Indian J. pure appl. Math., 31(6) : 675-686, June 2000

OPERATIONAL CALCULUS IN TWO VARIABLES AND SPECIAL FUNCTIONS

V. B. L. CHAURASIA AND ANJU GODIKA

Department of Mathematics, University of Rajasthan, Jaipur 302 004, India

(*Received* 26 *November* 1998; *accepted* 17 *May* 1999)

In this paper, we establish few operational relations between the original and the image for two dimensional Laplace transforms whose kernel involve the product of the general polynomials $S_{m_1, \ldots, m_R}^{l_1, \ldots, l_R}(x_1, \ldots, x_R)$, a general class of multivariable polynomials $S_{N_1}^{M_{r+1}, \ldots, M_{r'}}(-y_{r+1}, \ldots, -y_{r'})$, Fox's H-function and the multivariable H-function. A unification of the bivariate Laplace transform for the H-function given by Chaurasia[2, 3] is provided by the results established here. The importance of the present study lies in the fact that it unifies and extends the results of a large number of authors.

1. INTRODUCTION

The integral equation (Ditkin and Prudnikov[5])

$$F(w, v) = wv \int_0^\infty \int_0^\infty \exp(-wx - vy) f(x, y)\, dx\, dy, \ \mathrm{Re}(w, v) > 0 \qquad \ldots (1.1)$$

represents the Laplace-Carson transforms of a function $f(x, y)$.

$F(w, v)$ and $f(x, y)$ are said to be operationally related to each other, $F(w, v)$ is called the image and $f(x, y)$ the original.

Symbolically we can write

$$F(w, v) \doteqdot f(x, y) \text{ or } f(x, y) \doteqdot F(w, v). \qquad \ldots (1.2)$$

The series representation of Fox's H-function (Braaksma[1]; and Fox[6]) is

$$H_{P,Q}^{M,N}\left[z \,\middle|\, \begin{matrix} (e_p, E_p) \\ (f_Q, F_Q) \end{matrix} \right] = \sum_{G=0}^{\infty} \sum_{g=1}^{M} \frac{(-1)^G}{G!\, F_g} \phi(\eta_G)\, z^{\eta_G}, \qquad \ldots (1.3)$$

where

$$\phi(\eta_G) = \prod_{j=1, j\neq g}^{M} \Gamma(f_j - F_j \eta_G) \prod_{j=1}^{N} \Gamma(1 - e_j + E_j \eta_G)$$
$$\times \left\{ \prod_{j=1+M}^{Q} \Gamma(1 - f_j + F_j \eta_G) \prod_{j=1}^{P} \Gamma(e_j - E_j \eta_G) \right\}^{-1}$$

and

$$\eta_G = (f_g + G)/F_g. \qquad \ldots (1.4)$$

Srivastava and Panda[12] have defined the H-function of several complex variables (see also Srivastava *et al.*[11]).

$$H^{O, \lambda ; (u', v') ; \ldots ; (u^{(r)}, v^{(r)})}_{A, C : [B', D'] ; \ldots : [B^{(r)}, D^{(r)}]} \left(\begin{matrix} [(a) : \theta' : \ldots : \theta^{(r)}] : [(b') : \phi'] : \ldots : [(b^{(r)}) : \phi^{(r)}] : \\ [(c) : \psi' : \ldots : \psi^{(r)}] : [(d') : \delta'] : \ldots : [(d^{(r)}) : \delta^{(r)}] : \end{matrix} \; z_1, \ldots, z_r \right) \qquad \ldots (1.5)$$

The special cases of the above function, convergence condition, its defining integral and other details can be found in the paper referred to above.

For the sake of brevity, we write :

$$T_i = -\sum_{j=\lambda+1}^{A} \theta_j^{(i)} + \sum_{j=1}^{v^{(i)}} \phi_j^{(i)} - \sum_{j=v^{(i)}+1}^{B^{(i)}} \phi_j^{(i)}$$
$$-\sum_{j=1}^{C} \psi_j^{(i)} + \sum_{j=1}^{u^{(i)}} \delta_j^{(i)} - \sum_{j=u^{(i)}+1}^{D^{(i)}} \delta_j^{(i)} > 0 \;, \; i = 1, \ldots, r, \qquad \ldots (1.6)$$

$$\Delta_i = d_j^{(i)} / \delta_j^{(i)}, j = 1, \ldots, u^{(i)}, \qquad \ldots (1.7)$$

$$\nabla_i = (1 - b_j^{(i)}) / \phi_j^{(i)}, j = 1, \ldots, v^{(i)}, \; i = 1, \ldots, r, \qquad \ldots (1.8)$$

$$T = \sum_{j=1}^{N} E_j - \sum_{j=N+1}^{P} E_j + \sum_{j=1}^{M} F_j - \sum_{j=M+1}^{Q} F_j, \qquad \ldots (1.9)$$

$$\theta = f_l / F_l, \; l = 1, \ldots, M$$

and

$$\phi = (e_{l'} - 1)/E_{l'}, \; l' = 1, \ldots, N. \qquad \ldots (1.10)$$

We write the expression (1.5) in the following abbreviated form by using a contracted notation

$$H^{O,\,\lambda\,:\,(u',\,v')\,:\,\ldots\,:\,(u^{(r)},\,v^{(r)})}_{A,\,C\,:\,[B',\,D']\,:\,\ldots\,:\,[B^{(r)},\,D^{(r)}]}\,(z_1,\,\ldots,\,z_r). \qquad \ldots (1.11)$$

Srivastava[8] has defined and introduced the general polynomials

$$S^{l_1,\,\ldots,\,l_R}_{m_1,\,\ldots,\,m_R}\;(x_1,\,\ldots,\,x_R) = \sum_{\alpha_1=0}^{[m_1/l_1]} \cdots \sum_{\alpha_R=0}^{[m_R/l_R]} \frac{(-m_1)_{l_1\alpha_1}}{\alpha_1\,!},\,\ldots,$$

$$\frac{(-m_R)_{l_R\alpha_R}}{\alpha_R\,!}\;A_{m_1\alpha_1},\,\ldots,A_{m_R\,\alpha_R}\;x_1^{\alpha_1},\,\ldots,\,x_R^{\alpha_R}, \qquad \ldots (1.12)$$

where $l_1, \ldots, l_R$ are arbitrary positive integers and the coefficient $A_{m_1\alpha_1}, \ldots, A_{m_R\alpha_R}$ are arbitrary constants, real or complex.

A general class of multivariable polynomials (Srivastava *et al.*[10]) are defined as follows

$$S^{M_{r+1},\,\ldots,\,M_{r'}}_{N_1}\;[-y_{r+1},\,\ldots,\,-y_{r'}]\;M_{r+1}\,\alpha_{r+1}+\ldots+M_{r'}\,\alpha_{r'}\leq N_1$$

$$= \sum_{\alpha_{r+1},\,\ldots,\,\alpha_{r'}=0} (-N_1)_{M_{r+1}\,\alpha_{r+1}+\ldots+M_{r'}\,\alpha_{r'}}\,.$$

$$\times\; A_1(N_1 : \alpha_{r+1},\,\ldots,\,\alpha_{r'})\,\frac{(-y_{r+1})^{\alpha_{r+1}}}{\alpha_{r+1}\,!}\cdots\frac{(-y_{r'})^{\alpha_{r'}}}{\alpha_{r'}\,!}, \qquad \ldots (1.13)$$

where $M_{r+1}, \ldots, M_{r'}$ are arbitrary positive integers and the coefficients $A_1(N_1\,;\,\alpha_{r+1}, \ldots, \alpha_{r'})$, $(N_1\,;\,\alpha_{i'} \geq 0,\ i' = r+1, \ldots, r')$ are arbitrary constants, real or complex.

The main theorem of this paper provides a key formula from which we get many other theorems by specializing the parameters.

If we take $R = 1$ in eq. (1.12) and denote $A[m_1, \alpha_1, \ldots, m_R, \alpha_R]$ thus obtained by $A_{m_1\,\alpha_1}$, we arrive at the general class of polynomials $S^{l_1}_{m_1}\,[x_1]$ given by Srivastava[13].

By suitably specializing the coefficients $A_{m_1,\,\alpha_1}$ the polynomials $S^{l_1}_{m_1}\,[x_1]$ can be reduced to other classical orthogonal polynomials. Similarly we obtain a special case of a general class of multivariable polynomials by specializing the parameters.

We denote the transformed variables by w and v and the original variables by x and y. The notation employed are those of Ditkin and Prudnikov's operational calculus.

2. The Main Theorem

***Theorem* 1** — *With $T_i, \Delta_i, \nabla_i, T, \theta$ and ϕ as given by (1.6) through (1.10), let $T_i > 0, T > 0, |\arg(z_i)| < T_i \frac{\pi}{2}, |\arg(z)| < \frac{T\pi}{2}, h_i > 0,\ h > 0,\ h_{i'}' > 0, k_{i''} > 0, i = 1, \ldots, r,\ i' = r+1, \ldots, r',\ i'' = 1, \ldots, R, l_1, \ldots, l_R$ and $M_{r+1}, \ldots, M_{r'}$ be arbitrary positive integers and the coefficients $A_1(N_1 : \alpha_{r+1}, \ldots, \alpha_{r'})$ and $A_{m_1\alpha_1}, \ldots, A_{m_R\alpha_R}$ be arbitrary constants, real or complex, and*

$$(i)\ Re\left(\gamma - \beta - h\phi - \sum_{i=1}^{r} h_i \nabla_i\right) < \frac{3}{4},$$

$$(ii)\ Re(\gamma) > 0, \mathrm{Re}\left(\beta + h\theta + \sum_{i=1}^{r} h_i \Delta_i\right) > 0;$$

also let

$$0 \le N \le p, 0 \le M \le Q.$$

Then we have, for

$$Re(w) \ge 0,$$

$$W^{-\frac{1}{2}} (wv)^{\frac{\beta}{2} - \gamma + 1} \sum_{G=0}^{\infty} \sum_{g=1}^{M} \sum_{\alpha_1=0}^{[m_l/l_l]} \cdots \sum_{\alpha_R=0}^{[m_R/l_R]}$$

$$\times \sum_{\alpha_{r+1}, \ldots, \alpha_{r'}=0}^{M_{r+1}\alpha_{r+1} + \ldots + M_{r'}\alpha_{r'} \le N_1} \frac{(-1)^G}{G!\,F_g} \phi(\eta_G)\, z^{\eta_G} \frac{(-m_1)_{l_1\alpha_1}}{\alpha_1!}, \ldots,$$

$$\frac{(-m_R)_{l_R\alpha_R}}{\alpha_R!} A_{m_1\alpha_1}, \ldots, A_{m_R\alpha_R} (-1)^{h_{r+1}\alpha_{r+1} + \ldots + h_{r'}\alpha_{r'}}$$

$$\times \frac{(-N_1)_{M_{r+1}\alpha_{r+1} + \ldots + M_{r'}\alpha_{r'}}}{(\alpha_{r+1})!, \ldots, (\alpha_{r'})!} A_1(N_1 ; \alpha_{r+1}, \ldots, \alpha_{r'})$$

$$\times (wv)^{\frac{(h\eta_G + h_{r+1}\alpha_{r+1} + \ldots + h_{r'}\alpha_{r'} + k_1\alpha_1 + \ldots + k_R\alpha_R)}{2}}$$

$$\times H_{A, C : [B', D'] ; \ldots ; [B^{(r)}, D^{(r)}]}^{0, 0 : (u', v') ; \ldots ; (u^{(r)}, v^{(r)})} \left(z_1 (\sqrt{wv})^{h_1}, \ldots, z_2 (\sqrt{wv})^{h_r} \right)$$

$$\doteqdot \frac{(4xy)^{\gamma-\frac{\beta}{2}-\frac{1}{2}}}{\sqrt{\pi y}} \sum_{G=0}^{\infty} \sum_{g=1}^{M} \sum_{\alpha_1=0}^{[m_1/l_1]}, \ldots, \sum_{\alpha_R=0}^{[m_R/l_R]}$$

$$\times \sum_{\alpha_{r+1}, \ldots, \alpha_{r'}=0}^{M_{r+1}\alpha_{r+1}+\ldots+M_{r'}\alpha_{r'} \le N_1} \frac{(-1)^G}{G!\,F_g} \phi(\eta_G)\, z^{\eta_G} \frac{(-m_1)_{l_a\alpha_1}}{\alpha_1!}, \ldots,$$

$$\frac{(-m_R)_{l_R\alpha_R}}{\alpha_R!} A_{m_1\alpha_1}, \ldots, A_{m_R\alpha_R} (-1)^{h_{r+1}\alpha_{r+1}+\ldots+h_{r'}\alpha_{r'}}$$

$$\times \frac{(-N_1)_{M_{r+1}\alpha_{r+1}+\ldots+M_{r'}\alpha_{r'}}}{(\alpha_{r+1})!, \ldots, (\alpha_{r'})!} A_1(N_1; \alpha_{r+1}, \ldots, \alpha_{r'})$$

$$\times 4xy\left(\frac{-h\,\eta_G - h_{r+1}\alpha_{r+1} - \ldots - h_{r'}\alpha_{r'} - k_1\alpha_1 - \ldots - k_R\alpha_R}{2}\right)$$

$$\times H^{0,0:(u',v'),\ldots,(u^{(r)},v^{(r)})}_{A+1,C:[B',D'],\ldots,[B^{(r)},D^{(r)}]} \left(\begin{array}{l} [(a):\theta':\ldots:\theta^{(r)}]: \\ [(c):\psi':\ldots:\psi^{(r)}]: \end{array} \right.$$

$$[2\gamma - b\eta - h^{\eta_G} - h_{r+1}\alpha_{r+1} - \ldots - h_{r'}\alpha_{r'} - k_1\alpha_1 - \ldots - k_R\alpha_R : h_1, \ldots, h_r]] :$$

$$\left. \begin{array}{l} [(b');\phi'];\ldots;[(b^{(r)}):\phi^{(r)}]; \\ \qquad\qquad z_1(2\sqrt{xy})^{-h_1}, \ldots, z_r(2\sqrt{xy})^{-h_r}; \\ [(d');\delta'];\ldots;[(d^{(r)}):\delta^{(r)}] \end{array} \right). \qquad \ldots (2.1)$$

3. PROOF

The Laplace transform of the product of Fox's H-function, a general class of multivariable polynomials, general polynomials and the multivariable H-function is given by :

$$L\{ t^{b\eta-1} S_{N_1}^{M_{r+1},\ldots,M_{r'}} [-t^{h_{r+1}}, \ldots, -t^{h_{r'}}]$$

$$\times S^{l_1,\ldots,l_R}_{m_1,\ldots,m_R} [t^{k_1}, \ldots, t^{k_R}]\, H^{M,N}_{P,Q}\left[zt^h \,\middle|\, \begin{array}{l} (e_P, E_P) \\ (f_Q, F_Q) \end{array} \right]$$

$$\times H^{0,0:(u',v'):\ldots:(u^{(r)},v^{(r)})}_{A,C+1:[B',D']:\ldots:[B^{(r)},D^{(r)}]} \left(\begin{array}{l} [(a):\theta',\ldots,\theta^{(r)}]: \\ [(c):\psi',\ldots,\psi^{(r)}]: \end{array} \right.$$

$$[1 - \beta h^{\eta_G} - h_{r+1}\alpha_{r+1} - \ldots - h_{r'}\alpha_{r'} - k_1\alpha_1 - \ldots - k_R\alpha_R : h_1, \ldots, h_r]$$

$$\left. \begin{matrix} [(b') : \phi'] , \ldots , [(b^{(r)}) : \phi^{(r)}] ; \\ \quad z_1 t^{h_1} , \ldots , z_r t^{h_r} \\ [(d') : \delta'] , \ldots , [(d^{(r)}) \delta^{(r)}] ; \end{matrix} \right)$$

$$= (w)^{-\beta} \sum_{G=0}^{\infty} \sum_{g=1}^{M} \sum_{\alpha_1=0}^{[m_1/l_1]} , \ldots , \sum_{\alpha_R=0}^{[m_R/l_R]}$$

$$\sum_{\alpha_{r+1}, \ldots, \alpha_{r'}=0}^{M_{r+1}\alpha_{r+1} + \ldots + M_{r'}\alpha_{r'} \leq N_1}$$

$$\frac{(-1)^G}{G!\,F_g} \phi(\eta_G) z^{\eta_G} \frac{(-m_1)_{l_1\alpha_1}}{\alpha_1 !}, \ldots, \frac{(-m_R)_{l_R\alpha_R}}{\alpha_R !}$$

$$\times A_{m_1\alpha_1}, \ldots, A_{m_R\alpha_R} (-1)^{h_{r+1}\alpha_{r+1} + \ldots + h_{r'}\alpha_{r'}}$$

$$\times \frac{(-N_1)_{M_{r+1}\alpha_{r+1} + \ldots + M_{r'}\alpha_{r'}}}{(\alpha_{r+1})!, \ldots, (\alpha_{r'})!} A_1 (N_1 : \alpha_{r+1}, \ldots, \alpha_{r'})$$

$$\times (w)^{(-h\eta_G - h_{r+1}\alpha_{r+1} - \ldots - h_{r'}\alpha_{r'} - k_1\alpha_1 - \ldots - k_R\alpha_R)}$$

$$\times H^{0,0:(u',v'):\ldots:(u^{(r)},v^{(r)})}_{A,C:[B',D']:\ldots:[B^{(r)},D^{(r)}]}\left(z_1 w^{-h_1}, \ldots, z_r w^{-h_r} \right), \qquad \ldots (3.1)$$

where

$$Re(w) > 0, \ \mathrm{Re}\left(\beta + h\theta + \sum_{i=1}^{r} h_1 \Delta_i \right) > 0,$$

$$Re\left(b\eta + h\phi + \sum_{i=1}^{r} h_i \nabla_i \right) < 0, h_i > 0, h > 0, h_{i'} > 0,$$

$$k_{i''} > 0, i = 1, \ldots, r, i' = r+1, \ldots, r', i'' = 1, \ldots, R,$$

$|arg(z_i)| < \frac{T_i\pi}{2}$, $|arg(z)| < \frac{T\pi}{2}$, $i = 1, \ldots, r$, $l_1, \ldots, l_R$ and $M_{r+1}, \ldots, M_{r'}$ are arbitrary positive integers and the coefficients $A_{m_1, \alpha_1}, \ldots, A_{m_R, \alpha_R}$ and $A_1(N_1 ; \alpha_{r+1}, \ldots, \alpha_{r'})$ are arbitrary constants, real or complex.

The result in (3.1) can be obtained with the help of (1.12), (1.13) and a result recently obtained by Chaurasia[3].

Now we replace w by $(wv)^{-\frac{1}{2}}$ and multiply both sides of (3.1) by $w^{-1/2}(wv)^{1-\gamma}$. Interpreting it with the help of a known result (Ditkin[5]), we get

$$(4xy)^{\frac{\gamma}{2}-\frac{1}{4}} \int_0^\infty t^{\beta-\gamma-\frac{1}{2}} J_{2\gamma-1}[64(xyt^2)]^{\frac{1}{4}}$$

$$\times S_{N_1}^{M_{r+1},\ldots,M_{r'}}[-t^{h_{r+1}},\ldots,-t^{h_{r'}}]$$

$$\times S_{m_1,\ldots,m_R}^{l_1,\ldots,l_R}[t^{k_1},\ldots,t^{k_R}]\, H_{P,Q}^{M,N}\left[zt^h \,\middle|\, \begin{matrix}(e_P, E_P)\\ (f_Q, F_Q)\end{matrix}\right]$$

$$\times H_{A,C+1:[B',D']:\ldots:[B^{(r)},D^{(r)}]}^{(0,0:(u',v'):\ldots:(u^{(r)},v^{(r)})}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}]:\\ [(c):\psi',\ldots,\psi^{(r)}],\end{matrix}\right.$$

$$[1-\sigma-\beta-h\eta_G-h_{r+1}\alpha_{r+1}-\ldots-h_{r'}\alpha_{r'}-k_1\alpha_1-\ldots-k_R\alpha_R : h_1,\ldots,h_r]:$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}];\\ [(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}];\end{matrix}\; z_1 t^{h_1},\ldots,z_r t^{h_r}\right) dt$$

$$\doteq w^{-\frac{1}{2}}(wv)^{\frac{\beta}{2}-\gamma+1} \sum_{G=0}^{\infty}\sum_{g=1}^{M}\sum_{\alpha_1=0}^{[m_1/l_1]},\ldots,\sum_{\alpha_R=0}^{[m_R/l_R]}$$

$$\times \sum_{\alpha_{r+1},\ldots,\alpha_{r'}=0}^{M_{r+1}\alpha_{r+1}+\ldots+M_{r'}\alpha_{r'}\le N_1} \frac{(-1)^G}{G!\,F_g}\,\phi(\eta_G)\, z^{\eta_G}\frac{(-m_1)_{l_1\alpha_1}}{\alpha_1!},\ldots,$$

$$\frac{(-m_R)_{l_R\alpha_R}}{\alpha_R!} A_{m_1\alpha_1},\ldots,A_{m_R\alpha_R}(-1)^{h_{r+1}\alpha_{r+1}+\ldots+h_{r'}\alpha_{r'}}$$

$$\times \frac{(-N_1)_{M_{r+1}\alpha_{r+1}+\ldots+M_{r'}\alpha_{r'}}}{(\alpha_{r+1})!,\ldots,(\alpha_{r'})!} A_1(N_1:\alpha_{r+1},\ldots,\alpha_{r'})$$

$$\times (wv)^{\left(\frac{h\eta_G+h_{r+1}\alpha_{r+1}+\ldots+h_r\alpha_{r'}+k_1\alpha_1+\ldots+k_R\alpha_R}{2}\right)}$$

$$\times H_{A,C:[B',D'];\ldots;[B^{(r)},D^{(r)}]}^{0,0:(u',v');\ldots;(u^{(r)},v^{(r)})}\left(z_1(\sqrt{wv})^{h_1},\ldots,z_r(\sqrt{wv})^{h_r}\right)$$

Now, evaluating left hand side of (3.2) by the process mentioned in (3.1), we obtain the described result.

4. PARTICULAR CASES

(*i*) If, we take

$$A_1(N_1 : \alpha_{r+1}, \ldots, \alpha_{r'}) = \frac{\prod_{j=1}^{E_1} (e_j)_{\alpha_{r+1}\theta_j^{r+1} + \ldots + \alpha_{r'}\theta_j^{r'}}}{\prod_{j=1}^{G_1} (g_j)_{\alpha_{r+1}\psi_j^{r+1} + \ldots + \alpha_{r'}\psi_j^{r'}}}$$

$$\times \frac{\prod_{j=1}^{U_1^{r+1}} (u_j^{r+1})_{\alpha_{r+1}\phi_j^{r+1}} \cdots \prod_{j=1}^{U_1^{r'}} (u_j^{r'})_{\alpha_{r'}\phi_j^{r'}}}{\prod_{j=1}^{V_1^{r+1}} (v_j^{r+1})_{\alpha_{r+1}\tau_j^{r+1}} \cdots \prod_{j=1}^{V_1^{r'}} (v_j^{r'})_{\alpha_{r'}\tau_j^{r'}}}$$

in (3.1) $S_{N_1}^{M_{r+1}, \ldots, M_{r'}} [-t^{h_{r+1}}, \ldots, -t^{h_{r'}}]$ reduces to the generalized Lauricella function of Srivastava and Daoust[9] as follows :

$$S_{N_1}^{M_{r+1}, \ldots, M_{r'}} [-t^{h_{r+1}}, \ldots, -t^{h_{r'}}] = F_{G_1 \,:\, V_1^{r+1}, \ldots, V_1^{r'}}^{1+E_1 \,:\, U_1^{r+1}, \ldots, U_1^{r'}}$$

$$\left[\begin{matrix} (N_1 : M_{r+1}, \ldots, M_{r'}), (e : \theta^{r+1}, \ldots, \theta^{r'}) : ((u^{r+1}) : \phi^{r+1}) ; \ldots ; \\ (g : \psi^{r+1}, \ldots, \psi^{r'}) : ((v^{r+1}) : \tau^{r+1}) ; \ldots ; \end{matrix} \right.$$

$$\left. \begin{matrix} ((u^{r'}) : \phi^{r'}) ; \\ ((v^{r'}) : \tau^{r'}) ; \end{matrix} \; -t^{h_{r+1}}, \ldots, -t^{h_{r'}} \right] \qquad \ldots (4.1)$$

Theorem 2 — *With* $T_i, \Delta_i, \nabla_i, T, \theta$ *and* ϕ *given by (1.6) through (1.10) respectively, let* $T_i > 0, T > 0 \,|\, arg\,(z_i) \,|\, < T_i \frac{\pi}{2}, |\, arg\,(z) \,|\, < \frac{T\pi}{2}, \;\; h_i > 0, h > 0, k_{i''} > 0, i = 1, \ldots, r, i'' = 1, \ldots, R, l_1, \ldots, l_R$ *be arbitrary positive integers and the coefficients* $A_{m_1\alpha_1}, \ldots, A_{m_R\alpha_R}$ *be arbitrary constants, real or complex reducing a general class of multivariable polynomials into the generalized Lauricella function*

(i) $Re\left(\gamma - \beta - h\phi - \sum_{i=1}^{r} h_i \nabla_i \right) < \frac{3}{4}$ and

(ii) $Re(\gamma') > 0, Re\left(\beta + h\theta + \sum_{i=1}^{r} h_i \Delta_i\right) > 0;$

also let $0 \le N \le P, 0 \le M \le Q$. Then we have for $Re(w) \ge 0$.

$$w^{-\frac{1}{2}} (wv)^{\frac{\beta}{2} - \gamma + 1} \sum_{G=0}^{\infty} \sum_{g=1}^{M} \sum_{\alpha_1=0}^{[m_1/l_1]}, \ldots, \sum_{\alpha_R=0}^{[m_R/l_R]}$$

$$\times \sum_{\alpha_{r+1}, \ldots, \alpha_{r'}=0}^{M_{r+1}\alpha_{r+1} + \ldots + M_{r'}\alpha_{r'} \le N_1} \frac{(-1)^G}{G!\,F_g} \phi(\eta_G) z^{\eta_G} \frac{(-m_1)_{l_1\alpha_1}}{\alpha_1!}, \ldots, \frac{(-m_R)_{l_R\alpha_R}}{\alpha_R!}$$

$$\times A_{m_1\alpha_1}, \ldots, A_{m_R\alpha_R} (-1)^{h_{r+1}\alpha_{r+1} + \ldots + h_{r'}\alpha_{r'}}$$

$$\times \frac{(-N_1)_{\alpha_{r+1}M_{r+1}, \ldots, \alpha_{r'}M_{r'}}}{(\alpha_{r+1})!, \ldots, (\alpha_{r'})!} I(\theta)$$

$$\times (wv)^{\left(\frac{h\eta_G + h_{r+1}\alpha_{r+1} + \ldots + h_{r'}\alpha_{r'} + k_1\alpha_1 + \ldots + k_R\alpha_R}{2}\right)}$$

$$\times H^{0,0:(u',v'):\ldots:(u^{(r)},v^{(r)})}_{A,C:[B',D']:\ldots:[B^{(r)},D^{(r)}]}\left(z_1(\sqrt{wv})^{h_1}, \ldots, z_r(\sqrt{wv})^{h_r}\right)$$

$$\doteqdot \frac{(4xy)^{\gamma - \frac{\beta}{2} - \frac{1}{2}}}{\sqrt{\pi y}} \sum_{G=0}^{\infty} \sum_{g=1}^{M} \sum_{\alpha_1=0}^{[m_1/l_1]}, \ldots, \sum_{\alpha_R=0}^{[m_R/l_R]}$$

$$\times \sum_{\alpha_{r+1}, \ldots, \alpha_{r'}=0}^{M_{r+1}\alpha_{r+1} + \ldots + M_{r} - \alpha_{r'} \le N_1} \frac{(-1)^G}{G!\,F_g} \phi(\eta_G) z^{\eta_G} \frac{(-m_1)_{l_1\alpha_1}}{\alpha_1!}, \ldots, \frac{(-m_R)_{l_R\alpha_R}}{\alpha_R!}$$

$$\times A_{m_1\alpha_1}, \ldots, A_{m_R\alpha_R} (-1)^{h_{r+1}\alpha_{r+1} + \ldots + h_{r'}\alpha_{r'}}$$

$$\times \frac{(-N_1)_{M_{r+1}\alpha_{r+1}, \ldots, M_{r'}\alpha_{r'}}}{(\alpha_{r+1})!, \ldots, (\alpha_{r'})!} I(\theta)$$

$$\times (4xy)^{\left(\frac{-h\eta_G - h_{r+1}\alpha_{r+1} - \ldots - h_{r'}\alpha_{r'} - k_1\alpha_1 - \ldots - k_R\alpha_R}{2}\right)}$$

$$H^{0,0:(u',v'):\ldots:(u^{(r)},v^{(r)})}_{A+1,C:[B',D']:\ldots:[B^{(r)},D^{(r)}]}\left(\begin{matrix}[(a):\theta', \ldots, \theta^{(r)}];\\ [(c):\psi', \ldots, \psi^{(r)}];\end{matrix}\right.$$

$$[2\gamma' - \beta - h\eta_G - h_{r+1}\,\alpha_{r+1} - \ldots - h_{r'}\,\alpha_{r'} - k_1\alpha_1 - \ldots - k_R\alpha_R : h_1, \ldots, h_r];$$

$$\left.\begin{matrix} [(b') : \phi'] ; \ldots ; [(b^{(r)}) : \phi^{(r)}] ; \\ \qquad z_1\,(2\sqrt{xy})^{-h_1}, \ldots, z_r\,(2\sqrt{xy})^{-h_r} \\ [(d') : \delta'] ; \ldots ; [(d^{(r)}) : \delta^{(r)}] ; \end{matrix}\right) . \qquad \ldots (4.2)$$

Theorem **3** — $L\Big\{ t^{\beta-1}\, S^{l_1, \ldots, l_R}_{m_1, \ldots, m_R}\,[t_{k_1}, \ldots, t^{k_R}]$

$$\times\, F^{1+E_1 : U_1^{r+1}, \ldots, U_1^{r'}}_{G_1 : V_1^{r+1}, \ldots, V_1^{r'}}\left[\begin{matrix} (-N_1 : M_{r+1}, \ldots, M_r), (e : \theta^{r+1}, \ldots, \theta^{r'}) : \\ (g : \psi^{r+1}, \ldots, \psi^{r'}) : \end{matrix}\right.$$

$$\left.\begin{matrix} ((u^{r+1}) : \phi^{r+1} ; \ldots ; ((u^{r'}) : \phi^{r'}) : \\ \qquad -t^{h_{r+1}}, \ldots, -t^{h_{r'}} \\ ((v^{r+1}) : \tau^{r+1}) ; \ldots ; ((v^{r'}) : \tau^{r'}) : \end{matrix}\right]$$

$$\times\, H^{M,N}_{P,Q}\left[z\,t^h \left| \begin{matrix} (e_p, E_p) \\ (f_Q, F_Q) \end{matrix} \right.\right] H^{0,0:(u',v'):\ldots:(u^{(r)},v^{(r)})}_{A,C+1:[B',D']:\ldots:[B^{(r)},D^{(r)}]}$$

$$\left(\begin{matrix} [(a) : \theta', \ldots, \theta^{(r)}], [1 - \beta - h\eta_G - h_{r+1}\,\alpha_{r+1} - \ldots - h_{r'}\,\alpha_{r'} - k_1\alpha_1 - \ldots - k_R\alpha_R : h_1, \ldots, h_{r'}] : \\ [(c) : \psi', \ldots, \psi^{(r)}], \text{———————} \end{matrix}\right.$$

$$\left.\begin{matrix} [(b') : \phi'] ; \ldots ; [(b^{(r)}) : \phi^{(r)}] : \\ \qquad z_1\,t^{h_1}, \ldots, z_r\,t^{h_r} \\ [(d') : \delta'] ; \ldots ; [(d^{(r)}) : \delta^{(r)}] : \end{matrix}\right)$$

$$= w^{-b\eta} \sum_{G=0}^{\infty} \sum_{g=1}^{M} \sum_{\alpha_{r+1}, \ldots, \alpha_{r'}=0}^{M_{r+1}\alpha_{r+1} + \ldots + M_r - \alpha_r \le N_1} \sum_{\alpha_1=0}^{[m_1/l_1]}, \ldots, \sum_{\alpha_R=0}^{[m_R/l_R]}$$

$$\frac{(-1)^G}{G!\,F_g}\,\phi(\eta_G)\,z^{\eta_G}\,\frac{(-m_1)_{l_1\alpha_1}}{\alpha_1!}, \ldots, \frac{(-m_R)_{l_R\alpha_R}}{\alpha_R!}$$

$$\times\, A_{m_1,\alpha_1}, \ldots, A_{m_R\alpha_R}\,(-1)^{h_{r+1}\,\alpha_{r+1} \ldots + h_{r'}\,\alpha_{r'}}$$

$$\times\, \frac{(-N_1)_{\alpha_{r+1}M_{r+1} + \ldots + \alpha_{r'}M_{r'}}}{(\alpha_{r+1})!, \ldots, (\alpha_{r'})!}$$

$$\times\, I(\theta)\, w^{(-h\eta_G - h_{r+1}\alpha_{r+1} - \dots - h_{r'}\alpha_{r'} - k_1\alpha_1 - \dots - k_R\alpha_R)}$$

$$\times\, H^{0,0:(u',v');\dots;(u^{(r)},v^{(r)})}_{A,C:[B',D'];\dots;[B^{(r)},D^{(r)}]}\left(z_1 w^{-h_1}, \dots, z_r w^{-h_r} \right)$$

valid under the same conditions mentioned as in Theorem 2, with

$$I(\theta) = \frac{\prod_{j=1}^{E_1} (e_j)_{\alpha_{r+1}\theta_j^{r+1} + \dots + \alpha_{r'}\theta_j^{r'}}}{\prod_{j=1}^{G_1} (g_j)_{\alpha_{r+1}\psi_j^{r+1} + \dots + \alpha_{r'}\psi_j^{r'}}}$$

$$\times \frac{\prod_{j=1}^{U_1^{r+1}} (u_j^{r+1})_{\alpha_{r+1}\phi_j^{r+1}} \cdots \prod_{j=1}^{U_1^{r'}} (u_j^{r'})_{\alpha_{r'}\phi_j^{r'}}}{\prod_{j=1}^{V_1^{r+1}} (v_j^{r+1})_{\alpha_{r+1}\tau_j^{r+1}} \cdots \prod_{j=1}^{V_1^{r'}} (v_j^{r'})_{\alpha_{r'}\tau_j^{r'}}} \qquad \dots (4.4)$$

(*ii*) If we take $M_i = 0, t_i \to 0$ $(i = r+2, \dots, r')$, $N_1 \to 0$ and $m_2, \dots, m_R \to 0$, the results in (2.1) and (3.1) reduce to the known results obtained by Chaurasia and Girdhari Lal[4].

(*iii*) For $M_i = 0, t_i \to 0$ $(i = r+2, \dots, r')$, $N_1 \to 0$ and $m_1, \dots, m_R \to 0$ the results in (2.1) and (3.1) reduce to a result obtained by Chaurasia[3].

(*iv*) Letting $M_i = 0, t_i \to 0$ $(i = r+2, \dots, r')$, $N_1 \to 0$, $m_{i'} \to 0$ $(i' = 1, \dots, R)$, $h \to 0$ and $r = 2$, Theorem 1 reduces to a known result (Chaurasia[2]).

The importance of our results lies in its manifold generality. In view of the generality of the functions and polynomials of very general nature involved in the results, our results encompass several special cases of interest scattered hitherto in the literature.

ACKNOWLEDGEMENT

The authors are thankful to Professor H.M. Srivastava (University of Victoria, Canada) for his help and suggestions in the preparation of this paper.

REFERENCES

1. B. L. J. Braaksma, *Comp. Math.* **15** (19639), 239-341.
2. V. B. L. Chaurasia, *Math. Edu.* **26** (1982), 84-89.
3. V. B. L. Chaurasia, *Jñānābha* **20** (1990), 19-24.
4. V. B. L. Chaurasia and Girdhari Lal, *Proc. Indian Acad. Sci. (Math. Sci.)* **103** No. 1 (1993), 91-96.
5. V. A. Ditkin and A. P. Prudnikov, *Operational Calculus in Two Variables and its Applications*, London Pergamon press (1962).
6. C. Fox, *Trans. Amer. Math. Soc.* **98** (1961), 395-429.

7. A. M. Mathai and R. K. Saxena, *The H-function with Applications in Statistics and Other Disciplines*, New Delhi, Wiley Eastern Limited (1978).
8. H. M. Srivastava, *Pacific J. Math.* **117** (1985), 183-191.
9. H. M. Srivastava and M. C. Daoust, *Nederl. Acad. Wetensch Indag, Math.* **31** (1969), 449-457.
10. H. M. Srivastava and M. Garg, *Rev. Roumaine Phys.* **32** (1987), 685-692.
11. H. M. Srivastava, K. C. Gupta and S. P. Goyal, *The H-function of One and Two Variables with Applications*, South Asian Publishers, New Delhi (1982), 251-254.
12. H. M. Srivastava and R. Panda, *J. Reine Angew. Math.* **283/284** (1976), 265-274.
13. H. M. Srivastava and N. P. Singh, *Rend. Circ. Mat. Palermo*, **32** : 2 (1983), 157-187.