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ON PRIME LEFT IDEALS IN Γ-RINGS

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The concepts "left $g^1-\gamma$-prime left ideal" and left $g^1-\gamma$-prime radical were introduced in Γ-rings. These concepts were studied and finally an element wise characterization for left $g^1-\gamma$-prime radical was obtained.

Key Words : Prime Left Ideals; Γ-rings; Abelian Groups

INTRODUCTION

Nobusawa[4] introduced the concept of Γ-ring and Barnes[1] generalized this concept. Later several authors studied the Γ-ring in the sense of Barnes[1]. Barnes defined the Γ-ring as follows:

0.1 *Definition* — Let $(M, +)$, $(\Gamma, +)$ be additive Abelian groups. If there exists a mapping $M \times \Gamma \times M \to M$. (the image of (a, α, b) is denoted by $a\alpha b$ for $a, b \in M, \alpha \in \Gamma$) satisfying the following conditions: (*i*) $(x+y)\alpha z = x\alpha z + y\alpha z, x\alpha(y+z) = x\alpha y + x\alpha z$, $x(\alpha+\beta)z = x\alpha z + x\beta z$; and (*ii*) $x\alpha(y\beta z) = (x\alpha y)\beta z$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$, then M is called a Γ-ring. All Γ-rings considered in this paper are Γ-rings in the sense of Barnes.

0.2 *Examples* — (*i*) If G and G^1 are two additive Abelian groups, M = Hom (G, G^1), Γ = Hom (G^1, G) then M is a Γ-ring with respect to pointwise addition and composition of mappings.

(*ii*) Let R be a ring. Take $M = \Gamma = R$. For $a, b \in M, \alpha \in \Gamma$, suppose $a\alpha b$ is the product of $a, \alpha, b \in R$. Then M is a Γ-ring.

(*iii*) Let U, V be vector spaces over the same field F, M = Hom (U, V), Γ = Hom (V, U). Then M is a Γ-ring with respect to point wise addition and composition of mappings.

Throughout M denotes a Γ-ring. An additive subgroup $(A, +)$ of $(M, +)$ is said to be (*i*) a right ideal if $a\alpha m \in A$ for all $a \in A, \alpha \in \Gamma$, $m \in M$; (*ii*) a left ideal if $m\alpha a \in A$ for all $a \in A$, $\alpha \in \Gamma, m \in M$; and (iii) an ideal if it is both left and right ideal. The smallest ideal containing an element $a \in M$ is denoted by $\langle a \rangle$. The smallest ideal containing a subset X of M is denoted by $\langle X \rangle$. The smallest left ideal containing X is denoted by $\langle X \rangle_1$. $\langle \{a\} \rangle_1$ is denoted by $\langle a \rangle_1$.

For any set X of M, the set theoretic complement of X in M is denoted by $M \setminus X$. If $A \subseteq M, B \subseteq M, \gamma \in \Gamma, \Gamma^1 \subseteq \Gamma$ then the set $\{a\alpha b / a \in A, \alpha \in \Gamma^1, b \in B\}$ is denoted by $A\Gamma^1 B$; the set $\{a\gamma b / a \in A, b \in B\}$ is denoted by $A\gamma B$; and the set $\left\{\sum_{i=1}^{n} a_i \alpha_i b_i / a_i \in A, \alpha_i \in \Gamma, b_i \in B\right.$ and n is any positive integer$\}$ is denoted by AB.

0.3 *Definition* — (Booth[2]) A strong right unity of a Γ-ring M is a pair (δ, d) of $\Gamma \times M$ such that $x\delta d = x$ for all $x \in M$.

0.4 *Note* — If M has a strong right unity (δ, d) then $m = m\delta d \in m\delta M$ for all $m \in M$.

Following Hsu[3] we suppose that g is a function of M into the family of ideals of M such that for every n, $m \in M$, (*i*) $m \in g(m)$; and (*ii*) $n \in g(m) + A$ and A is an ideal of $M \Rightarrow g(n) \subseteq g(m) + A$.

0.5 *Definition* — (Hsu[3]) (*i*) A subset X of M is said to be an m-system if either $X = \phi$ or it satisfies the condition that $a, b \in X \Rightarrow$ there exists $\alpha \in \Gamma, a^1 \in \langle a \rangle, b^1 \in \langle b \rangle$ such that $a^1 \alpha b^1 \in X$.

(*ii*) A subset X of M is said to be a g-system if there exists an m-system X^* such that $X^* \subseteq X$ and $g(a) \cap X^* \neq \phi$ for all $a \in X$.

(*iii*) An ideal P of M is said to be a g-prime ideal if $M \setminus P$ is a g-system. An ideal P of M is said to be a prime ideal of M if it satisfies the condition that A, B are ideals of M such that $AB \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$.

Every prime ideal is a g-prime ideal, but the converse need not be true. Hsu provided an example of a g-prime ideal which cannot be a prime ideal. Barnes[1] and Booth[2], studied prime ideals and prime radical for Γ-rings. Hsu[3] and Satyanarayana[5] studied g-prime radical for Γ-rings.

In this paper, the concepts left g^1-prime, left $g^1 = \gamma$-prime, left $g^1 = \gamma$-system, left g^1-system, left $g^1 = \gamma$-prime radical were introduced. In section-2, equivalent conditions for an ideal (respectively, left ideal) to be a left $g^1 = \gamma$-prime ideal (respectively, left ideal) obtained. Some conditions, under which the concepts left $g^1 = \gamma$-prime and left g^1-prime coincide were presented. In section-3, an element wise characterization for left $g^1 = \gamma$-prime radical was obtained.

1. Ideals

Definition 1.1. — Fix $\gamma \in \Gamma$. A subgroup I of $(M, +)$ is said to be (*i*) a γ-right ideal if $I\gamma M \subseteq I$, (ii) a γ-left ideal if $M\gamma I \subseteq I$, (iii) a γ-ideal if I is both γ-right and γ-left ideal.

1.2 *Note* — It is clear that I is an ideal $\Leftrightarrow$ I is γ-ideal for all $\gamma \in \Gamma$.

1.3 *Definition* — Let $\gamma \in \Gamma$. M is said to be γ-commutative Γ-ring if $a\gamma b = b\gamma a$ for all $a, b \in M$.

1.4 *Note* — (*i*) If M is γ-commutative then a subgroup A of $(M, +)$ is a right γ-ideal if and only if it is a left γ-ideal.

(ii) If M is γ-commutative then every left ideal of M is a γ-right ideal of M.

1.5 *Definition* — A (left/right) ideal P is said to be *IFP* (insertion of factors property) ideal if it satisfies the following condition: $a, b \in M, \gamma \in \Gamma, a\gamma b \in P \Rightarrow a\Gamma M\gamma b \subseteq P$.

1.6 ***Theorem*** — *Fix $\gamma \in \Gamma$. J is a left ideal. For a left ideal P, write $(P : \gamma J) = \{x \in M / x\gamma J \subseteq P\}$. Then*

(*i*) $(P : \gamma J)$ is a left ideal of M,

(*ii*) $(P : \gamma J)$ is a γ- ideal of M,

(*iii*) If $x\gamma M = x\Gamma M$ for all $x \in (P : \gamma J)$ then $(P : \gamma J)$ is an ideal,

(*iv*) If M is a commutative Γ-ring then $(P : \gamma J)$ is an ideal

and

(*v*) If P is an IFP ideal then $(P : \gamma J)$ is an ideal.

PROOF : Write $I = (P : \gamma J)$. Take $x, y \in I \Rightarrow x\gamma J \subseteq P$ and $y\gamma J \subseteq P$. Consider $(x+y)\gamma J \subseteq x\gamma J + y\gamma J \subseteq P + P = P \Rightarrow x + y \in I$. $x\gamma J \subseteq P \Rightarrow -(x\gamma J) \subseteq P \Rightarrow (-x)\gamma J \subseteq P \Rightarrow -x \in I$. Therefore $(I, +)$ is a subgroup of $(M, +)$.

(*i*) To show I is a left ideal, take $\alpha \in \Gamma$ and $m \in M$. $(m\alpha x)\gamma J = m\alpha(x\gamma J) \subseteq m\alpha P \subseteq P$ (since P is a left ideal) $\Rightarrow m\alpha x \in I$. Therefore, I is a left ideal.

(*ii*) Since I is a left ideal, we have that I is a γ-left ideal. To show that I is a γ-right ideal, take $m \in M$. Consider $(x\gamma m)\gamma J = x\gamma(m\gamma J) \subseteq x\gamma J$ (since J is left ideal) $\subseteq P \Rightarrow x\gamma m \in I$. Therefore, $x\gamma M \subseteq I$. Hence, I is γ-right ideal.

(*iii*) Suppose $x\gamma M = x\Gamma M$ for all $x \in I$. Let $x \in I, \alpha \in \Gamma, m \in M$. Consider $x\alpha m \in x\Gamma M$. Now $x\alpha m \in x\Gamma M = x\gamma M \Rightarrow x\alpha m = x\gamma m^1$ for some $m^1 \in M$. Consider $(x\alpha m)\gamma J = (x\gamma m^1)\gamma J = x\gamma(m^1\gamma J) \subseteq x\gamma J \subseteq P \Rightarrow x\alpha m \in I$. This is true for all $\alpha \in \Gamma$ and $m \in M$. Therefore, $x\Gamma M \subseteq I$. This shows that I is a right ideal. Hence, I is an ideal (by (*i*)).

(*iv*) Suppose M is commutative Γ-ring. By (*i*), we have that I is a left ideal. Since M is commutative we have that I is a right ideal. Hence I is an ideal.

(*v*) Suppose P is an *IFP* ideal. Let $x \in I \Rightarrow x\gamma J \subseteq P \Rightarrow x\Gamma M\gamma J \subseteq P$ (since P is an IFP ideal) $\Rightarrow x\Gamma M \subseteq \{a \in M / a\gamma J \subseteq P\} = I$. Therefore, I is a right ideal. By (*i*), I is a left ideal. Hence, I is an ideal.

1.7 *Note* — If $n^1 \in P + \langle n \rangle$ then $g(n^1) \subseteq P + g(n)$.

(Verification : $n^1 \in P + \langle n \rangle \subseteq P + g(n) \Rightarrow g(n^1) \subseteq P + g(n)$).

2. LEFT $g^1 - \gamma$-PRIME LEFT IDEALS

Definition 2.1 — Let $\gamma \in \Gamma$. (*i*) A left ideal (respectively ideal) P of M is said to be a left $g^1 = \gamma$-prime left ideal (respectively, left $g^1 = \gamma$-prime ideal) if it satisfies the condition that $m, n \in M, m\,\gamma g(n) \subseteq P$ imply either $m \in P$ or $n \in P$.

(*ii*) A left ideal (respectively ideal) P of M is said to be a left g^1-prime left ideal (respectively, left g^1-prime ideal) if it satisfies the condition that $m, n \in M, m\Gamma g(n) \subseteq P$ implies either $m \in P$ or $n \in P$.

Note 2.2 — (*i*) Every left $g^1 = \gamma$-prime left ideal is a left g^1-prime left ideal; (*ii*) Every left $g^1 - \gamma$-prime ideal is a left g^1-prime ideal; and (*iii*) The converse of (*i*) and (*ii*) need not be true.

Example 2.3 — Take Z = the additive group of integers, $M = 2Z$, $\Gamma = 3Z$. For any $a, b \in M$ and $\alpha \in \Gamma, a\alpha b$ is the usual product of integers. Clearly M is a Γ-ring. Write $g(a) = \langle\{a, 10\}\rangle$ for any $a \in M$. $P = 14Z$ is an ideal of the Γ-ring M. Now we show that P is a left g^1-prime ideal but not a left $g^1 - \gamma$-prime ideal for some fixed $\gamma \in \Gamma$. For this, take $\gamma = 21 \in \Gamma$. Suppose $m\Gamma g(n) \subseteq P$ for some $m, n \in M \Rightarrow m.\ 3.\ n.\ 10 \in m \cdot \Gamma \cdot \langle\{n, 10\}\rangle \subseteq P = 14Z \Rightarrow 14$ divides $m3.n.10$ $\Rightarrow 7$ divides m or 7 divides n. Since $m, n \in M = 2Z$ we have 2 divides m and 2 divides n. Therefore, 14 divides m or 14 divides $n \Rightarrow m \in 14Z = P$ or $n \in 14Z = P$. Therefore, P is a left g^1-prime ideal. To verify that P is not a left $g^1 - \gamma$-prime ideal for $\gamma = 21$, take $m = 2$, $n = 2$. Now $m.21.g(n) = 2.21.\ g(2) = 2.21.\ \langle\{2, 10\}\rangle \subseteq 42\ Z \subseteq\ 14\ Z = P$, but $m = 2 \notin P$ and $n = 2 \notin P$. This shows that P is a left g^1-prime ideal (or left ideal) which is not a left g^1-γ-prime ideal (or left ideal),

2.4 *Definition* — (*i*) A subset X of M is said to be a left g^1-system if either $X = \phi$ or for any $a, b \in X$ there exists $\alpha \in \Gamma, b^1 \in g(b)$ such that $a\alpha b^1 \in X$.

(*ii*) Fix $\gamma \in \Gamma$. A subset X of M is said to be a left g^1-γ-system if either $X = \phi$ or for any $a, b \in X$ there exists $b1 \in g(b)$ such that $a\gamma b^1 \in X$.

2.5. *Note* — Every left $g^1 - \gamma$-system is a left g^1-system, but the converse need not be true.

2.6 *Example* — Suppose $M = Z = \Gamma$. Then M is a Γ-ring. Take $g(a) = \langle a \rangle$ for all $a \in M$. Write $X = \{2, 2^2, 2^3, ...\}$ is a left g^1-system. Take $\gamma = 5$. In a contrary way, suppose X is a left g^1-γ-system. Now $2, 4 \in X \Rightarrow \exists\, b^1 \in \langle 4 \rangle = 4Z$ such that $2\gamma b^1 \in X \Rightarrow 2.5(4.k) \in X$ for some $k \Rightarrow 2.5.4.\ k = 2^m$ for some m, a contradiction. Hence, X is not a left $g^1 - \gamma$-system when $\gamma = 5$.

2.7 **Theorem** — *Let P be a ideal (respectively left ideal) of M and $\gamma \in \Gamma$. Then the following conditions are equivalent: (i) P is a left $g^1 - \gamma$-prime ideal (respectively, left $g^1 - \gamma$-prime left ideal); (ii) $a, b \in M$ such that $\langle a \rangle_1\ \gamma g(b) \subseteq P \Rightarrow a \in P$ or $b \in P$; (iii) A is a left ideal of M and B is an ideal of M such that $A\gamma(B + g(o)) \subseteq P \Rightarrow A \subseteq P$ or $B \subseteq P$; and (iv) $M \backslash P$ is a left $g^1 - \gamma$-system.*

PROOF : (*i*) $\Rightarrow$ (*ii*) Suppose P is a left $g^1 - \gamma$-prime ideal. To show (*ii*), take $a \in M, b \in M$ such that $\langle a \rangle_1\ \gamma g(b) \subseteq P$. Now $a\gamma g(b) \subseteq \langle a \rangle_1\ \gamma g\ (b) \subseteq P \Rightarrow a \in P$ or $b \in P$.

(*ii*) $\Rightarrow$ (*iii*) : Suppose (*ii*). In a contrary way, suppose that the condition (*iii*) is not true. Then there exist a left ideal A and an ideal B of M such that $A\gamma(B + g(o)) \subseteq P, A \not\subseteq P$ and $B \not\subseteq P$. Take $a \in A\ P$, $b \in B \backslash P$. Clearly $\langle a \rangle_1 \subseteq A$ and $\langle b \rangle \subseteq B \Rightarrow \langle a \rangle_1 \subseteq A$.

Since $b \in \langle b \rangle + g(o)$, we have $g(b) \subseteq \langle b \rangle + g(o) \subseteq B + g(o)$. Therefore, $\langle a \rangle_1 \gamma g(b) \subseteq A\gamma(B + g(o)) \subseteq P \Rightarrow a \in P$ or $b \in P$, a contradiction. Hence, condition (*iii*) is true.

(*iii*) $\Rightarrow$ (*i*) : Suppose (*iii*). Take $a, b \in M$ such that $a\gamma g(b) \subseteq P$. Write $J = g(b) = \langle b \rangle + g(o)$. Now $a\gamma J \subseteq P \Rightarrow a \in (P : \gamma J)$. By theorem 1.6, $\langle a \rangle_1 \subseteq (P : \gamma J) \Rightarrow \langle a \rangle_1 \gamma J \subseteq P \Rightarrow \langle a \rangle_1 \gamma(\langle b \rangle + g(o)) = \langle a \rangle_1 \gamma g(b) \subseteq \langle a \rangle_1 \gamma J \subseteq P \Rightarrow a \in P$ or $b \in P$. Therefore, (*iii*) $\Rightarrow$ (*i*) is true.

(*i*) $\Rightarrow$ (*iv*) : Suppose (*i*). To show $M \setminus P$ is a left $g^1 - \gamma$-system, take $a, b \in M \setminus P \Rightarrow a \notin P, b \notin P \Rightarrow a\gamma g(b) \not\subseteq P \Rightarrow \exists\, b^1 \in g(b)$ such that $a\gamma b^1 \notin P \Rightarrow a\gamma b^1 \in M \setminus P$. Therefore, $M \setminus P$ is a left $g^1 - \gamma$-system.

(*iv*) $\Rightarrow$ (*i*): Suppose $M \setminus P$ is left $g^1 - \gamma$-system. In a contrary way, suppose that P is not a $g^1 - \gamma -$prime ideal. Then there exists $a, b \in M \setminus P$ such that $a\gamma g(b) \subseteq P$. Since $a, b \in M \setminus P$ and $M \setminus P$ is a left $g^1 - \gamma -$system there exists $b^1 \in g(b)$ such that $a\gamma b^1 \in M \setminus P$. Now $a\gamma b^1 \in a\gamma g(b) \subseteq P \Rightarrow a\gamma b^1 \in P$, a contradiction. Hence, P is a $g^1 - \gamma$-prime ideal.

2.8 *Note* — It is easy to verify the following :

(*i*) Let P be a left ideal. Then P is left g^1-prime if and only if $M \setminus P$ is a left g^1-system.

(*ii*) Let P be an ideal. Then P is left $g^1 = \gamma$-prime $\Rightarrow M \setminus P$ is left $g^1 = \gamma$-system $\Rightarrow M \setminus P$ is left g^1-system $\Rightarrow P$ is left g^1-prime.

(*iii*) Every g-system may not be left g^1-system.

Example 2.9 — Let $M = \Gamma = Z$, the additive group of integers. Then M is a Γ-ring. Let p, q be two distinct prime numbers. Define $g(x) = \langle\{x, q\}\rangle$ for all $x \in M$. Write $S^* = \{q, q^2, q^3, \ldots\}$. S^* is an m-system. Write $S = \{P\} \cup S^*$. S is a g-system with kernel S^*. But S cannot be a left g^1-system (because $p, q \in S$ and there is no $q^1 \in g(q)$ and $\gamma \in \Gamma$ such that $p\gamma q^1 \in S$). Hence, S is a g-system which is not a left g^1-system.

Theorem 2.10 — *(i) If there exists $\gamma \in \Gamma$ such that $m \in m\gamma M$ for all $m \in M$ then a left ideal P of M is left g^1-prime if and only if it is left $g^1 - \gamma$-prime.*

(*ii*) If there exist $\gamma \in \Gamma, d \in M$ such that (γ, d) is a strong right unity then a left ideal P of M is left g^1-prime if and only if it is left $g^1 - \gamma$-prime.

PROOF : It is clear that every left $g^1 = \gamma$-prime left ideal is a left g^1-prime left ideal. Conversely suppose that P is left g^1prime. In a contrary way, suppose that P is not a left g^1-γ-prime ideal $\Rightarrow$ there exist $m, n \in M \setminus P$ such that $m\gamma g(n) \subseteq P$. Let $m^1 \in M$. Now $m\gamma m^1 \Gamma g(n) \subseteq m\gamma M \Gamma g(n) \subseteq m\gamma g(n)$ (since $g(n)$ is an ideal) $\subseteq P \Rightarrow m\gamma m^1 \in P$. Therefore, $m \in m\gamma M \subseteq P$, a contradiction. Therefore, P is left $g^1 - \gamma$-prime. This completes the proof of (*i*).

(ii) follows from (i).

Theorem 2.11 — *Suppose that there exists $\gamma \in \Gamma$ such that $M\gamma M = M$. For an IFP left ideal P the following conditions are equivalent:*

(*i*) P is left g^1-prime; and (ii) P is left $g^1-\gamma$-prime.

PROOF : (*ii*) $\Rightarrow$ (*i*) is clear.

(*i*) $\Rightarrow$ (*ii*). Suppose P is left g^1-prime. In a contrary way, suppose that P is not left g^1-γ-prime. Then there exist $m, n \in M \setminus P$ such that $m\gamma g(n) \subseteq P$. Since P is *IFP*, we have $m\Gamma M\gamma g(n) \subseteq P$. Since $g(n)$ is left ideal we have $m\Gamma M\gamma M\Gamma g(n) \subseteq m\Gamma M\gamma g(n) \subseteq P \Rightarrow m\Gamma M\Gamma g(n) = m\Gamma M\gamma M\Gamma g(n)$ (since $M = M\gamma M$) $\subseteq P$. For any $\beta_1, \beta_2 \in \Gamma$ and $m_1 \in M$ we have that $m\beta_1 m_1 \Gamma g(n) \subseteq P \Rightarrow m\beta_1 m_1 \in P$ (since P is left g^1-prime). Therefore, $m\Gamma M \subseteq P \Rightarrow m\Gamma g(n) \subseteq P$ $\Rightarrow m \in P$ or $n \in P$, a contradiction. Hence, P is left g^1-γ-prime.

3. THE LEFT $g^1-\gamma$-PRIME RADICAL

Definitions 3.1 — (*i*) The intersection of all left $g^1 = \gamma$-prime left ideals of M is called the left $g^1-\gamma$-prime radical of M and it is denoted by $P_{g^1\gamma}(M)$.

(*ii*) The intersection of all left g^1-prime left ideals of M is called the left g^1-prime radical of M and it is denoted by $P_{g^1}(M)$.

Note 3.2. — (*i*) Since every left g^1-γ-prime left ideal is a left g^1-prime left ideal, we have that $P_{g^1}(M) \subseteq P_{g^1\gamma}(M)$, for every $\gamma \in \Gamma$.

(*ii*) If there exist $\gamma \in \Gamma$ such that $m \in m\gamma M$ for all $m \in M$, then by using Theorem 2.10 we get that $P_{g^1}(M) = P_{g^1\gamma}(M)$.

(*iii*) If there exists $\gamma \in \Gamma$ such that $M\gamma M = M$ and every left g^1-prime left ideal is IFP then by using Theorem 2.11, we get that $P_{g^l}(M) = P_{g^1\gamma}(M)$.

Theorem 3.3 — *Let $\gamma \in \Gamma$. Suppose every left ideal of M is a γ-right ideal. Then* $P_{g^1\gamma}(M)$ $= \{x \in M$ / *every left* $g^1-\gamma$*-system containing x contains o*$\}$.

PROOF : Write $B = \{x \in M$ / every left $g^1 = \gamma$-system containing x contains $o\}$.

Let $x \in B$. Now we have to show that $x \in P_{g^1\gamma}(M)$. In a contrary way suppose that $x \notin P_{g^1\gamma}(M) \Rightarrow$ there exists a left $g^1 = \gamma$-prime left ideal P in M such that $x \notin P \Rightarrow x \in M \setminus P$. By theorem 2.7, $M \setminus P$ is a left $g^1 = \gamma$-system. Since $x \in B$ we have that $o \in M \setminus P \Rightarrow o \notin P$, a contradiction (because P is an ideal). Conversely, take $x \in P_{g^1\gamma}(M)$. Now we show that $x \in B$. In a contrary way, suppose $x \notin B$. Then there exists a left $g^1 = \gamma$-system X in M such that $x \in X$ and $o \notin X$. Write $S = \{I$ / I is a left ideal of M such that $I \cap X = \phi\}$. Clearly $o \in S$. It is easy to verify that every chain $\{I_\alpha\}_{\alpha \in \Delta}$ of elements from S possess an upper bound $\bigcup_{\alpha \in \Delta} I_\alpha$. So by zorn's lemma,

S contains a maximal element, say P. Now we show that P is a left $g^1 = \gamma$-prime left ideal. In a contrary way, suppose that P is not left g^1-γ-prime. Then there exist $m, n \in M \setminus P$ such that $m\gamma g(n) \subseteq P$. By Theorem 1.6 we have $\langle m \rangle_1 \gamma g(n) \subseteq P$. Since $m \notin P$ we have $P + \langle m \rangle_1 \notin S \Rightarrow (P + \langle m \rangle_1) \cap X \neq \phi$. Similarly $(P + \langle n \rangle_1) \cap X \neq \phi$. Take $m^1 \in (P + \langle m \rangle_1) \cap X$ and $n^1 \in (P + \langle n \rangle) \cap X$. Now $m^1, n^1 \in X \Rightarrow \exists\, n^* \in g(n^1) \ni m^1\gamma n^* \in X$. Now $m^1\gamma n^* \in m^1\gamma g(n^1)$ $\subseteq m^1\gamma(P + g(n))$ (by note 1.6) $\subseteq (P + \langle m \rangle_1)\gamma(P + g(n)) \subseteq P\gamma P + P\gamma g(n) + \langle m \rangle_1 \gamma P + \langle m \rangle_1 \gamma g(n) \subseteq$ $P + P + P + P = P$. Therefore $m^1\gamma n^* \in P \cap X = \phi$, a contradiction. This shows that P *is left* $g^1 = \gamma$*-prime. Now* $x \in X$ *and* $P \cap X = \phi \Rightarrow x \notin P \Rightarrow x \notin P_{g^1\gamma}(M)$. *This completes the proof.*

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