*Indian J. pure appl. Math.*, **31**(5) : 541-544, May 2000


# ON FUZZY ALMOST COMPLETELY SEMI-CONTINUOUS FUNCTIONS

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(*Received* 9 *September* 1998; *after final revision* 18 *June* 1999; *Accepted* 5 *November* 1999)

The aim of this paper is to introduce the concepts of fuzzy almost completely semi-continuous functions as a generalization of fuzzy completely semi-continuous functions introduced by the author in an earlier paper. A comparative study regarding the interrelations among the fuzzy completely semi-continuous Fcsc, fuzzy almost continuous Fac and fuzzy continuous Fc functions along with fuzzy almost completely semi-continuous Facsc functions is made. Preservation of some fuzzy topological structures are examined under these functions. Finally, a few situations are mentioned where these functions may be applied.

**Key Words : Fuzzy Topology; Fuzzy Almost Completely Semi-continuous; Fuzzy S-closedness; Fuzzy Nearly Compact; Fuzzy Regular Open; Fuzzy Regular Semi Open Set**

## INTRODUCTION

The concept of fuzzy completely semi-continuous functions was introduced by the author[6]. This paper is devoted to the introduction and study of the concept of fuzzy almost completely semi continuous function. In section 2 it is seen that fuzzy completely semi-continuity implies fuzzy almost completely semi continuity but not conversely. Our endeavour is to correlate the fuzzy completely semi-continuity[6] fuzzy almost continuity[1] and fuzzy continuity[3] with fuzzy almost completely semi-continuity so as to determine their mutual relationships under suitably defined conditions. Finally the preservation of some fuzzy topological structures are examined under these functions.

Let $X$ be a non-empty (ordinary) set and $I$ the closed unit interval [0, 1]. We denote by $F_R$ (resp $F_\phi$), the fuzzy topology on $X$ which has the set of all fuzzy regular open subsets (resp. of all fuzzy regular semi open subsets) as a subbase. Some definitions and results which will be needed in this paper are recalled here;

*Definitions* 1.1 — (a) A fuzzy subset $\lambda$ of a fuzzy topological space (ft space $(X, F)$ is called

(i) fuzzy regular open [1] iff $\lambda = \text{Int}\ (\text{Cl}\ \lambda)$.

(ii) Fuzzy regular semi open [8] iff there exists a fuzzy regular open set $\alpha$ of $X$ such that $\alpha \leq \lambda \leq Cl\ (\alpha)$.

(b) Let $f: (X, F) \to (Y, K)$ be a function between two ft spaces. Then $f$ is called

(i) Fuzzy continuous [3] iff $f^{-1}(\lambda) \in F$ for each $\lambda \in K$.

(ii) Fuzzy almost continuous [1] iff $f^{-1}(\lambda) \in F$ for each fuzzy regular set $\lambda$ of $Y$.

(iii) Fuzzy completely semi-continuous[6] iff $f^{-1}(\lambda)$ is fuzzy regular semi open in $X$ for each $\lambda \in K$.

(iv) Fuzzy faintly continuous[5] iff $f^{-1}(\lambda) \in F$ for each fuzzy $\theta$-open subset $\lambda$ in $Y$.

*Definition* 1.2 — A fuzzy topological space $(X, F)$ is called

(i) Fuzzy s-closed[7], iff every fuzzy cover of $X$ by fuzzy regular semi open subsets has a finite subcover.

(ii) Fuzzy nearly compact[4], iff every fuzzy regular open cover of $X$ has a finite sub cover.

(iii) Fuzzy extremally disconnected (FED)[2] if the closure of each fuzzy open subset of $X$ is fuzzy open.

## 2. Fuzzy Almost Completely Semi-Continuous Functions

In this section a new class of functions - fuzzy almost completely semi-continuous functions is introduced as a generalization of fuzzy completely semi-continuous functions. Some of its properties are also studied.

*Definition* 2.1 — A function $f: (X, F) \to (Y, K)$ from a ft space $(X, F)$ to another ft space $(Y, K)$ is called Facsc iff $f^{-1}(\alpha)$ is a fuzzy regular semi open subset of $X$ for each fuzzy regular open subset $\alpha$ of $Y$. Equivalently, $f: (X, F) \to (Y, K)$ is Facsc iff $f^{-1}\beta$ is a fuzzy regular semi-closed subset of $X$ for each fuzzy regular closed subset $\beta$ of $Y$.

Every Fcsc function is Facsc but the converse is not true which can be seen from the following example:

*Example* 2 — Consider fuzzy subsets $\mu_1$ and $\mu_2$ of a closed unit interval $I = [0, 1]$, which are defined as follows:

$$\mu_1 = 0, 0 \leq x \leq 1/2$$

$$2x - 1, 1/2 \leq x \leq 1,$$

$$\mu_2(x) = 1, 0 \leq x \leq 1/4$$

$$-4x + 2, 1/4 \leq x \leq 1/2$$

$$0, 1/2 \leq x \leq 1.$$

We take $F = \{0, \mu_2, \mu_2, \mu_1 \cup \mu_2, 1\}$ then $(I, F)$ is a fuzzy topological spaces. Let $f : (I, F) \to (I, F)$ be a function defined by $f(x) = x$ for all $x \in I$. Here, $f^{-1}(0) = 0, f^{-1}(1) = 1$, $f^{-1}(\mu_1) = \mu_1, f^{-1}(\mu_2) = \mu_2, f^{-1}(\mu_1 \cup \mu_2) = \mu_1 \cup \mu_2$.

Here both $\mu_1$ and $\mu_2$ are fuzzy regular open in $(I, F)$ but $\mu_1 \cup \mu_2$ is not fuzzy regular open on $(I, F)$. Clearly, $f$ is not Fcsc, Since $f^{-1}(\mu_1 \cup \mu_2) = \mu_1 \cup \mu_2$ is not fuzzy regular semi open in $(I, F)$., But $f$ is Facsc.

In Example 2.1 of [6] it was shown that every fuzzy completely continuous Fcc function is Fcsc but not conversely. Thus we have,

$$\text{Fcc} \to \text{Fcsc} \to \text{Facsc}$$

*Conditions for equivalence of fuzzy almost completely semi-continuous functions with other fuzzy functions:*

We study the conditions for equivalence of Facsc functions with other fuzzy functions. Let $(X, F)$ be a ft space, $F_R$ and $F_\phi$ be the fuzzy topologies on $X$ defined above.

***Theorem*** 2.3 — *If $f$ is a function from a ft space $(X, F)$ to another ft space $(Y, K)$ then the following are equivalent :*

*(a) $f: (X, F) \to (Y, K)$ is a Facsc function.*

*(b) $f: (X, F) \to (Y, K_R)$ is a Fcsc function.*

*(c) $f: (X, F_\phi) \to (Y, K)$ is a Fac function.*

*(d) $f: (X, F_\phi) \to (Y, K_R)$ is a Fc function.*

PROOF : (a) →(b) Let $\alpha$ be a fuzzy open subset of $(Y, K_R)$ then $\alpha$ is a fuzzy regular open subset of $(Y, K)$. By (a) $f^{-1}(\alpha)$ is fuzzy regular semi open in $(X, F)$, which shows that $f: (X, F) \to (Y, K_R)$ is Fcsc.

(b) →(c) Let $\beta$ be a fuzzy regular open subset of $(Y, K)$ then $\beta$ is fuzzy open in $(Y, K_R)$. By (b) $f^{-1}(\beta)$ is fuzzy regular semi open in $(X, F)$. Thus $f^{-1}(\beta)$ is fuzzy open in $(X, F_\phi)$, which shows that $f: (X, F_\phi) \to (Y, K)$ is a Fac function.

(c) → (d) Let $\gamma$ be a fuzzy open subset in $(Y, K_R)$ then $\gamma$ is fuzzy regular open in $(Y, K)$. By (c) $f^{-1}(\gamma)$ is fuzzy in $(X, F_\phi)$ which shows that $f: (X, F_\phi) \to (Y, K_R)$ is a Fc function.

(d) → (a) Let $\mu$ be a fuzzy regular open subset of $(Y, K)$ then $\mu$ is fuzzy open in $(Y, K_R)$. By (d) $f^{-1}(\mu)$ is fuzzy open in $(X, F_\phi)$ i.e. $f^{-1}(\mu)$ is fuzzy regular semi open in $(X, F)$, which shows that $f: (X, F) \to (Y, K)$ is a Facsc function.

It was proved in [2] that in a FED space $(X, F)$ every fuzzy regular open subset of $X$ is fuzzy $\theta$-open. Now, we have the following theorem.

***Theorem*** 2.4 — *If a function $f: (X, F) \to (Y, K)$ be a fuzzy almost completely semi-continuous and $Y$ is a fuzzy extremally disconnected space then $f: (X, F_\phi) \to (Y, K)$ is a fuzzy faintly continuous function.*

PROOF : $f: (X, F) \to (Y, K)$ be a facsc function and $Y$ is a FED space. Let $\alpha$ be a regular open subject in $(Y, K)$ then $\alpha$ is fuzzy $\theta$-open in $(Y, K)$. Now $f^{-1}(\alpha)$·is fuzzy regular semi open in $(X, F)$, hence $f^{-1}(\alpha)$ is fuzzy open in $(X, F_\phi)$. Thus $f: (X, F_\phi) \to (Y, K)$ is a fuzzy faintly continuous function.

*Definition* 2.5 — A function $f: (X, Y) \to (Y, K)$ from ft space $(X, F)$ to another ft space $(Y, K)$ is said to be fuzzy almost semi-open if the image of every fuzzy regular semi-open ubset is fuzzy open.

Now we have the following theorem :-

***Theorem*** 2.6 — *If $f: (X, F) \to (Y, F_1)$ is a onto fuzzy almost semi open and Facsc function and $g: (Y, F_1) \to (Z, F_2)$ is a function such that $g$ of is Facse than $g$ is Fac.*

PROOF : Let $\alpha$ be any fuzzy regular open subset of $Z$ then $(g \circ f)^{-1}(\alpha)$ is a fuzzy regular semi-open subset of $X$. But $f(g \circ f)^{-1}(\alpha) = g^{-1}(\alpha)$ is a fuzzy open subset of $Y$. Hence $g$ is a Fac function.

***Theorem* 2.7** — *If $f: X \rightarrow Y$ be a Facsc onto function and $X$ is a fuzzy s-closed space, then $Y$ is a fuzzy nearly compact space.*

PROOF : Let $\{\alpha_i : i \in \Lambda\}$ be a fuzzy regular open cover of $Y$, then $f^{-1}(\alpha_i) : i \in \Lambda)\}$ is a fuzzy regular semi open cover of $X$. By fuzzy s-closedness of $X$ there is a finite subfamily of $(f^{-1}(\alpha_i))$ such that $\bigcup_{j=1}^{n} (f^{-1}(\alpha_{ij})) = 1_X$.

Now $1_Y = f(1_X) = f\left(\bigcup_{j=1}^{n} f^{-1}(\alpha_{ij})\right) \subset \bigcup_{j=1}^{n} \alpha_{ij}$; which implies that $Y$ is a fuzzy nearly compact space.

## ACKNOWLEDGEMENT

The author is grateful to the referee for his valuable comments which helped in the modification of the paper.

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