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ANISOTROPIC BRANS-DICKE COSMOLOGY

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In this paper Bianchi Type I Cosmological models have been studied for (1) constant and (2) variable deceleration parameters in the framework of Brans-Dicke theory. In the first case, two types (singular and non-singular) of models are shown. But in the second case, only singular models are presented.

Key Words : Bianchi Type I Cosmological Models; Brans-Dicke Theory

1. INTRODUCTION

The study of Brans-Dicke (BD) cosmology has attracted much attention in the recent years. The inflationary models[1], extended inflation[2, 3], hyperextended inflation and extended chaotic inflation[4] based on BD theory and general scalar tensor theories are now fairly active fields of investigation. It is observed that all the known cosmological models of BD theory with $k = 0$, including the inflationary model[1] are models with constant deceleration parameter, q. Johri *et al.* have presented the flat FRW models in BD cosmology with constant q[5]. In addition to the models with constant q, models with variable deceleration parameters are also possible. In this paper we study the Bianchi I models in BD theory with both constant and variable deceleration prameters.

The classical evolution equations (in General Relativity and Brans-Dicke theory) are purely adiabatic and reversible; consequently they cannot provide by themselves an explanation of the origin of cosmological entropy which might have been generated through irreversible processes during the cosmic expansion. Prigogine *et al.*[6, 7] have investigated the role of irreversible processes with creation of matter out of gravitational energy in the context of General Relativity. It was shown by Prigogine[8,9] and Prigogine and Glansdorff[10] that thermodynamics of open systems, when applied to cosmology, leads to a reinterpretation, in Einstein's equations, of the matter-energy stress tensor[6]. Here the universe starts from random vacuum fluctuations and the cosmic expansion is driven by the creation of matter particles. In this scenario, different regions of the universe might evolve in entirely different ways depending on the mode of particle creation. Hence, the particle creation function $N(t)$ must be regarded as an initial condition in the particle creation scenario.

Before we discuss our work based on Prigogine's hypothesis at length, we briefly survey the work done by other authors on particle production. The idea of particle production in cosmology has been discussed by a number of authors. Schrödinger[11] and later de Witt[12] indicated the possibility of particle creation by the vacuum fluctuation of quantized fields embedded in nonflat, especially nonstationary, classical space-times. Parker[13] and Zel'dovich[14] have considered particle creation in an expanding universe on the basis of the general covariant Klein-Gordon theory. Parker[15], Audretsch[16] and Isham *et al.*[17] have dealt with particle creation in an expanding universe by considering particles with spin 1/2. Quantized free Dirac fields in an expanding universe with arbitrary expansion law and spherically symmetric space have been used by Schäfer and Dehnen[18] to obtain a creation rate of particles with spin 1/2. Obregon and Pimentel[19] have considered the creation of spin 1/2 particles in the framework of Brans-Dicke theory. It has been shown by Brout, Englert and Gunzig[20-22] and Brout, Englert and Spindel[23] that the energy of the particles produced quantum-mechanically can be extracted from that of the gravitational field. In fact, it was Brout, Englert and Gunzig[20] who gave most explicitly the idea of matter creation out of gravitational energy. Prigogine, Geheniau, Gunzig and Nardone[6-8] have further emphasised this fact in their work on thermo-dynamics of open systems in the framework of cosmology and have given a quantitative expression for the creation of particles out of gravitational energy.

We have presented the modified field equations of BD theory for open systems incorporating the creation pressure term in §2. In §3, the models with constant deceleration parameter are derived. We have presented models with variable deceleration parameter in §4. We conclude in §5.

2. Field Equations of BD Theory with Creation of Matter

Let us consider the universe as an open system with N particles initially. Suppose a random fluctuation in curvature induces a transformation of gravitational energy into matter energy, creating an additional number of particles dN. This increase in the number of particles from N to $N + dN$ gives rise to a negative pressure p_c (discussed in [5]) which drives the expansion of the universe. The negative pressure p_c is a supplementary pressure to the thermodynamic pressure p. Hence, the effective energy-momentum tensor of the cosmic fluid in the presence of creation of matter, includes the creation pressure term p_c and is given by

$$T_{ab} = (\rho + p + p_c)\, u_a u_b - (p + p_c)\, g_{ab} \qquad \text{... (1)}$$

Accordingly the modified field equations of BD theory are given by

$$G_{ab} = \frac{8\pi}{\phi} T_{ab} + \frac{\omega}{\phi^2}\left[\phi_{;a}\phi_{;b} - \frac{1}{2} g_{ab}\, \phi_{;c}\, \phi^{;c} \right] - \frac{1}{\phi}\left[\phi_{;a;b} - g_{ab}\, \Box^2 \phi\right] \qquad \text{... (2)}$$

and

$$\Box^2 \phi = \frac{8\pi}{\phi} T^a_a \qquad \text{... (3)}$$

Let us consider a Bianchi I universe (assuming that the universe might have had anisotropy at the beginning)

$$ds^2 = dt^2 - a_1^2(t)\, dx^2 - a_2^2(t)\, dy^2 - a_3^2(t)\, dz^2 \qquad \text{... (4)}$$

The BD field eqs. (2)-(3) with the above metric are

$$\frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_2\dot{a}_3}{a_2a_3}+\frac{\dot{a}_3\dot{a}_1}{a_3a_1}-\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2+\left(\frac{\dot{a}_1}{a_1}+\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3}\right)\frac{\dot{\phi}}{\phi}=\frac{8\pi\rho}{\phi} \quad \text{... (5)}$$

$$\frac{\dot{a}_2\dot{a}_3}{a_2a_3}+\frac{\ddot{a}_2}{a_2}+\frac{\ddot{a}_3}{a_3}+\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2+\frac{\ddot{\phi}}{\phi}+\frac{\dot{\phi}}{\phi}\left(\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3}\right)=-\frac{8\pi}{\phi}(p+p_c) \quad \text{... (6)}$$

$$\frac{\dot{a}_3\dot{a}_1}{a_3a_1}+\frac{\ddot{a}_3}{a_3}+\frac{\ddot{a}_1}{a_1}+\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2+\frac{\ddot{\phi}}{\phi}+\frac{\dot{\phi}}{\phi}\left(\frac{\dot{a}_3}{a_3}+\frac{\dot{a}_1}{a_1}\right)=-\frac{8\pi}{\phi}(p+p_c) \quad \text{... (7)}$$

$$\frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2}+\frac{\ddot{a}_1}{a_1}+\frac{\ddot{a}_2}{a_2}+\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2+\frac{\ddot{\phi}}{\phi}+\frac{\dot{\phi}}{\phi}\left(\frac{\dot{a}_1}{a_1}+\frac{\dot{a}_2}{a_2}\right)=-\frac{8\pi}{\phi}(p+p_c) \quad \text{... (8)}$$

$$\frac{\ddot{\phi}}{\phi}+\frac{\dot{\phi}}{\phi}\left(\frac{\dot{a}_1}{a_1}+\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3}\right)=\frac{1}{(3+2\omega)}\frac{8\pi}{\phi}[\rho-3(p+p_c)] \quad \text{... (9)}$$

Let us take the anisotropy σ^2 as

$$6\sigma^2=\left(\frac{\dot{a}_1}{a_1}-\frac{\dot{a}_2}{a_2}\right)^2+\left(\frac{\dot{a}_2}{a_2}-\frac{\dot{a}_3}{a_3}\right)^2+\left(\frac{\dot{a}_3}{a_3}-\frac{\dot{a}_1}{a_1}\right)^2 \quad \text{... (10)}$$

From (10), we have

$$\sigma^2=3H^2-\left(\frac{\dot{a}_1\dot{a}_2}{a_1a_2}+\frac{\dot{a}_2\dot{a}_3}{a_2a_3}+\frac{\dot{a}_3\dot{a}_1}{a_3a_1}\right) \quad \text{... (11)}$$

Here we have taken $R^3=a_1a_2a_3$ i.e., $3\frac{\dot{R}}{R}=\left(\frac{\dot{a}_1}{a_1}+\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3}\right)$, $H\,(=\dot{R}/R)$ being the Hubble parameter.

Eqs. (5) and (11) give

$$3H^2-\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2+\frac{3\dot{R}}{R}\frac{\dot{\phi}}{\phi}=\frac{8\pi\rho}{\phi}+\sigma^2 \quad \text{... (12)}$$

Using eq. (11), equations (6)-(8) give

$$2\dot{H}+3H^2+\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2+\frac{\ddot{\phi}}{\phi}+\frac{2\dot{R}}{R}\frac{\dot{\phi}}{\phi}=-\frac{8\pi}{\phi}(p+p_c)-\sigma^2 \quad \text{... (13)}$$

Eq. (9) becomes

$$\frac{\ddot{\phi}}{\phi}+\frac{3\dot{R}}{R}\frac{\dot{\phi}}{\phi}=\frac{1}{(3+2\omega)}\frac{8\pi}{\phi}[\rho-3(p+p_c)] \quad \text{... (14)}$$

Eqs. (12)-(14) lead to the continuity equation

$$\frac{8\pi\dot{\rho}}{\phi}+\frac{8\pi}{\phi}(\rho+p+p_c)\left[3-\frac{(\sigma^2)}{(H^2)}\right]H$$

$$+\frac{(\sigma^2)}{(H^2)}\left[-\omega\left(\frac{\dot{\phi}}{\phi}\right)^2-\frac{\ddot{\phi}}{\phi}+H\frac{\dot{\phi}}{\phi}-2\sigma^2\right]H+2\sigma^2\left(3H+\frac{\dot{\phi}}{\phi}\right)=0, \qquad \text{... (15)}$$

where

$$p_c=-(1+\gamma)\frac{\rho}{N}\frac{dN}{dt}\frac{1}{3H}\ \text{[vide [5]].} \qquad \text{... (16)}$$

3. Models with Constant Deceleration Parameter

In the above, eqs. (12)-(14) are 3 independent equations having 6 (e.g., R, ϕ, ρ, p, p_c and σ^2) unknowns. To solve them we take the ansatz :

$$\phi=\phi_0 R^{\alpha},\ \phi_0 = \text{constant},\ \alpha = \text{power index} \qquad \text{... (17)}$$

$$p=\gamma\rho,\ 0\le\gamma\le 1 \qquad \text{... (18)}$$

and

$$\sigma^2=\sigma_0^2 H^2,\ \sigma_0^2 = \text{constant.} \qquad \text{... (19)}$$

Eqs. (12)-(14) and (17)-(19) give

$$-\left(\frac{\dot{H}}{H^2}+1\right)=\beta=\frac{\omega\alpha^2+4\omega\alpha-6-2\sigma_0^2}{2(\omega\alpha-3)} = \text{deceleration parameter } (q)$$

$$= \text{constant.}$$

Eqs. (12), (17) and (19) lead to

$$\rho=\frac{\phi_0}{8\pi}H^2R^{\alpha}\left[3-\frac{\omega\alpha^2}{2}+3\alpha-\sigma_0^2\right] \qquad \text{... (21)}$$

Eq. (15) with (17)-(19) and (21) leads to

$$p_c=-K\rho, \qquad \text{... (22)}$$

where

$$K=\left[\frac{1}{(3-\sigma_0^2)}\left\{(-2-2\beta+\alpha)+(1+\gamma)(3-\sigma_0^2)\right\}\right.$$

$$+\frac{\sigma_0^2}{(3-\sigma_0^2)}\frac{(-\omega\alpha^2+\alpha(1+\beta)-\alpha^2+3\alpha+6-2\sigma_0^2)}{\left(3-\frac{\omega\alpha^2}{2}+3\alpha-\sigma_0^2\right)}\Bigg] \qquad \dots (23)$$

Eqs. (16) and (22) give

$$\frac{1}{N}\frac{dN}{dt}=aH, \qquad \dots (24)$$

where

$$a=\frac{3K}{1+\gamma} = \text{constant}.$$

Eq. (24) gives

$$N=N_\circ R^a, N(0)\equiv N_\circ. \qquad \dots (25)$$

Hence, the rate of creation as given by (24) is the only one permitted by models with constant deceleration parameter.

Conversely, consider a rate of creation of particles of the form

$$\frac{1}{N}\frac{dN}{dt}=aH, \text{ a } = \text{constant} > 0 \qquad \dots (26)$$

From (12) and (13) using (18), we get

$$2\frac{\ddot{R}}{R}+(1+3\gamma)\left(\frac{\dot{R}}{R}\right)^2+(2+3\gamma)\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi}+\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2(1-\gamma)$$

$$+\frac{\ddot{\phi}}{\phi} = -\frac{8\pi p_c}{\phi}-\sigma^2(1-\gamma) \qquad \dots (27)$$

Eliminating ρ, p, p_c from (12)-(14), we get

$$\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi}=\frac{1}{\omega}\frac{\ddot{R}}{R}+\frac{1}{\omega}\left(\frac{\dot{R}}{R}\right)^2+\frac{1}{6}\left(\frac{\dot{\phi}}{\phi}\right)^2-\frac{1}{3}\frac{\ddot{\phi}}{\phi}+\frac{2\sigma^2}{6\omega} \qquad \dots (28)$$

From eqs. (27) and (28), we get

$$\frac{[2\omega+(2+3\gamma)]}{\omega}\frac{\ddot{R}}{R}+\frac{[\omega(1+3\gamma)+(2+3\gamma)]}{\omega}\left(\frac{\dot{R}}{R}\right)^2$$

$$+\frac{(1-3\gamma)}{3}\frac{\ddot{\phi}}{\phi}+\frac{[3\omega(1-\gamma)+(2+3\gamma)]}{6}\left(\frac{\dot{\phi}}{\phi}\right)^2+\sigma^2(1-\gamma)=-\frac{8\pi p_c}{\phi}. \qquad \dots (29)$$

Now using (24) in (16) leads to

$$p_c = -(1+\gamma)\,\rho\cdot\frac{a}{3}. \qquad \dots (30)$$

Eq. (29) in conjunction with (12), (28), (19) and (30) gives

$$\frac{[2\omega+(2+3\gamma)-(1+\gamma)a]}{\omega}\frac{\ddot{R}}{R}+\left[\frac{(1+3\gamma)\omega+(2+3\gamma)-(1+\gamma)(\omega+1)}{\omega}\right.$$

$$\left.+\ \sigma_0^2\cdot\frac{27\omega(1-\gamma)+a(1+\gamma)(9\omega-1)}{27\,\omega}\right]\left(\frac{\dot{R}}{R}\right)^2$$

$$+\ \frac{[(1-3\gamma)+(1+\gamma)a]}{3}\frac{\ddot{\phi}}{\phi}+\frac{[3\omega(1-\gamma)+(2+3\gamma)+(1+\gamma)a(\omega-1)]}{6}\left(\frac{\dot{\phi}}{\phi}\right)^2=0 \qquad \dots (31)$$

Obviously, (31) admits of a particular solution of the type

$$\phi=\phi_0\,R^{\alpha}, \qquad \dots (32)$$

whence

$$\frac{\ddot{R}}{R}+f(\alpha)\left(\frac{\dot{R}}{R}\right)^2=0.$$

Hence the deceleration parameter, $q=-\dfrac{\ddot{R}}{RH^2}=f(\alpha)$ = constant. Thus the mode of particle creation given by (26) using (19) leads to cosmological models with constant q.

Creation of matter from gravitational energy takes place in the expanding phase ($H>0$) of the universe provided $a>0$.

Now equation (20) gives for $\beta\neq-1$, the exact solution

$$R(t)=(D+Ct)^{1/(1+\beta)}, \qquad \dots (33)$$

where C and D are constants of integration.

For physically viable models, we must have $1+\beta\geq 0, a\geq 0$.

(i) Non-singular Models

From eq. (33),

$$R(t)=(1+t)^{1/(1+\beta)} \text{ [Vide [5]]} \qquad \dots (34)$$

Using eq. (34), eqs. (17), (19), (21), (18), (22) and (25) give

$$\phi=\phi_0\,(1+t)^{\alpha/(1+\beta)} \qquad \dots (35)$$

$$\sigma^2=\frac{\sigma_0^2}{(1+\beta)^2}(1+t)^{-2}, \qquad \dots (36)$$

$$\rho = \frac{\phi_0}{8\pi}\left[3 - \frac{\omega\alpha^2}{2} + 3\alpha - \sigma_0^2\right]\frac{1}{(1+\beta)^2}(1+t)^{\frac{\alpha}{(1+\beta)} - 2}, \quad \text{... (37)}$$

$$p = \gamma p, 0 \leq \gamma \leq 1, \quad \text{... (38)}$$

$$p_c = -K\frac{\phi_0}{8\pi}\left[3 - \frac{\omega\alpha^2}{2} + 3\alpha - \sigma_0^2\right]\frac{1}{(1+\beta)^2}(1+t)^{\frac{\alpha}{(1+\beta)} - 2} \quad \text{... (39)}$$

and

$$N(t) = N_0 (1+t)^{a/(1+\beta)}. \quad \text{... (40)}$$

In the above,

$$\beta = \frac{\omega\alpha^2 + 4\omega\alpha - 6 - 2\sigma_0^2}{2(\omega\alpha - 3)}, \quad \text{... (41)}$$

$$K = \left[\frac{1}{(3-\sigma_0^2)}[(-2-2\beta+\alpha) + (1+\gamma)(3-\sigma_0^2)] + \frac{\sigma_0^2}{(3-\sigma_0^2)}\frac{\left\{-\omega\alpha^2 + \alpha(1+\beta) - \alpha^2 + 3\alpha + 6 - 2\sigma_0^2\right\}}{\left(3 - \frac{\omega\alpha^2}{2} + 3\alpha - \sigma_0^2\right)}\right] \quad \text{... (42)}$$

and

$$a = \frac{3}{(1+\gamma)\left(3 - \frac{\omega\alpha^2}{2} + 3\alpha - \sigma_0^2\right)(2\omega\alpha - 6)}[2(3-\sigma_0^2)\ \{\omega\alpha(\gamma - 1) + (1 - 3\gamma)\}$$

$$+ \omega\alpha^3 + (1-\gamma)\,\omega^2\gamma^3 + (9\gamma - 7)\,\omega\alpha^2 - 6\alpha^2 + 2\alpha(\sigma_0^2 - 9\gamma)],$$

$$\left[2\omega\alpha - 6 \neq 0, 3 - \frac{\omega\alpha^3}{2} + 3\alpha - \sigma_0^2 > 0, 3 - \sigma_0^2 \neq 0\right] \quad \text{... (43)}$$

Some models with some numerical values of the parameters are presented here.

MODEL 1

$\gamma = 1$ (stiff matter), $\beta = 1/2$ (decelerating expansion), $\sigma_0^2, = 2\alpha = 3/2$, $\omega = 28/27$

$$R = (1+t)^{2/3}$$

$$\phi = \phi_0 (1+t)$$

$$\sigma^2 = \frac{8}{9}(1+t)^{-2}$$

$$\rho = \frac{\phi_0}{8\pi} \times \left(\frac{52}{27}\right)(1+t)^{-1}$$

$$p = \rho$$

$$p_c = -2.42\,\rho$$

$$N = N_0\,(1+t)^{2.42}$$

MODEL 2

$\gamma = 1,\ \beta = -\frac{1}{2}$ (accelerating expansion), $\sigma_0^2 = 2,\ \alpha = \frac{1}{2},\ \omega = \frac{52}{11}$.

$$R = (1+t)^2$$

$$\phi = \phi_0\,(1+t)$$

$$\sigma^2 = 8(1+t)^{-2}$$

$$\rho = \frac{\phi_0}{8\pi} \times \left(\frac{84}{11}\right)(1+t)^{-1}$$

$$p = \rho$$

$$p_c = -(55/14)\,\rho$$

$$N = N_0\,(1+t)^{11.79}$$

MODEL 3

$\gamma = \frac{1}{2}$ (hard universe), $\beta = \frac{1}{2}$ (deceleration), $\sigma_0^2 = 2,\ \alpha = 2,\ \omega = 7/10$.

$$R = (1+t)^{2/3}$$

$$\phi = \phi_0\,(1+t)^{4/3}$$

$$\sigma^2 = \frac{8}{9}(1+t)^{-2}$$

$$\rho = \frac{\phi_0}{8\pi} \times (112/45)\,(1+t)^{-(2/3)}$$

$$p = \frac{1}{2}\rho$$

$$p_c = -2\rho$$

$$N = N_0 (1+t)^{8/3}.$$

MODEL 4

$\gamma = \frac{1}{2}$, $\beta = -\frac{1}{2}$ (acceleration), $\sigma_0^2 = 2$, $\alpha = 1/2$, $\omega = 52/11$.

$$R = (1+t)^2$$

$$\phi = \phi_0 (1+t)$$

$$\sigma^2 = 8(1+t)^{-2}$$

$$\rho = \frac{\phi_0}{8\pi} \times \left(\frac{84}{11}\right)(1+t)^{-1}.$$

$$p = \frac{1}{2}\rho$$

$$p_c = -(24/7)\,\rho$$

$$N = N_0 (1+t)^{96/7}.$$

MODEL 5

$\gamma = 1/3$ (radiation universe), $\beta = 1/3$ (deceleration), $\sigma_0^2 = 2$, $\alpha = 2$, $\omega = 3/4$.

$$R = (1+t)^{3/4}$$

$$\phi = \phi_0 (1+t)^{3/2}$$

$$\sigma^2 = (9/8) \times (1+t)^{-2}$$

$$\rho = \frac{\phi_0}{8\pi} \times \left(\frac{99}{32}\right)(1+t)^{-1/2}$$

$$p = \frac{1}{3}\rho$$

$$p_c = -2\rho$$

$$N = N_0 (1 + t)^{(27/8)}$$

MODEL 6

$\gamma = 1/3$, $\beta = -1/3$ (acceleration), $\sigma_0^2 = 2$, $\alpha = 1/2$, $\omega = 144/31$

$$R = (1 + t)^{3/2}$$

$$\phi = \phi_0 (1 + t)^{3/4}$$

$$\sigma^2 = (9/2) \times (1 + t)^{-2}$$

$$\rho = \frac{\phi_0}{8\pi} \times \left(\frac{1071}{248} \right) (1 + t)^{-(5/4)}$$

$$p = \frac{1}{3} \rho$$

$$p_c = -3.38 \rho$$

$$N = N_0 (1 + t)^{10.21}.$$

For $\alpha = -2 + \left(\frac{[2(3 + \sigma_0^2) + 4\omega]}{\omega} \right)^{1/2}$, $\beta = 0$. Then, $R = (1 + t)$ and we see that R grows linearly with time without depending upon ω.

For $\beta = -1$, $\dot{H} = 0$ (from eq. (20)), i.e, $H = H_0$ = constant. For this value of H, σ^2 becomes constant. So, no Bianchi I model can be physical with $\beta = -1$.

(ii) Singular Models with Expansion driven by Big Bang.

In the absence of creation of matter (i.e., $p_c = 0$) the cosmic expansion may be due to big-bang impulse and the eqs. (12)-(14) with $p = \gamma\rho$ will reduce to

$$3H^2 - \frac{\omega}{2} \left(\frac{\dot{\phi}}{\phi} \right)^2 + 3\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} = \frac{8\pi\rho}{\phi} + \sigma^2, \quad \text{... (44)}$$

$$2\dot{H} + 3H^2 + \frac{\omega}{2} \left(\frac{\dot{\phi}}{\phi} \right)^2 + \frac{\ddot{\phi}}{\phi} + \frac{2\dot{R}}{R}\frac{\dot{\phi}}{\phi} = -\frac{8\pi\gamma\rho}{\phi} - \sigma^2 \quad \text{... (45)}$$

and

$$\frac{\ddot{\phi}}{\phi} + 3\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} = \frac{1}{(3 + 2\omega)} \frac{8\pi}{\phi} [1 - 3\gamma] \rho. \quad \text{... (46)}$$

Eqs. (44)-(46) will lead to the continuity equation

$$\frac{8\pi\dot{\rho}}{\phi} + \frac{8\pi}{\phi}(1+\gamma)\rho\left[3 - \frac{(\dot{\sigma^2})}{(\dot{H^2})}\right]H$$

$$+ \frac{(\dot{\sigma^2})}{(\dot{H^2})}\left[-\omega\left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{\ddot{\phi}}{\phi} + H\frac{\dot{\phi}}{\phi} - 2\sigma^2\right]H + 2\sigma^2\left(3H + \frac{\dot{\phi}}{\phi}\right) = 0. \qquad \text{... (47)}$$

Values of ϕ and σ^2 assumed in (17) and (19) respectively are again considered. Then

$$\phi = \phi_0 R^{\alpha} \qquad \text{... (48)}$$

and

$$\sigma^2 = \sigma_0^2 H^2. \qquad \text{... (49)}$$

Now, eqs. (44)-(46), (48), (49) lead once again to

$$-\left(\frac{\dot{H}}{H^2} + 1\right) = \frac{\omega\alpha^2 + 4\omega\alpha - 6 - 2\sigma_0^2}{2(\omega\alpha - 3)} = q \text{ (deceleration parameter)}$$

$$= \text{constant}. \qquad \text{... (50)}$$

Eq. (50) gives

$$R(t) = t^{1/(1+\beta)} \text{ [vide [5]]} \qquad \text{... (51)}$$

Now eqs. (48), (49), (44) give

$$\phi = \phi_0 t^{\alpha/(1+\beta)}, \qquad \text{... (52)}$$

$$\sigma^2 = \frac{\sigma_0^2}{(1+\beta)^2} t^{-2} \qquad \text{... (53)}$$

and

$$\rho = \rho_0 t^{[\alpha/(1+\beta) - 2]}, \left[\begin{array}{l} \rho_0 = \dfrac{\phi_0}{8\pi(1+\beta)^2}\left[3 - \dfrac{\omega\alpha^2}{2} + 3\alpha - \sigma_0^2\right] \\ \rho_0 \equiv \rho(0) \end{array}\right] \qquad \text{... (54)}$$

Again

$$p = \gamma\rho, 0 \le \gamma \le 1 \qquad \text{... (55)}$$

Here α, β are constrained by the no-creation condition $a = 0$, i.e.,

$$[\{\alpha - 2(1+\beta)\} + (1+\gamma)(3 - \sigma_0^2)$$

$$+ \sigma_0^2 \frac{\left\{-\omega\alpha^2 + \alpha(1+\beta) - \alpha^2 + 3\alpha + 6 - 2\sigma_0^2\right\}}{\left(3 - \frac{\omega\alpha^2}{2} + 3\alpha - \sigma_0^2\right)}\Bigg] = 0 \quad \text{... (56)}$$

or,

$$[2(3-\sigma_0^2)\ \{\omega\alpha(\gamma-1) + (1-3\gamma)\} + \omega\alpha^3 + (1-\gamma)\,\omega^2\alpha^3$$

$$+ (9\gamma-7)\,\omega\alpha^2 - 6\alpha^2 + 2\alpha(\sigma_0^2 - 9\gamma)] = 0, \text{ [using (50) in (56)]} \quad \text{... (57)}$$

From eq. (57), for $\gamma = 1$, we get a value of ω which, when put in the expression of ρ (eq. (54)), gives $\rho = 0$.

So, we cannot obtain any singular Bianchi I stiff matter model with non-zero energy-density in BD cosmology with constant q.

From (57), for $\gamma \neq 1$, we have

$$\omega = -\frac{1}{\alpha(1-\gamma)}[\alpha + (3\gamma - 1)] \quad \text{... (58)}$$

Here, non-zero energy-density can be obtained.

Some models with some numerical values of the different parameters are presented here.

Model 1

$\gamma = \frac{1}{2}$ (hard universe), $\beta = 19/10$ (decelerating expansion), $\sigma_0^2 = 2$, $\alpha = 1/2$, $\omega = -4$.

$$R = t^{10/29}$$

$$\phi = \phi_0\, t^{5/29}$$

$$\rho = \frac{\phi_0}{8\pi} \times (0.36)\, t^{-1.83}$$

$$\sigma^2 = (0.24)\, t^{-2}$$

$$p = \frac{1}{2}\rho.$$

Model 2

$\gamma = \frac{1}{3}$ (radiation universe), $\beta = 143/60$ (deceleration), $\sigma_0^2 = 2$, $\alpha = 1/2$, $\omega = -3/2$

$$R = t^{(60/203)}$$

$$\phi = \phi_0\, t^{(30/203)}$$

$$\rho = \frac{\phi_0}{8\pi} \times (0.24) \times t^{-0.85}$$

$$\sigma^2 = 0.17\, t^{-2}$$

$$p = (1/3)\ \rho$$

MODEL 3

$\gamma = 1/4$, β (729/440) = (deceleration), $\sigma_0^2 = 2$, $\alpha = 1/5$, $\omega = 1/3$.

$$R = t^{0.38}$$

$$\phi = \phi_0\, t^{0.08}$$

$$\rho = \frac{\phi_0}{8\pi} \times (0.005)\, t^{-1.92}$$

$$\sigma^2 = 0.28\, t^{-2}$$

$$p = \frac{1}{4}\rho.$$

4. VARIABLE DECELERATION PARAMETER

Rewriting the eqs. (11)-(14), we get

$$3H^2 - \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + 3\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} = \frac{8\pi\rho}{\phi} + \sigma^2, \qquad \ldots (59)$$

$$2\dot{H} + 3H^2 + \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\ddot{\phi}}{\phi} + \frac{2\dot{R}}{R}\frac{\dot{\phi}}{\phi} = -\frac{8\pi}{\phi}(p + p_c) - \sigma^2 \qquad \ldots (60)$$

$$\frac{\ddot{\phi}}{\phi} + 3\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} = \frac{1}{(3+2\omega)}\frac{8\pi}{\phi}[\rho - 3(p + p_c)] \qquad \ldots (61)$$

and

$$\frac{8\pi\dot{\rho}}{\phi} + \frac{8\pi}{\phi}[\rho + p + p_c]\left[3 - \frac{\dot{(\sigma^2)}}{\dot{(H^2)}}\right]H + \frac{\dot{(\sigma^2)}}{\dot{(H^2)}}\left[-\omega\left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{\ddot{\phi}}{\phi} + H\frac{\dot{\phi}}{\phi} - 2\sigma^2\right]$$

$$+ \; 2\sigma^2 \left(3H + \frac{\dot{\phi}}{\phi} \right) = 0. \qquad \text{... (62)}$$

Here, again we see that (59)-(61) are 3 independent equations having 6 (e.g., $R, \phi, \rho, p, p_c, \sigma^2$) unknowns. We take the following ansatz :

$$p = \gamma\rho, \; 0 \leq \gamma \leq 1, \qquad \text{... (63)}$$

$$\phi = At^{-m} (t + t_*)^{-m} \qquad \text{... (64)}$$

and

$$\rho = Bt^{-(m+1)} (t + t_*)^{-m+1)}, \qquad \text{... (65)}$$

where A, B, m, t_* are constants.

From (59)-(61), we get

$$3\dot{H} + 9H^2 - \omega \frac{\ddot{\phi}}{\phi} + 3\,(1 - \omega) \frac{\dot{R}}{R} \frac{\dot{\phi}}{\phi} = \frac{8\pi\rho}{\phi} \qquad \text{... (66)}$$

We take a trial solution for (66) as

$$R = R_0\, t^k\, (t + t_*)^k, \; R_0, \; k(>0) \text{ are constants} \qquad \text{... (67)}$$

From (67), we get

Deceleration parameter,

$$q = -\frac{\ddot{R}/R}{(\dot{R}/R)^2} = -\frac{1}{k} \left[2 \frac{t(t + t_*)}{(2t + t_*)^2} + (k - 1) \right] \equiv f(t, t_*).$$

Eq. (66) with eqs. (64), (65), (67) leads

$$k = \frac{m + 1}{3}$$

and

$$B = \frac{2A}{8\pi} (1 + m + \omega m). \qquad \text{... (68)}$$

Using (64), (65), (67) and (68) in (59), we get

$$m = -\frac{1}{4 + 3\omega} \qquad \text{... (69)}$$

and

$$\sigma^2 = \frac{2\omega + 3}{2(3\omega + 4)} \frac{t_*^2}{(t(t + t_*))^2} \qquad \text{... (70)}$$

Now, eq. (61) with eqs. (63)-(65), (67), (68) and (69) gives

$$p_c = -\gamma p = -p \qquad \text{... (71)}$$

Eqs. (16) and (71) give

$$N = CR^a,\ C = \text{Constant},\ a = \frac{3\gamma}{1+\gamma} = \text{constant} \qquad \text{... (72)}$$

Finally, (63)-(65), (67), (68), (69), (70), (71) and (72) become

$$\phi = At^{1/(4+3\omega)}\,(t+t_*)^{1/(4+3\omega)} \qquad \text{... (73)}$$

$$\rho = \frac{2A}{8\pi}\frac{(3+2\omega)}{(4+3\omega)}\,t^{-3\frac{(1+\omega)}{(4+3\omega)}}\,(t+t_*)^{-\frac{(1+\omega)}{(4+3\omega)}}, \qquad \text{... (74)}$$

$$p = \gamma p,\ 0 \le \gamma \le 1, \qquad \text{... (75)}$$

$$p_c = -p, \qquad \text{... (76)}$$

$$R = R_0\, t^{\frac{1+\omega}{4+3\omega}}\,(t+t_*)^{\frac{1+\omega}{4+3\omega}}, \qquad \text{... (77)}$$

$$\sigma^2 = \frac{(3+2\omega)}{2(4+3\omega)}\,\frac{t_*^2}{(t(t+t_*))^2} \qquad \text{... (78)}$$

and

$$N = CR_0^a\, t^{\left(\frac{1+\omega}{4+3\omega}\right)^a}\,(t+t_*)^{\left(\frac{1+\omega}{4+3\omega}\right)^a}, \qquad \text{... (79)}$$

where

$$a = \frac{3\gamma}{1+\gamma} = \text{constant } (>0) \qquad \text{... (80)}$$

Values of coupling parameter (ω) should be as follows : $\omega > -1,\ \omega < -\frac{5}{3}$.

From (78), we can call t_* as the 'anisotropy time' as σ^2 exists if $t_* \neq 0$. For $t >> t_*$, the results turn into those of a flat FRW model in BD cosmology with constant deceleration parameter.

MODEL

One model is shown below with $\omega = 1$.

$$\phi = At^{1/7}\,(t+t_*)^{1/7}$$

$$\rho = \frac{2A}{8\pi} \times \left(\frac{5}{7}\right) t^{-6/7}\,(t+t_*)^{-6/7}$$

$$R = R_0 t^{2/7} (t + t_*)^{2/7}$$

$$\sigma^2 = \frac{5}{14}\left[\frac{t_*^2}{(t(t+t_*))^2}\right]$$

$$p = \gamma \rho$$

$$p_c = -p$$

$$N = CR_0^a t^{2a/7} (t + t_*)^{2a/7}, \; a = \frac{3\gamma}{1+\gamma}.$$

5. Conclusion

In the paper, we have discussed Bianchi I models in BD cosmology with both constant and variable deceleration parameters.

In the first case, by making use of Prigogine's hypothesis of creation of matter out of gravitational energy, we have obtained non-singular models with both accelerating and decelerating expansions driven by creation of matter particles in stiff matter ($\gamma = 1$) universe, hard universe ($\gamma = 1/2$) and in radiation universe ($\gamma = 1/3$). We used only positive values of the coupling parameter (ω).

In this same case, we have obtained singular models also. Here, in the case of $\gamma = 1$, no model is obtained with non-zero energy density.

For $\frac{1}{3} \leq \gamma < 1$, we have not found models with $\omega > 0$. We again find that no accelerating model with singularity and with constant q can be obtained for $0 \leq \gamma < 1$.

In the second case, we have found singular models only. Models can be obtained for $\gamma = 1$, $\gamma = 1/2$, $\gamma = 1/3$, 'ω' may have both positive values ($\omega > -1$, $\omega < -5/3$).

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