{B}$. If, in addition, $x_a \leq A$, for a fuzzy set A, then $\mathcal{B}$ is said to have a fuzzy δ_p-cluster point in A.

Listed below are some results which will be needed frequently in the sequel.

Theorem 1.1 [2] — *In an fts X, the following hold :*

(a) A fuzzy set A in X is δ-preopen (δ-preclosed) iff A = δ-pint A (resp. A = δ-pcl A).

(b) δ-pcl (1-A) = 1-δ-pint A, for any fuzzy set A in X.

(c) $\bigcup_{i=1}^{n} \delta\text{-pcl } A_i = \delta\text{-pcl } \bigcup_{i=1}^{n} A_i$ *, for any finite collection* $\{A_1, \ldots, A_n\}$ *of fuzzy sets* $A_1, \ldots, A_n$ *in X.*

(d) Union of any family of fuzzy δ-preopen sets in X is again fuzzy δ-preopen.

(e) δ-pcl (δ-pcl A) = δ-pcl A, for any fuzzy set A in X.

2. Fuzzy δ_p-Almost Compactness : Characterizations

Definition 2.1 — A collection $\mathcal{U}$ of fuzzy sets in an fts (X, τ) is said to be a fuzzy cover of X [3] if $\sup_{U \in \mathcal{U}} U(x) = 1$, for each $x \in X$. If, in addition, the members of $\mathcal{U}$ are fuzzy δ-preopen, then $\mathcal{U}$ is called a fuzzy δ-preopen cover of X.

If A is any fuzzy set in X, then a collection $\mathcal{U}$ of fuzzy (δ-preopen) sets in X is said to be fuzzy (δ-preopen) cover of A if $\sup_{u \in \mathcal{U}} U(x) = 1$, for each $x \in$ supp A.

Definition 2.2 — A fuzzy cover $\mathcal{U}$ of a fuzzy set A in an fts X is said to have (a) a finite subcover $\mathcal{U}_0$ if $\mathcal{U}_0$ is a finite subcollection of $\mathcal{U}$ such that $\bigcup \mathcal{U}_0 \geq A$ [6],

(b) a finite δ_p-proximate subcover $\mathcal{U}_0$ if $\mathcal{U}_0$ is a finite subcollection of $\mathcal{U}$ such that $\bigcup$ $\{\delta$-pcl $U : U \in \mathcal{U}_0\} \geq A$.

Clearly if $A = 1_X$ in particular, then the requirements on $\mathcal{U}_0$ in (a) and (b) above are respectively $\bigcup \mathcal{U}_0 = 1_X$ and $\bigcup$ $\{\delta$-pcl $U : U \in \mathcal{U}_0\} = 1_X$.

Definition 2.3 — A fuzzy set A in an fts X is said to be a fuzzy δ_p-almost compact set if every fuzzy δ-preopen cover of A has a finite δ_p-proximate subcover. If in particular $A = 1_X$, we say that the fts X is a fuzzy δ_p-almost compact space.

We need a few more definitions before we give the characterizations of δ_p-almost compactness of fuzzy sets and fts's.

Definition 2.4 — A fuzzy point x_α in an fts X is said to be in the p-closure of a fuzzy set A in X, denoted by $x_\alpha \leq p$-cl$A$, if for every fuzzy δ-preopen q-nbd V of x_α, δ-pcl V q A.

Definition 2.5 — Let x_α be a fuzzy point in an fts X. A prefilterbase $\mathcal{F}$ on X is said

(*i*) to p-adhere at x_α, written as $x_\alpha \leq p$-ad $\mathcal{F}$, if for each fuzzy δ-preopen q-nbd U of x_α and each $F \in \mathcal{F}$, F q δ-pcl U, i.e., $x_\alpha \leq p$-cl F, for each $F \in \mathcal{F}$;

(*ii*) to p-converge to x_α, written as $\mathcal{F} \xrightarrow{p} x_\alpha$, if to each fuzzy δ-preopen q-nbd U of x_α, there corresponds some $F \in \mathcal{F}$ such that $F \leq \delta$-pcl U.

Definition 2.6 — Let x_α be a fuzzy point in an fts X. A fuxxy net $\{S_n : n \in (D, \geq)\}$ is said

(*i*) to p-adhere at x_α, denoted by $x_\alpha \leq p$-ad(S_n), if for each fuzzy δ-preopen q-nbd U of x_α and each $n \in D$, there exists $m \in D$ with $m \geq n$ such that $S_m\, q(\delta$-pcl $U)$;

(*ii*) to p-converge to x_α, denoted by $S_n \xrightarrow{p} x_\alpha$, if for each fuzzy δ-preopen q-nbd W of x_α, there exists $m \in D$ such that $S_n\, q(\delta$-pcl $W)$, for all $n \geq m$ $(n \in D)$.

Theorem 2.7 — *For a fuzzy set A in a fts X, the following are equivalent :*

(a) A is a fuzzy δ_p*-almost compact set.*

(b) For every prefilterbase $\mathcal{B}$ *in X,* $[\bigcap\{\delta$*-pcl* $B : B \in \mathcal{B}\}] \bigcap A = 0_X$ *implies that there exists a finite subcollection* $\mathcal{B}_0$ *of* $\mathcal{B}$ *such that* $\bigcap$ $\{\delta$*-pint* $B : B \in \mathcal{B}_0\}$ $\not{q}$ A.

(c) For any family $\mathcal{F}$ *of fuzzy δ-preclosed sets in X with* $\bigcap$ $\{F : F \in \mathcal{F}\} \bigcap A = 0_X$, *there*

exists a finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$ such that $\bigcap$ {δ-pint $F : F \in \mathcal{F}_0$ $\underline{q}$ A}.

(d) Every prefilterbase on X, each member of which is q-coincident with A, p-adheres at some fuzzy point in A.

PROOF : (a) ⇒ (b) : Let $\mathcal{B}$ be a prefilterbase in X such that $[\bigcap\{\delta\text{-pcl } B : B \in \mathcal{B}\}] \bigcap A = 0_x$. Then for every $x \in supp\, A$, $\sup_{B \in \mathcal{B}} [(\delta\text{-pcl } B)'(x)] = 1$. Thus $\{(\delta\text{-pcl } B)' : B \in \mathcal{B}\}$ is a. δ-preopen cover of A. By (a), there is a finite δ_p-proximate subcover $\{(\delta\text{-pcl } B_1)', ..., (\delta\text{-}pcl\, B_n)'\}$ (say) of it for A. Thus $A \le \bigcup_{i \in I}^{n} \delta\text{-}pcl\ (\delta\text{-pcl } B_i)' = \bigcup_{i=1}^{n} (1\text{-}\delta\text{-pint } (\delta\text{-pcl } B_i))$ so that $1 - A \ge 1 - \bigcup_{i=1}^{n} (1 - \delta\text{-pint } (\delta\text{-pcl } B_i)) = \bigcap_{i=1}^{n} \delta\text{ -pint } (\delta\text{-pcl } B_i)$. Thus $A = \bigcap_{i=1}^{n} (\delta\text{-pint } (\delta\text{-pcl } B_i))$ so that $\bigcap_{i=1}^{n} (\delta\text{-pint } B_i)\ \underline{q}\ A$.

(b) ⇒ (a) : Let the condition (b) hold, but there exists a fuzzy δ-preopen cover $\mathcal{U}$ of A having no finite δ_p-proximate subcover for A. Then for every finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$, there exists $x \in$ supp A such that $\sup_{u \in \mathcal{U}_0} [(\delta - pcl\ U)(x)] < A(x)$, i.e., $\inf_{u \in \mathcal{U}_0} [(\delta\text{ -pcl } U)'(x)] > 1 - A(x) \ge 0$. Thus $\{\bigcap_{u \in \mathcal{U}_0} (\delta - pcl\ U)' : \mathcal{U}_0$ is a finite subcollection of $\mathcal{U}\}$ (= $\mathcal{B}$, say) is a prefilterbase in X. If there exists a finite subcollection $\{U_1, ..., U_n\}$ (say) of $\mathcal{U}$ such that $\bigcap_{i=1}^{n} \delta\text{-pint } (\delta\text{-}pcl\ U_i)' A$, then $A \le 1 - \bigcap_{i=1}^{n} \delta - \text{pint } (\delta\text{-pcl } U_i)' = \bigcup_{i=1}^{n} (1\text{-}\delta\text{-pint } (\delta\text{-pcl } U_i)') = \bigcup_{i=I}^{n} \delta - pcl\ U_i$. Thus $\mathcal{U}$ has a finite δ_p-proximate subcover for A, contradicting our hypothesis. Hence for every finite subcollection $\left\{\bigcap_{u \in \mathcal{U}_1} (\delta\text{-}pcl\ U)', ..., \bigcap_{u \in \mathcal{U}_k} (\delta\text{-}pcl\ U)'\right\}$ of $\mathcal{B}$ where $\mathcal{U}_1, ..., \mathcal{U}_k$ are finite subsets of $\mathcal{U}$, we have $\left[\bigcap_{U \in \mathcal{U}_1 \cup ... \cup \mathcal{U}_k} \delta\text{-}pint\ (\delta - pcl\ U)'\right] qA$. Then by the given condition, $[\bigcap\{\delta\text{-pcl } (\delta\text{-pcl}$

$U)' : U \in \mathcal{U}\}] \cap A \neq 0_X$. Then there exists $x \in$ supp A such that $\inf_{U \in \mathcal{U}} [\delta\text{-}pcl\ (\delta\text{-pcl}\ U)'](x) > 0$, i.e., $\sup_{U \in \mathcal{U}} [\delta\text{-pcl}\ (\ \delta\text{-pcl}\ \ U)']'(x) < 1$. Thus $\sup_{U \in \mathcal{U}} U(x) \leq \sup_{U \in \mathcal{U}} (\delta\text{-pint}\ (\delta\text{-pcl}\ U))\ (x) < 1$, which contradicts the fact that $\mathcal{U}$ is a fuzzy δ-preopen cover of A.

(a) $\Rightarrow$ (c) : Let $\mathcal{F}$ be a family of fuzzy δ-preclosed sets in X such that $\cap\ \{F : F \in \mathcal{F}\}$ $\cap\ A = 0_x$. Then for each $x \in$ supp A and for each positive integer n, there exists some $F_n \in \mathcal{F}$ such that $F_n(x) < \frac{1}{n}$, i.e., $1 - F_n(x) > 1 - \frac{1}{n}$. Thus $\sup_{F \in \mathcal{F}} [(1 - F)(x)] = 1$. Consequently, $\{1 - F : F \in \mathcal{F}\}$ is a fuzzy δ-preopen cover of A, so that by (a), $A \leq \bigcup_{F \in \mathcal{F}_0} \delta\text{-}pcl\ (1\ -\ F)$, for some finite subset $\mathcal{F}_0$ of $\mathcal{F}$. Then $1 - A \geq 1 - \bigcup_{F \in \mathcal{F}_0} \delta\text{-}pcl\ (1\ -\ F)\ =\ \bigcap_{F \in \mathcal{F}_0} \{1\ -\ \delta\text{-pcl}\ (1\ -\ \text{F})\}\ =\ \cap\ [\delta\text{-pint}$ $F : F \in \mathcal{F}_0\}$. Hence $q \left[\bigcap_{F \in \mathcal{F}_0} \delta\text{-pint}\, F \right]$, where $\mathcal{F}_0$ is a finite subcollection of $\mathcal{F}$.

(c) $\Rightarrow$ (d) : Let $\mathcal{B}$ be a prefilterbase in X such that $[\cap\{\delta\text{-pcl}\ B : B \in \mathcal{B}\}] \cap A = 0_x$. Then the family $\mathcal{F}\ =\ \{\delta\text{-pcl}\ B : B \in \mathcal{B}\}$ is a family of fuzzy δ-preclosed sets in X with $(\cap\ \mathcal{F}) \cap A = 0_x$. Hence by (c), there is a finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$ such that $[\cap\ \{\delta\text{-pint}\ (\delta\text{-pcl}$ $B)\ :\ B \in \mathcal{B}_0\}]\ \underline{q}\ A$. This implies that $\left(\bigcap_{B \in \mathcal{B}_0} \delta\text{-pint}\, B \right) A$.

(a) $\Rightarrow$ (d) : Let $\mathcal{F}$ be a prefilterbase in X, each member of which is q-coincident with A. Suppose, if possible, $\mathcal{F}$ does not p-adhere at any fuzzy point in A. For each $x \in$ supp A there exists $n_x \in N$ (= the set of natural numbers) such that $x_{1/n_x} \leq A$. Then there are a δ-preopen set $U^x_{n_x}$ and a member $F^x_{n_x}$ of $\mathcal{F}$ such that $x_{1/n_x}\, q\, U^x_{n_x}$ and δ-pcl $U^x_{n_s}\ \underline{q}\ F^x_{n_x}$. Thus $U^x_{n_x}(x) > 1 - \frac{1}{n_x}$ so that sup $\{U^x_n(x) : n \in N$ and $n \geq n_x\}\ =\ 1$. Then $\{U^x_n : n \geq n_x, n \in N$ and $x \in$ supp $A\}$ forms a fuzzy δ-preopen cover of A. By (a), there exist finitely many $x_1, x_2, \ldots, x_k \in$ supp A and finitely many $n_1, \ldots, n_k \in N$ such that $A \leq \bigcup_{i=1}^{K} \delta\text{-}pcl\, U^{x_i}_{n_i}$. Choose $F \in \mathcal{F}$ such that $F \leq \bigcap_{i=1}^{K} F^{x_i}_{n_i}$. Then F $\left[\bigcup_{i=1}^{K} \delta - pcl\, U^{x_i}_{n_i} \right]$, i.e., $F\ \underline{q}\ A$, a contradiction.

(d) $\Rightarrow$ (a) : If possible, let there exist a fuzzy δ-preopen cover u of A such that for every finite subset u_0 of u, $\bigcup\{\delta\text{-pcl } U : U \in u_0\} \not\geq A$. Then $\mathcal{F} = \{1 - \bigcup_{U \in u_0} \delta\text{-pcl } U : u_0$ is a finite subset of $u\}$ is a prefilterbase on X such that $F \, q \, A$, for each $F \in \mathcal{F}$. By (d), $\mathcal{F}$ p-adheres at some fuzzy point $x_\alpha \leq A$. As u is a fuzzy cover of A, $\sup_{U \in u} U(x) = 1$ and hence there exists $U_0 \in u$ such that $U_0(x) > 1 - \alpha$. Then $x_\alpha \, q \, U_0$. As $x_\alpha \leq p\text{-ad } \mathcal{F}$ and $1\text{-}\delta\text{-pcl } U_0 \in \mathcal{F}$, we have $\delta\text{-pcl } U_0 q(1\text{-}\delta\text{-pcl } U_0)$ which is a contradiction.

Some sufficient conditions for fuzzy set A to be fuzzy δ_p-almost compact are given by the following theorem.

***Theorem* 2.8** — *For a fuzzy set A in an fts X, the following implications hold :*

(a) Every fuzzy net in A p-adheres at some fuzzy point in A.

$\Leftrightarrow$ *(b) Every fuzzy net in A has a p-convergent fuzzy subnet.*

$\Leftrightarrow$ *(c) Every prefilterbase in A p-adheres at some fuzzy point in A.*

$\Rightarrow$ *(d) For every family* $\{B_\alpha : \alpha \in \Lambda\}$ *of non-null fuzzy sets with*

$\left[\left(\bigcap_{\alpha \in \Lambda} \text{pcl } B_\alpha\right)\right] \bigcap A = 0_X$, there is a finite subset Λ_0 of Λ such that

$$\left(\bigcap_{\alpha \in \Lambda_0} B_\alpha\right) \bigcap A = 0_X.$$

$\Rightarrow$ *(e) A is fuzzy* δ_p*-almost compact.*

PROOF : (a) $\Rightarrow$ (b) : Let a fuzzy net $\{S_n : n \in (D, \geq)\}$, where $(D, \geq)$ is a directed set, in A p-adhere at a fuzzy point $x_\alpha \leq A$. Let Q_{x_α} denote the set of the fuzzy δ-preclosures of all fuzzy δ-preopen q-nbds of x_α. For any $B \in Q_{x_n}$, there can be chosen some $n \in D$ such that $S_n \, qB$.. Let E denote the set of all ordered pairs (n, B) with the property that $n \in D, B \in Q_{x_n}$ and $S_n qB$. Then $(E, >>)$ is a directed set where $(m, C) >> (n, B)$ iff $m \geq n$ in D and $C \leq B$. Then $T : (E >>) \to (X, \tau)$ given by $T(n, B) = S_n$, is a subnet of $\{S_n : n \in (D, \geq)\}$.

Let V be any fuzzy δ-preopen q-nbd of x_α. Then there is $n \in D$ such that $(n, \delta\text{-pcl } V) \in E$ and hence $S_n \, q \, \delta\text{-pcl } V$. Now, for any $(m, U) >> (n, \delta\text{-pcl } V)$, $T(m, U) = S_m \, qU \leq \delta\text{-pcl } V$ $\Rightarrow T(m, U) \, q \, \delta\text{-pcl } V$. Hence $T \xrightarrow{p} x_\alpha$.

(b) $\Rightarrow$ (a) : If a fuzzy net $\{S_n : n \in (D, \geq)\}$ does not p-adhere at a fuzzy point x_α, then there is a fuzzy δ-preopen q-nbd U of x_α and an $n \in D$ such that S_m (δ-pcl U), for all $m \geq n$. Then obviously no fuzzy subnet of the fuzzy net can p-converge to x_α.

(a) $\Rightarrow$ (c) : Let $\mathcal{F} = \{F_\alpha : \alpha \in \wedge\}$ be a prefilterbase in A. For each $\alpha \in \wedge$, choose a fuzzy point $x_{F_\alpha} \leq F_\alpha$, and construct the fuzzy net $S = \{x_{F_\alpha} : F_\alpha \in \mathcal{F}\}$ in A with $(\mathcal{F}, >>)$ as domain, where for two members $F_\alpha, F_\beta \in \mathcal{F}, F_\alpha >> F_\beta$ iff $F_\alpha \leq F_\beta$. By (a), the fuzzy net p-adheres at some fuzzy point $x_t (0 < t \leq 1) \leq A$. Then for any fuzzy δ-preopen q-nbd U of x_t and any $F_\alpha \in \mathcal{F}$, there exists $F_\beta \in \mathcal{F}$ such that $F_\beta >> F_\alpha$ and $x_{F_\beta} q$ (δ-pcl U). Then $F_\beta q$ (δ-pcl U) and hence $F_\alpha q$ (δ-pcl U). Thus $\mathcal{F}$ p-adheres at x_t.

(c) $\Rightarrow$ (a) : Let $\{S_n : n \in (D, \geq)\}$ be a fuzzy net in A. Consider the prefilterbase $\mathcal{F}$ = $\{T_n : n \in D\}$ generated by the net, where $T_n = \{S_m : m \in D$ and $m \geq n\}$. By (c), there exists a fuzzy point $a_\alpha \leq A \cap p$-ad $\mathcal{F}$. Then for each fuzzy δ-preopen q-nbd U of a_α and each $F \in \mathcal{F}$, δ-pcl U q F, i.e., (δ-pcl U)q T_n, for all $n \in D$. Hence the given fuzzy net p-adheres at a_α.

(c) $\Rightarrow$ (d) : Let $\mathcal{B} = \{B_\alpha : \alpha \in \wedge\}$ be a family of fuzzy sets in X such that for every finite subset $\wedge_0$ of $\wedge$, $\left(\bigcap_{\alpha \in \wedge_0} B_\alpha\right) \cap A \neq 0_X$. Then $\mathcal{F} = \left\{\left(\bigcap_{\alpha \in \wedge_0} B_\alpha\right) \cap A : \wedge_0 \text{ a finite subset of } \wedge\right\}$ is a prefilterbase in A. By (c), there is a fuzzy point $a_t \leq A \cap$ p-ad $\mathcal{F}$ (where $0 < t \leq 1$). Then for each $\alpha \in \wedge$ and each fuzzy δ-preopen q-nbd U of a_t, $B_\alpha q$ (δ-pcl U), i.e., $a_t \leq p$-cl B_α, for each $\alpha \in \wedge$. Consequently, $\left(\bigcap_{\alpha \in \wedge} p\text{-}cl\, B_\alpha\right) \cap A \neq 0_x$.

(d) $\Rightarrow$ (e) : Let $\{U_\alpha : \alpha \in \wedge\}$ be a fuzzy δ-preopen cover of a fuzzy set A. Then $A \cap \left[\bigcap_{\alpha \in \wedge} (1 - U_\alpha)\right] = A \cap \left[1 - \bigcup_{\alpha \in \wedge} U_\alpha\right] = 0_x$. If for some $\alpha \in \wedge$, 1-δ-pcl $U_\alpha = 0_x$, then (e) follows. If 1-δ-pcl U_α (= B_α, say) $\neq 0_X$, for each $\alpha \in \wedge$, then $\mathcal{B} = \{B_\alpha : \alpha \in \wedge\}$ is a family of non-null fuzzy sets. We show that $\bigcap \{p\text{-cl } B_\alpha : \alpha \in \wedge\} \leq \bigcap \{1 - U_\alpha : \alpha \in \wedge\}$. In fact, let $x_t (0 < t \leq 1)$ be a fuzzy point such that $x_t \leq p$-cl $B_\alpha = p$-cl (1-δ-pcl U_α). If $x_t\, q\, U_\alpha$, then (δ-pcl U_α) q (1-δ-pcl U_α) which is impossible. Hence x_t $\not{q}$ U_α so that $x_t \leq 1 - U_\alpha$. Thus $\left[\bigcap_{\alpha \in \wedge} (p\text{-}cl\, B_\alpha)\right] \cap A \leq A \cap \left[\bigcap_{\alpha \in \wedge} (1 - U_\alpha)\right] = 0_x$. By (d), there exists a finite subset $\wedge_0$ of $\wedge$

such that $A \cap \left(\bigcap_{\alpha \in \wedge_0} B_\alpha \right) = 0_X$, i.e., $A \leq 1 - \bigcap_{\alpha \in \wedge_0} B_\alpha = \bigcup_{\alpha \in \wedge_0} \delta - pcl\ U_\alpha$, and (e) follows.

Definition 2.9 — A fuzzy set A in an fts X will be called fuzzy regularly δ_p-closed if δ-pcl (δ-pint A) = A. The complement of such a fuzzy set will be called fuzzy regularly δ_p-open.

Theorem 2.10 — *A fuzzy set A in an fts X is fuzzy δ_p-almost compact iff for each prefilterbase $\mathcal{F}$ in X which is such that for each set of finitely many members $F_1, \ldots, F_n$ from $\mathcal{F}$ and for any fuzzy regularly δ_p-closed set C containing A, one has $(F_1 \cap \ldots \cap F_n)\, qC$, $\mathcal{F}$ has a fuzzy δ_p-cluster point in A.*

PROOF : Let A be a fuzzy δ_p-almost compact set and suppose $\mathcal{F}$ is a prefilterbase in X such that

$$A \cap [\cap\{\delta\text{-pcl } F : F \in \mathcal{F}\}] = 0_X. \qquad \ldots (1)$$

Let $x \in$ supp A. Consider any $n \in N$ (= the set of natural numbers) such that $\frac{1}{n} < A(x)$, i.e., $x_{1/n} \leq A$. By (1), there is a fuzzy δ-preopen q-nbd U_x^n of $x_{1/n}$ and an $F_x^n \in \mathcal{F}$ such that $U_x^n \not{q}\, F_x^n$. Now, $U_x^n(x) > 1 - \frac{1}{n}$ and hence $\sup\left\{ U_x^n(x) : \frac{1}{n} < A(x), n \in N \right\} = 1$. Thus the collection u of all such U_x^n for $n \in N$ and $x \in$ supp A forms a fuzzy δ-preopen cover of A such that for U_x^n, there is $F_x^n \in \mathcal{F}$ with $U_x^n \not{q}\, F_x^n$. Since A is fuzzy δ_p-almost compact, there exist finitely many members $U_{x_1}^{n_1}, \ldots, U_{x_k}^{n_k}$ of u such that $A \leq \bigcup_{i=1}^{k} \delta$ -pcl $U_{x_i}^{n_i} = \delta\text{-}pcl\left(\bigcup_{i=1}^{k} U_{x_i}^{n_i} \right) U$ (say). Now, $F_{x_1}^{n_1}, \ldots, F_{x_k}^{n_k} \in \mathcal{F}$ such that $U_{x_i}^{n_i} \not{q}\, F_{x_i}^{n_i}$ for i = 1, 2, ..., k. Hence U is a fuzzy regularly δ_p-closed set containing A such that $U \not{q} \left[F_{x_1}^{n_1} \cap \ldots \cap F_{x_k}^{n_k} \right]$.

Conversely, let $\mathcal{B}$ be a prefilterbase in X having no fuzzy δ_p-cluster point in A. Then by hypothesis, there is a fuzzy regularly δ_p-closed set C containing A such that for some finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$, $(\cap \mathcal{B}_0) \not{q}\, C$. Then $(\cap \mathcal{B}_0) \not{q}\, A$ and consequently, by "(b) $\Rightarrow$ (a)" of Theorem 2.7, A becomes fuzzy δ_p-almost compact.

From Theorems 2.7, 2.8 and 2.10, we now obtain the following characterizations of a fuzzy δ_p-almost compact space.

Theorem 2. 11 — *For an fts X, the following statements are equivalent :*

(a) X is fuzzy δ_p-almost compact.

(b) Every fuzzy net in X p-adheres at some fuzzy point in X.

(c) Every fuzzy net in X has a p-convergent fuzzy subnet.

(d) Every prefilterbase in X p-adheres at some fuzzy point in X.

(e) For every family $\{B_\alpha : \alpha \in \wedge\}$ *of non-null fuzzy sets with* $\bigcap \{pcl\ B_\alpha : \alpha \in \wedge\} = 0_x$, *there is a finite subset* Λ_0 *of* $\wedge$ *such that* $\bigcap \{B_\alpha : \alpha \in \wedge_0\} = 0_X$.

(f) For every prefilterbase $\mathcal{B}$ *in X with* $\bigcap \{\delta\text{-}pcl\ B : B \in \mathcal{B}\} = 0_X$, *there exists a finite subcollection* $\mathcal{B}_0$ *of* $\mathcal{B}$ *such that* $\bigcap \{\delta\text{-}pint\ B : B \in \mathcal{B}_0\} = 0_X$.

(g) For any family $\mathcal{F}$ *of fuzzy δ-preclosed sets in X with* $\bigcap \mathcal{F} = 0_X$, *there exists a finite subcollection* $\mathcal{F}_0$ *of* $\mathcal{F}$ *such that* $\bigcap \{\delta\text{-}pint\ F : F \in \mathcal{F}_0\} = 0_X$.

Theorem 2.12 — *An fts X is fuzzy* δ_p*-almost compact iff for any collection* $\{F_\alpha : \alpha \in \wedge\}$ *of fuzzy δ-preopen sets in X having the finite intersection property,* $\bigcap \{\delta\text{-}pcl\ F_\alpha : \alpha \in \wedge\} \neq 0_X$.

PROOF : Let X be a fuzzy δ_p-almost compact space and $\mathcal{F} = \{F_\alpha : \alpha \in \wedge\}$ be a collection of fuzzy δ-preopen sets in X with finite intersection property. Suppose $\bigcap \{\delta\text{-pcl}\ F_\alpha : \alpha \in \wedge\} = 0_X$. Then $\{1\text{-}\ \delta\text{-pcl}\ F_\alpha : \alpha \in \wedge\}$ is a fuzzy δ-preopen cover of X. By hypothesis, there is a finite subset $\wedge_0$ of $\wedge$ such that $1_X = \bigcup \{\delta\text{-pcl}\ (1\text{-}\delta\text{-pcl}\ F_\alpha) : \alpha \in \wedge_0\} = \bigcup \{1\text{-}\delta\text{-pint}\ (\delta\text{-pcl}\ F_\alpha) : \alpha \in \wedge_0\} \leq \bigcup \{1 - F_\alpha : \alpha \in \wedge_0\} = 1 - \bigcap_{\alpha \in \wedge_0} F_\alpha$. Thus $\bigcap_{\alpha \in \wedge_0} F_\alpha = 0_X$, contradicting the finite intersection property of $\mathcal{F}$.

Conversely, suppose X is not fuzzy δ_p-almost compact. Then there is a fuzzy δ-preopen cover $\mathcal{F} = \{F_\alpha : \alpha \in \wedge\}$ of X such that for every finite subset $\wedge_0$ of $\wedge$, $\bigcup \{\delta\text{-pcl}\ F_\alpha : \alpha \in \wedge_0\} \neq 1_X$. Then $\bigcap_{\alpha \in \wedge_0} (1-\delta\text{-}pcl\ F_\alpha) = 1 - \bigcup_{\alpha \in \wedge_0} \delta - pcl\ F_\alpha \neq 0_x$, for every finite subset $\wedge_0$ of $\wedge$. Thus $\{1\text{-}\delta\text{-pcl}\ F_\alpha : \alpha \in \wedge\}$ is a collection of fuzzy δ-preopen sets with finite intersection property. By hypothesis, $\bigcap_{\alpha \in \wedge} \delta\ \text{-pcl}\ (1\text{-}\delta\text{-pcl}\ F_\alpha) \neq 0_x$, i.e., $1 - \bigcup_{\alpha \in \wedge} \delta - pint\ (\delta\text{-pcl}\ F_\alpha) \neq 0_x$ so that $\bigcup_{\alpha \in \wedge} \delta - pint\ (\delta\text{-pcl}\ F_\alpha) \neq 1_X$. Hence $\bigcup_{\alpha \in \wedge} F_\alpha \neq 1_X$, a contradiction as $\mathcal{F}$ is a fuzzy δ-preopen cover of X.

Definition 2.13 — Let $\{U_n : n \in D\}$ be a fuzzy net of fuzzy δ-preopen sets in X, i.e., for each member n of a directed set $(D, \geq)$, U_n is a fuzzy δ-preopen set in X. A fuzzy point x_α in X is said to be a fuzzy δ_p-cluster point of the fuzzy net if for every $n \in D$ and every fuzzy δ-preopen q-nbd V of x_α, there exists $m \in D$ with $m \geq n$ such that $U_m\, q\, V$.

Theorem 2.14 — *An fts X is fuzzy* δ_p*-almost compact iff every fuzzy net of fuzzy δ-preopen sets in X has a fuzzy* δ_p*-cluster point in X.*

PROOF : Let $\mathcal{U} = \{U_n : n \in (D, \geq)\}$ be a fuzzy net of fuzzy δ-preopen sets in a fuzzy δ_p-almost compact space X. For each $n \in D$, let $F_n = \delta\text{-pcl}\ [\bigcup \{U_m : m \in D \text{ and } m \geq n\}]$. Then $\mathcal{F}$

$= \{F_n : n \in D\}$ is a family of fuzzy δ_p-closed sets with the condition that for every finite subcollection $\mathcal{F}_0$ and $\mathcal{F}$, $\bigcap \{\delta\text{-pint } F : F \in \mathcal{F}_0\} \neq 0_X$. Then by Theorem 2.11 ((a) $\Rightarrow$(g)), $\bigcap_{n \in D} F_n \neq 0_X$. Let $x_\lambda \in \bigcap_{n \in D} F_n$. Then for any fuzzy δ-preopen q-nbd A of x_λ and any $n \in D$, $A\, q\, [\bigcup\{U_m : m \geq n\}]$. Then there is some $m \in D$ with $m \geq n$ such that $A\, q\, U_m$. This shows that x_λ is a fuzzy δ_p-cluster point of u.

Conversely, let $\mathcal{F}$ be a collection of fuzzy δ-preclosed sets in X with the condition that for every finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$, $\bigcap \{\delta\text{-pint } F : F \in \mathcal{F}_0\} \neq 0_X$. Let $\mathcal{F}^*$ denote the family of all finite intersections of members of $\mathcal{F}$ directed by the relation '>>' such that for $F_1, F_2 \in \mathcal{F}^*$, $F_1 >> F_2$ iff $F_1 \leq F_2$. Let F^* denote the δ-preinterior of F, for each $F \in \mathcal{F}^*$. Then $F^* \neq 0_X$. Consider the fuzzy net $u = \{F^* : F^* \in (\mathcal{F}^*, >>)\}$ of non-null fuzzy δ-preopen sets in X. By the hypothesis, there exists a fuzzy point x_λ which is a fuzzy δ_p-cluster point of u. We now show that $x_\lambda \in \bigcap \mathcal{F}$. In fact, let $F \in \mathcal{F}$ be arbitrary and A be any fuzzy δ-preopen q-nbd of x_λ. Since $F \in \mathcal{F}^*$ and x_λ is a fuzzy δ_p-cluster point of u, there is $G \in \mathcal{F}^*$ such that $G >> F$ (i.e., $G \leq F$) and $G^*\, q\, A$. Hence $F\, q\, A$. Thus $x_\lambda \leq \delta\text{-pcl } F = F$, for each $F \in \mathcal{F}$, i.e., $\bigcap \mathcal{F} \neq 0_X$. Hence by Theorem 2.11 ((g) $\Rightarrow$ (a)), X becomes fuzzy δ_p-almost compact.

Definition 2.15 — A fuzzy cover u of fuzzy δ-preclosed sets of an fts X will be called a fuzzt δ_p-cover of X if for each fuzzy point $x_\alpha (0 < \alpha < 1)$ in X there is some $U \in u$ such that U is a fuzzy δ-preopen nbd of x_α.

Theorem 2.16 — *An fts X is fuzzy δ_p-almost compact iff every fuzzy δ_p-cover of X has a finite subcover.*

PROOF : Let X be fuzzy δ_p-almost compact and u be any fuzzy δ_p-cover of X. Then for each $n \in N$ (= the set of naturals) with $n > 1$, there are a $U_x^n \in u$ and a δ-preopen set V_x^n in X such that $x_{1-1/n} \leq V_x^n \leq U_x^n$. Then $V_x^n(x) = 1 - \frac{1}{n}$ and hence $\sup_{n \in N} V_x^n(x) = 1$. It then follows that $\{V_x^n : x \in X, n \in N, n > 1\}$ is a fuzzy δ-preopen cover of X. Since X is fuzzy δ_p-almost compact, there exist finitely many points $x_1, \ldots, x_m \in X$ and $n_1, n_2, \ldots, n_m \in N \setminus \{1\}$ such that $1_x = \bigcup_{i=1}^{m} \delta - pcl\ V_{x_i}^{n_i}$.

Conversely, let u be a fuzzy δ-preopen cover of X. For any fuzzy point $x_\alpha (0 < \alpha < 1)$ in X, as $\sup_{U \in u} U(x) = 1$, there exists some $U_{x_\alpha} \in u$ such that $U_{x_n}(x) \geq \alpha$ (since $0 < \alpha < 1$). Thus $\{\delta\text{-pcl } U : U \in u\}$ is a fuzzy cover of X and the rest is clear.

The following theorem gives a necessary condition for an fts to be fuzzy δ_p-almost compact.

Theorem 2.17 — *If an fts X is fuzzy δ_p-almost compact, then every prefilterbase on X with atmost one fuzzy p-adherent point is p-convergent.*

PROOF : Let $\mathcal{F}$ be a prefilterbase with at most one p-adherent point in a fuzzy δ_p-almost compact fts X. Then by Theorem 2.11, $\mathcal{F}$ has at least one p-adherent point. Let x_α be the unique p-adherent point of $\mathcal{F}$ and if possible, let $\mathcal{F}$ do not p-converge to x_α. Then for some fuzzy δ-preopen q-nbd U of x_α and for each $F \in \mathcal{F}, F \not\leq \delta$-pcl U, so that $F \cap (1\text{-}\delta\text{-pcl } U) \neq 0_X$. Then $\mathcal{G} = \{F \cap (1\text{-}\delta\text{-pcl U}) : F \in \mathcal{F}\}$ is a prefilterbase in X and hence has a fuzzy p-adherent point y_t (say) in X. Now, δ-pcl $U = G$, for all $G \in \mathcal{G}$ so that $x_\alpha \neq y_t$. Again, for each fuzzy δ-preopen q-nbd V of y_t and and each $F \in \mathcal{F}$, δ-pcl $V \, q \, (F \cap (1 - \delta\text{-pcl } U)) \Rightarrow \delta$-pcl $V \, q \, F \Rightarrow y_t$ is a fuzzy p-adherent point of $\mathcal{F}$, where $x_\alpha \neq y_t$. This contradicts the fact that x_α is the only p-adherent point of $\mathcal{F}$.

3. Fuzzy δ_p-Almost Compactness : Some Associated Results

Having characterized δ_p-almost compactness of fuzzy sets and fts's in the last section, we now turn our attention to some pertinent results that follow.

Theorem 3.1 — *Let X be an fts.*

(a) If A is a fuzzy δ_p-almost compact set, then so is δ-pcl A.

(b) Union of two fuzzy δ_p-almost compact fuzzy sets is also so.

(c) If X is fuzzy δ_p-almost compact, then every fuzzy regularly δ_p-closed set A in X is a fuzzy δ_p-almost compact set.

PROOF (a) : Let u be a fuzzy δ-preopen cover of δ-pcl A. Then u is also a fuzzy δ-preopen cover of A and hence by fuzzy δ_p-almost compactness of A, there exists a finite subset u_0 of u such that $A \leq \bigcup\{\delta\text{-pcl } U : U \in u_0\} = \delta\text{-pcl } [\bigcup\{U : U \in u_0\}]$. Thus δ-pcl $A \leq \bigcup \{\delta\text{-pcl } U : U \in u_0\}$ (since δ-preclosure is an idempotent operator). Hence δ-pcl A is also a fuzzy δ_p-almost compact set.

(b) Obvious.

(c) Let $u = \{U_\alpha : \alpha \in \wedge\}$ be a fuzzy δ-preopen cover of a fuzzy regularly δ_p-closed set A in X. Since for each $x \notin \text{supp } A$, $(1 - A)(x) = 1$, $u \bigcup \{(1 - A)\}$ is a fuzzy δ-preopen cover of X. Since X is fuzzy δ_p-almost compact, there exist finitely many sets $U_1, \ldots, U_n$ in u such that $[(\delta\text{-pcl } U_1) \bigcup \cdots \bigcup (\delta\text{-pcl } U_n)] \bigcup [\delta\text{-pcl } (1 - A)] = 1_X$. We claim that $\delta\text{-pint } A \leq (\delta\text{-pcl } U_1) \bigcup \cdots \bigcup (\delta\text{-pcl } U_n)$. If not, there exists a fuzzy point $x_\beta \leq \delta$-pint A such that $\beta > \max [(\delta\text{-pcl } U_1)(x), .., (\delta\text{-pcl } U_n)(x)]$. Since $[(\delta\text{-pcl } U_1) \bigcup \cdots \bigcup (\delta\text{-pcl } U_n)] \bigcup (1 - \delta\text{-pint } A) = 1_X$, we have $(1\text{-}\delta\text{-pint } A)(x) = 1$ so that $(\delta\text{-pint } A)(x) = 0$, a contradiction. Hence our claim is established. Then $A = \delta\text{-pcl } (\delta\text{-pint } A) \leq [(\delta\text{-pcl } U_1) \bigcup \cdots \bigcup (\delta\text{-pcl } U_n)]$ (as $\delta\text{-pcl } (\delta\text{-pcl } U_i) = \delta\text{-pcl } U_i$) so that A becomes a fuzzy δ_p-almost compact set.

In order to ascertain the type of functions under which fuzzy δ_p-almost compactness remains invariant, we recall the following definition and the result thereafter from [2].

Definition 3.2 — A function $f : X \to Y$ (where X and Y are fts's) is said to be fuzzy

δ^*-almost continuous if the inverse image of every fuzzy δ-preopen set in Y is fuzzy δ-preopen in X.

Lemma 3.3 — For a function $f: X \to Y$, the following are equivalent :

(a) f is fuzzy δ^*-almost continuous.

(b) For each fuzzy set A in X, $f(\delta\text{-pcl } A) \leq \delta\text{-pcl } f(A)$.

(c) For each fuzzy point x_α in X and for each fuzzy δ-preopen q-nbd V of $f(x_\alpha)$ in Y, there is a fuzzy δ-preopen q-nbd W of x_α in X such that $f(W) \leq V$.

Theorem 3.4 — *Fuzzy δ^*-almost continuous image of a fuzzy δ_p-almost compact space is fuzzy δ_p-almost compact.*

PROOF : Let f be a fuzzy δ^*-almost continuous surjection from a fuzzy δ_p-almost compact space X to an fts Y, and let v be a fuzzy δ-preopen cover of Y. Let $x \in X$ and $f(x) = y$. Since sup $\{V(y) : V \in v\} = 1$, for each $n \in N$ (= the set of natural numbers) there exists some $V_n^x \in v$ with $V_n^x(y) > 1 - \frac{1}{n}$, so that $y_{1/n}\, q\, V_n^x$. By fuzzy δ^*-almost continuity of f, we have using Lemma 3.3 (c) that $f(U_n^x) \leq V_n^x$, for some fuzzy δ-preopen set U_n^x in X, q-coincident with $x_{1/n}$. Since $U_n^x(x) > 1 - \frac{1}{n}$, sup $\{U_n^x(x) : n \in N\} = 1$. Then $u = \{U_n^x : n \in N$ and $x \in X\}$ is a fuzzy δ-preopen cover of X. By fuzzy δ_p-almost compactness of X, $\bigcup_{i=1}^{k} \delta-pcl\,(U_{n_i}^{x_i}) = 1_X$, for some finite subcollection $\left\{U_{n_1}^{x_1}, \ldots, U_{n_k}^{x_k}\right\}$ of u. Then $1_Y = f\left(\bigcup_{i=1}^{k} \delta-pcl\,(U_{n_i}^{x_i})\right) = \bigcup_{i=1}^{k} f(\delta-pcl\, U_{n_i}^{x_i}) \leq \bigcup_{i=1}^{k} \delta\text{ -pcl } [(U_{n_i}^{x_i})]$ (by Lemma 3.3) $\leq \bigcup_{i=1}^{k} \delta-pcl\, V_{n_i}^{x_i}$ and hence Y becomes fuzzy δ_p-almost compact.

The concept of fuzzy almost compact space was introduced by Diconcilio and Gerla [4] and subsequently many others investigated such a space more thoroughly. As defined in [4], an fts X is called fuzzy almost compact if every fuzzy open cover u of X has a finite subcollection u_0 such that $\bigcup$ (cl $U : U \in u_0\} = 1_X$. It then follows at once that

Theorem 3.5 — *Every fuzzy δ_p-almost compact space is fuzzy almost compact.*

Remark 3.6 : That the converse of the above theorem is false can be observed from the simple fact that in the particular setting of a topological space, such a converse fails. For example, one may consider an infinite set with cofinite topology. Then the space is compact and hence obviously almost compact, but not δ_p-almost compact because the collection of δ-preopen sets in X becomes the power set of X.

It is now our aim to formulate a condition under which the aforesaid converse may hold. To this end, we observe that under the axiom of regularity on a topological space, the concepts of compactness coincides with some of its weaker forms like almost compactness and near compactness. Here we shall employ a sort of fuzzy separation axiom introduced in[2], to achieve the desired result. We need to recall the following definition and result from[2].

Definition 3.7 — An fts X is said to be fuzzy δ-preregular if for each fuzzy δ-preclosed set F in X and each fuzzy point x_α with $x_\alpha \underline{q}$ (1-F), there exist a fuzzy open set U and a fuzzy δ-pre-open set V such that $x_\alpha q\, U, F \leq V$ and $U \underline{q}\ V$.

Lemma 3.8 — An fts X is fuzzy δ-preregular iff for each fuzzy δ-preclosed set F in X, $\bigcap$ {cl $V : F \leq V$ and V is fuzzy δ-preopen in X} = F.

We are now in a position to prove the converse of Theorem 3.5 as follows.

Theorem 3.9 — *A fuzzy δ-preregular, almost compact fts X is fuzzy δ_p-almost compact.*

PROOF : In a fuzzy δ-preregular space we have by Lemma 3.8 that every fuzzy δ-preclosed set is fuzzy closed, i.e., every fuzzy δ-preopen set is fuzzy open. Thus if u is a fuzzy δ-preopen cover of X, then it is a fuzzy open cover of X. By fuzzy almost compactness of X, there is a finite subset u_0 of u such that $\bigcup$ {cl $U : U \in u_0$} = $\bigcup$ {δ-pcl $U : U \in u_0$} = 1_X, proving X to be fuzzy δ_p-almost compact.

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