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STUDY OF A MATHEMATICAL MODEL RELATED TO TOTAL BIOMASS OF TREES

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Growth data based on actual felling of *Leucaena leucocephala* trees, with in the DBH range of 10-25 cm, planted at the boundary of fields were recorded. Prediction of total above ground biomass were attempted through allometric regressions of the type $Y = b^* X^c$; where Y is the total above ground biomass and X is diameter at 30% of the total height. The estimates of the parameters of the nonlinear model were computed using Gauss-Newton exact derivatives. For assessing the certainty of the parameters estimates, Wald confidence region and Cook and Weisberg confidence curves were plotted. The effect of over parameterization in the full model was solved by fixing one parameter and estimating the other for which C-W curve shows asymetry in distribution. The model : Biomass = 0.365* $(\text{dia.30})^{2.259}$ with R^2 = 0.95 was found optimum to provide maximum possible prediction.

Key Words : *Leucaena leucocephala*; Biomass prediction; Nonlinear models

1. INTRODUCTION

Studies on tree taper, volume and biomass models have often considered measurements on total height of the tree and diameter at breast height. Researchers have tried out various permutations and combinations of these two growth attributes (Pathak *et al.*, 1988[7], Khan *et al.*, 1993[5]) for deriving the prediction equations for total above gound biomass. Various combinations tried out includes, height alone, diameter at breast height alone, height * diameter at breast height & height * square of diameter at breast height etc. Circular growth attributes (CD, DBH) plays a dominant role in determination of total biomass (Ajit *et al.*, 1998[1]) as compared to polar growth (height). However, considering only one diameter point (DBH) along the main bole for prediction of total biomass is debatable. Keeping this in mind, several diameter measurements along the main bole viz at 10%, 20%, 30%, 40% & 50% of the total height of the tree along with DBH are considered to determine the best predictor diameter value for determination of total biomass.

2. MATERIAL AND METHODS

Site description

National Research Centre for Agroforestry, Jhansi is situated between latitude 24°11'-26°27' N and longitude 78°17'-81°34' E with mean annual precipitation 936 mm of which 80-90% is received

during July-September. The climatic pattern of Jhansi is true representative of semi-arid region with weekly mean maximum temperature ranging from 22 °C-47 °C and minimum from 2.5 °C to 29.9 °C. The soil of the experimental site is black containing organic carbon 0.29%, *p*H 7.07 and available NPK 82.3 kg/ha, 3.16 kg/ha, 83.66 kg/ha respectively.

Methodology

A sample of 11 trees having DBH values in the range of 10-25 cm have been recorded. Observations were made on main bole weight, leaf weight, branch weight, total height of tree, CD, DBH, diameter values at 10%, 20%, 30%, 40%, 50% of total height of the tree. Relationship of total above ground biomass with growth parameters were established and the model of the type Biomass = b^* (dia 30)^c was attempted for the study. The estimates of the parameters of the non-linear model were computed using Gauss-Newton exact derivatives. To avoid local minima, Simplex method (O'Neill[6] & Griffiths and Hills[4]), which is the most robust against local minimum and Quasi-Newton method (Fletcher[3]) were also evaluated to be sure of absolute convergence. For assessing the certainty of the parameters estimate, Wald confidence region and C-W confidence curves (Cook and Weisberg[2]) were plotted. The effect of over parameterization in the full model was solved by fixing one parameter and estimating the other for which C-W curve shows asymetry in distribution. Coefficient of determination R^2 was used to get the idea of fitness among the data points.

3. Results and Discussion

To evolve a prediction, the model should be based on field measurable data from basic growth attributes. The basic assumption in a model is that the relationship between any two characteristics of a tree can be estimated by allometric regressions. The full model leaded to the estimates of '*b*' and '*c*' [Figs. 2 & 3] that depicted high correlation and consequently in the partial model '*c*' was fixed and '*b*' was optimized. The analysis of variance for the effect of fixing the parameter $H_0 : c = 2.3$ reflected F-value = 0.053 with p value = 0.824 indicating that there was no significant difference between the two models, looking to the similarities of the values of estimate (Table I). However the most striking difference was in the Wald confidence interval for 'b' which tremendously reduced in the later case, moreover the C-W curve coincided almost with the Wald's region [Fig. 4] indicating the optimization of the model Biomass = 0.365^* (dia. 30)$^{2.259}$ [Fig. 1].

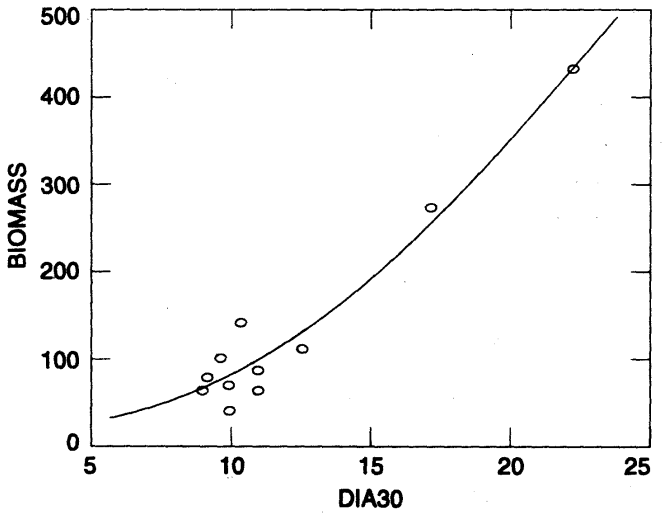

FIG. 1. Curve of the fitted model

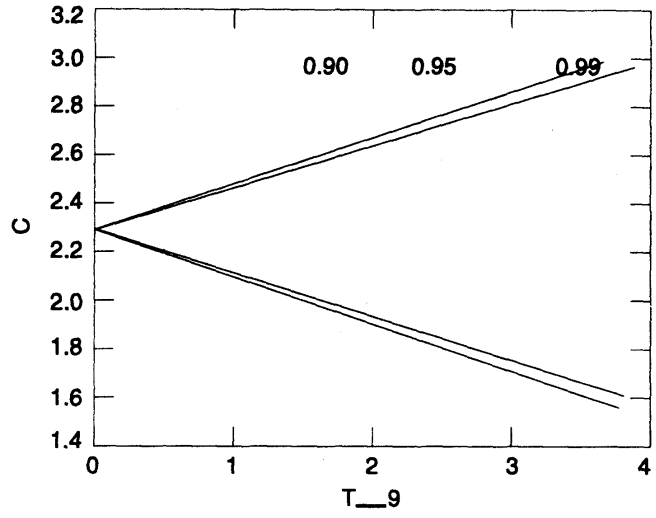

FIG. 2. Wald/Cook & Weisberg Curve for C

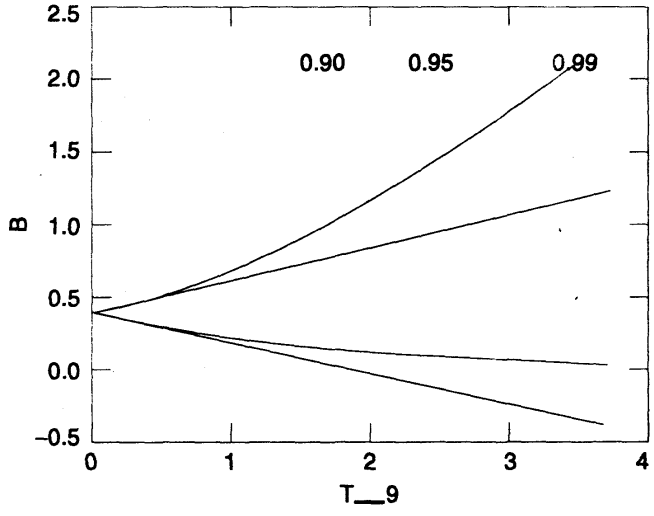

FIG. 3. Wald/Cook & Weisberg Curve for B

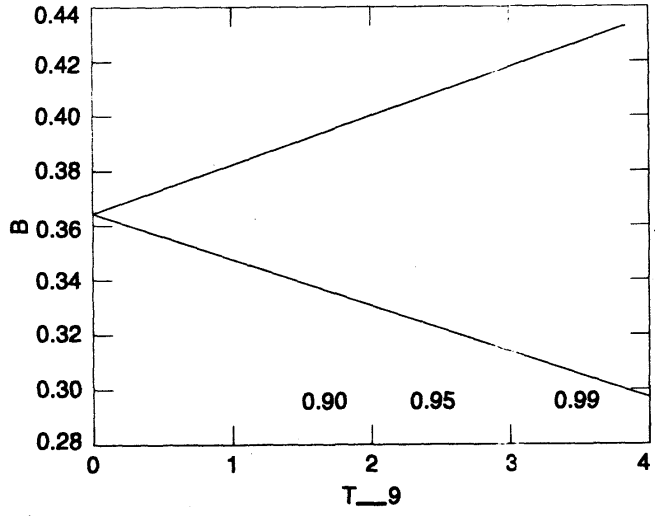

FIG. 4. Wald/Cook & Weisberg Curve for B after fixing C

TABLE I. Analysis of the effect of fixing parameter

Parameters	Estimates			
	Original model	Wald Confidence Interval	Partial model 'c' fixed	Wald Confidence Interval
'b'	0.411	–0.069 to 0.892	0.365	0.327 to 0.403
'c'	2.259	1.855 to 2.663	2.300	
R^2	0.947		0.947	

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